AGH University of Krakow
Faculty of Physics and Applied Computer Science

Chapter 1

Electric Charge and Coulomb's Law

Chapter 1 Lecture Notes

Hamzeh Khanpour
hamzeh.khanpour@agh.edu.pl
Faculty of Physics and Applied Computer Science
AGH University of Krakow

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Contents

Preface

This chapter opens the second volume of *Physics for Technical and Engineering Students* by introducing electric charge as an intrinsic property of matter and Coulomb's law as the first quantitative law of electrostatics. It develops the conceptual, mathematical and experimental foundations needed for later chapters on electric fields, electric potential, capacitance, current, circuits, magnetism, induction, electromagnetic waves, optics, relativity and quantum physics.

The transition from Physics 1 to Physics 2 is a conceptual transition. In mechanics the central objects were usually masses, positions, velocities, accelerations, forces, energies and momenta. In electromagnetism we add a new intrinsic property, electric charge. Charge is not a label attached only to unusual laboratory objects. It is a property of electrons, protons and many elementary particles, and it is responsible for the structure of atoms, the bonding of molecules, the behaviour of solids, the operation of electronic devices, the release of sparks and the damage caused by electrostatic discharge in semiconductor laboratories.

The purpose of this chapter is not merely to learn the formula for a force between two charged particles. The purpose is to build a disciplined way of thinking about charge. We shall distinguish a neutral object from an object with no charge carriers; distinguish charge transfer from charge creation; distinguish the magnitude of a force from its vector direction; distinguish a chosen object from the objects exerting forces on it; and distinguish the simple point-charge law from the richer field language that begins in the next chapter.

The chapter is written for students in Technical Physics, engineering, computer science, applied science, space science and related programmes. It therefore combines conceptual foundations with technical interpretation. Static electricity, conductors, insulators, grounding, charge sharing, electrostatic discharge, charge conservation and numerical force summation are not separate stories. They are all manifestations of one central idea: charged particles interact, and the net interaction on a chosen particle must be modelled as a vector sum.

All explanations, examples, exercises and figures in this chapter are original teaching material prepared for this lecture-note series. Standard references are listed at the end for comparison of conventions and for further study.

The Guideline

Treat an electrostatic problem first as a modelling problem and only then as an algebra problem. Identify the charged objects, decide which object is the object of interest, choose a coordinate system, draw the force vectors on the correct object, state whether the point-charge approximation is valid, and then apply Coulomb's law. A correct force diagram is part of the solution, not decoration.

Pedagogical boundary

This chapter works directly with electric force. The electric field, electric flux, Gauss's law, electric potential and capacitance are deliberately postponed. They are not optional refinements: they are the natural next language of electrostatics. However, students understand that language better if they first master charge, Coulomb force, sign conventions and vector superposition.

Conventions and notation

Vectors are written in bold notation using the macro `\vect{}`. Cartesian unit vectors are written as $\hat{\boldsymbol{i}}$, $\hat{\boldsymbol{j}}$ and $\hat{\boldsymbol{k}}$. The symbol q denotes electric charge, and Q is used when a total charge or a charge on a larger object is meant. Charge is measured in coulombs, abbreviated C. The elementary charge is denoted by e.

The force notation used throughout the chapter is important. The symbol

$$\boldsymbol{F}_{12} \tag{0.1}$$

means *the force on charge 1 due to charge 2*. The first subscript tells which object feels the force. The second subscript tells which object produces that force. With this convention $\boldsymbol{F}_{21}$ is a different vector: it is the force on charge 2 due to charge 1. Newton's third law gives

$$\boldsymbol{F}_{21} = -\boldsymbol{F}_{12} \tag{0.2}$$

for the interaction between the same two point charges.

For the vector form of Coulomb's law, if charge 1 is at position $\boldsymbol{r}_1$ and charge 2 is at position $\boldsymbol{r}_2$, define

$$\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2, \qquad r_{12} = |\boldsymbol{r}_{12}|, \qquad \hat{\boldsymbol{r}}_{12} = \frac{\boldsymbol{r}_{12}}{r_{12}}. \tag{0.3}$$

Thus $\hat{\boldsymbol{r}}_{12}$ points from charge 2 toward charge 1. This convention is used consistently in this chapter.

Symbol	Meaning	SI unit or dimension
Q, q	electric charge	C
e	elementary charge, $1.602\,176\,634 \times 10^{-19}$ C	C
n	integer in $q = ne$	dimensionless
I	electric current, introduced only as charge flow rate	$\mathrm{A} = \mathrm{C\ s^{-1}}$
ε_0	vacuum permittivity	$\mathrm{C^2\ N^{-1}\ m^{-2}}$
k_e	Coulomb constant, $1/(4\pi\varepsilon_0)$	$\mathrm{N\ m^2\ C^{-2}}$
$\boldsymbol{F}_{12}$	force on charge 1 due to charge 2	N
$\boldsymbol{r}_i$	position vector of charge i	m
$\boldsymbol{r}_{12}$	displacement from charge 2 to charge 1	m
r_{12}	distance between charges 1 and 2	m
$\hat{\boldsymbol{r}}_{12}$	unit vector from charge 2 to charge 1	dimensionless
λ	linear charge density, used only in advanced preview	$\mathrm{C\ m^{-1}}$
ρ	volume charge density, used only in advanced preview	$\mathrm{C\ m^{-3}}$
G	Newtonian gravitational constant, used for comparison	$\mathrm{N\ m^2\ kg^{-2}}$

Note

A charge q can be positive, negative or zero. A force magnitude $F = |\boldsymbol{F}|$ is non-negative. Do not use the sign of q as if it were the sign of a force magnitude. The sign of charge determines force direction only after the geometry and the object of interest are specified.

Note

The Coulomb constant is written as k_e in this chapter. This avoids confusion with the spring constant k used in mechanics. The two constants have different units and describe completely different physical situations.

Chapter 1

Electric Charge and Coulomb's Law

1.1 Motivation and roadmap

Most objects around us are almost electrically neutral, and this can make charge seem like a special laboratory curiosity. It is not. Charge is one of the basic properties of matter. The normal force between a table and a book, the tension in a fibre, the rigidity of a metal beam, the adhesion of dust to a surface, the discharge that damages a microchip, the binding of ions in a crystal and the chemical structure of matter all ultimately involve electromagnetic interactions among charged particles.

At the same time, the first observations of electrostatics are familiar and almost playful. A rubbed plastic rod attracts small pieces of paper. A balloon rubbed on hair may stick to a wall. A metal can may roll toward a charged object even though the can has zero net charge. A spark can jump from a finger to a door handle after walking on a carpet in dry air. These examples are not magic. They are entry points to the idea that charged particles interact through forces and that charges in matter can move, separate or polarise.

Figure 1.1 shows the conceptual roadmap of the chapter. We begin with the meaning of charge, then study how charge is distributed and transferred in materials, then introduce conservation and quantization, and finally build Coulomb's law as a vector force law.

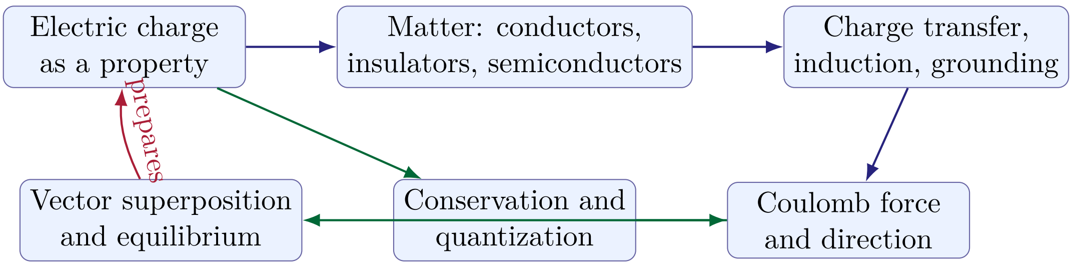

Figure 1.1: Conceptual roadmap for Chapter 1. Charge, material response, conservation, quantization and Coulomb force form the foundation for the field language developed in the following chapters.

Learning goals

After studying this chapter, the student should be able to:

- explain electric charge as an intrinsic property of matter;
- distinguish positive, negative, neutral and excess charge;
- explain the difference between conductors, insulators and semiconductors at an introductory level;
- describe charging by contact, conduction, induction and grounding;
- apply charge conservation to isolated systems and charge-sharing problems;
- use charge quantization, $q = ne$, to connect macroscopic charge to the number of elementary charges;

- state Coulomb's law in magnitude form and in vector form;
- determine the direction of electrostatic forces for like and unlike charges;
- draw correct electrostatic free-body diagrams;
- calculate net electrostatic forces by vector superposition;
- solve simple equilibrium problems involving point charges;
- state the assumptions and validity limits of the point-charge model;
- connect electrostatics to grounding, electrostatic discharge, materials and elementary computational modelling.

Key message

Charge is not introduced because sparks are interesting, although they are. Charge is introduced because electromagnetic interactions dominate the structure and behaviour of ordinary matter. Mechanics taught us how forces affect motion; electrostatics introduces a new kind of force and a new property that sources it.

1.1.1 Why the chapter starts with force rather than field

The modern formulation of electromagnetism is field-based. Nevertheless, a first chapter on electrostatics usually begins with charge and force. The reason is pedagogical. Students already know from Physics 1 how to reason with forces, free-body diagrams and vector sums. Coulomb's law therefore provides a bridge from familiar mechanics to the new electromagnetic concepts.

The bridge must be used carefully. Electrostatic force is not just another mechanical contact force. It can act between objects that do not touch. Later we shall explain this interaction locally by introducing the electric field. In this chapter we concentrate on the measurable force and the charge property that controls it.

1.2 Electric charge as an intrinsic property of matter

A physical object can have mass, energy, momentum, angular momentum and electric charge. Charge is intrinsic in the sense that it is a property carried by particles. An electron is not negatively charged because something has been painted onto it; negative charge is one of its defining properties. A proton carries positive charge. A neutron has zero net charge, although it has internal charged constituents at a more advanced level.

The two signs of charge are called positive and negative. The names are conventional, but the distinction is physical. Two particles with charges of the same sign repel each other. Two particles with charges of opposite signs attract each other. If the total positive charge in an object equals the total negative charge, the object is electrically neutral.

Definition 1.1: Electric charge, neutral object and excess charge

Electric charge is an intrinsic property of matter that determines the strength and sign of electrostatic interactions. An object is electrically neutral when its total positive charge and total negative charge sum to zero. An object has excess charge when this balance is disturbed and the net charge is nonzero.

A neutral object should not be imagined as an object without charge. A neutral copper sphere contains enormous amounts of positive and negative charge. It is neutral because these contributions cancel in the total. This distinction is essential for understanding why a neutral conductor can be attracted to a charged rod.

Key message

Neutral does not mean charge-free. It means zero net charge. A neutral atom, a neutral metal sphere and a neutral human body contain charged particles whose total charge cancels to a very high precision.

1.2.1 Microscopic charge carriers

For introductory electrostatics in ordinary matter, the most important charge carriers are electrons and protons. Electrons carry charge $-e$, protons carry charge $+e$, and neutrons carry zero net charge. Atoms are neutral when the number of electrons equals the number of protons. Ions are atoms or molecules with an electron imbalance.

In a solid metal, some outer electrons are mobile throughout the material. These mobile electrons are responsible for the conducting behaviour of the metal. The positive ion cores are not free to drift through the solid in ordinary electrostatic situations. In an electrolyte or plasma, both positive and negative charge carriers may move, but this chapter will focus mainly on solids and simple conductors.

Caution

In an ordinary metal, the mobile charge carriers are electrons. A positively charged metal object is usually not a metal object containing extra mobile protons. It is a metal object with a deficit of electrons compared with the neutral state.

1.3 Positive, negative, neutral and excess charge

If an object has net charge $Q > 0$, we say that it is positively charged. If it has net charge $Q < 0$, we say that it is negatively charged. If $Q = 0$, the object is neutral. These statements refer to total charge, not to the absence or presence of charged particles inside the object.

Charge imbalances in ordinary laboratory situations are often tiny compared with the total amount of positive or negative charge contained in matter. Removing even a small fraction of the mobile electrons from a macroscopic object can produce a measurable electrostatic effect. This is one reason electrostatics can appear surprising: the net charge is small compared with the charge content of matter, yet the electrostatic force can be large.

Figure 1.2 illustrates the bookkeeping. If electrons move from object A to object B, object A is left with a deficit of negative charge and therefore becomes positively charged. Object B gains negative charge and becomes negatively charged. The total charge of the two-object system is unchanged if the system is isolated.

Guided checks

Check 1. A metal sphere has equal total positive and negative charge. Is it charge-free?
Answer. No. It contains charged particles, but the total charge is zero.

Check 2. If a neutral object loses electrons, what is the sign of its net charge?
Answer. It becomes positively charged, because negative charge has been removed.

Check 3. Can two objects both contain many electrons and still have opposite net charges?
Answer. Yes. Net charge is the imbalance between positive and negative charge, not the mere presence of electrons.

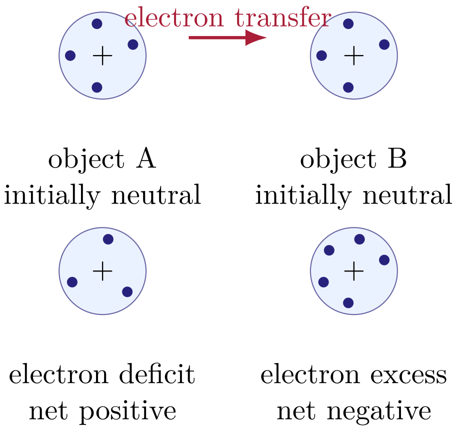

Figure 1.2: A schematic charge-transfer picture. The symbols are not literal atomic models; they show the important bookkeeping idea. Moving electrons from one object to another can leave one object with positive excess charge and the other with negative excess charge.

1.4 Conductors, insulators and semiconductors

Materials differ in how easily charge can move through them. A conductor contains mobile charge carriers that can rearrange over macroscopic distances. Metals are the standard examples. An insulator contains charges that are bound much more strongly to atoms or molecules; charge cannot move freely through the material under ordinary conditions. A semiconductor has behaviour between these limits and can be engineered by doping, temperature, illumination or electric fields. Semiconductors will be treated more fully in the later chapter on electrical conduction in solids.

Definition 1.2: Conductors, insulators and semiconductors

A conductor is a material in which some charge carriers can move through the material relatively freely. An insulator is a material in which charge carriers are not free to move over macroscopic distances under ordinary conditions. A semiconductor is a material whose conductivity is intermediate and highly controllable by impurities, temperature, illumination or device structure.

The distinction is not absolute. A material that behaves as an insulator at low voltage may break down at high voltage. A semiconductor can behave nearly insulating in one situation and conducting in another. A human body can act as a conductor for electrostatic discharge, especially through moist skin. Air is normally an insulator, but it becomes conducting during a spark when molecules are ionized.

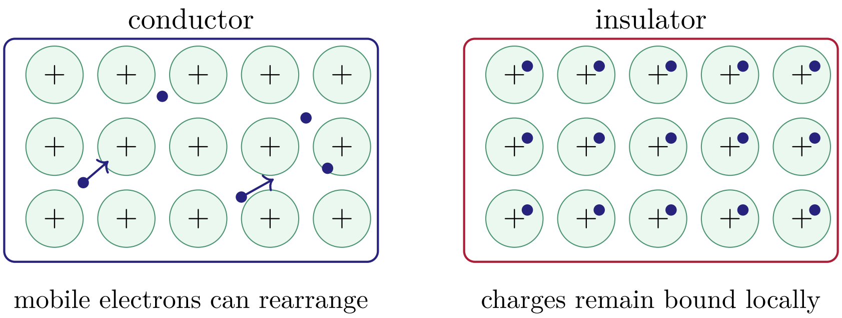

Figure 1.3: Introductory microscopic distinction between a conductor and an insulator. In a metal, some electrons are mobile across the sample. In an insulator, charges may shift slightly inside atoms or molecules, but they do not move freely through the material.

1.4.1 Material response and induced separation

When a charged object is brought near a conductor, mobile electrons in the conductor rearrange. This redistribution can make the near side of the conductor have a charge imbalance opposite in sign to the nearby external charge. The far side then has the opposite imbalance. The conductor as a whole can remain neutral while still being attracted to the charged object.

In an insulator, charges cannot move freely through the whole object, but microscopic positive and negative charges can shift slightly relative to one another. This effect is called polarization. It is weaker than free charge motion in a conductor but still explains why small neutral paper pieces can be attracted to a charged plastic rod.

Key message

A neutral object can be attracted to a charged object because charge can separate or polarise. Attraction does not require the neutral object to have a nonzero total charge.

Caution

Do not say that a neutral conductor is attracted because it somehow becomes oppositely charged as a whole. During induction the near and far regions acquire opposite imbalances, while the total charge of the isolated conductor may remain zero.

1.5 Charging by contact, conduction, induction and grounding

Charging means producing a nonzero net charge on an object. It does not mean creating charge from nothing. In ordinary electrostatic experiments, charging involves transferring charge from one object to another or allowing charge to enter or leave through a conducting path.

1.5.1 Charging by contact and conduction

If a charged conductor touches another conductor, mobile charge can flow during contact. The final distribution depends on object shape, size and environment. For two identical isolated conducting spheres, symmetry gives a simple result: after contact and separation, they share the total charge equally.

1.5.2 Charging by induction

Charging by induction uses a nearby charged object to rearrange charge in a conductor, followed by grounding or separation steps that leave a net charge. A typical induction process is:

1. bring a charged rod near a neutral conductor without touching it;
2. allow charges in the conductor to separate;
3. connect the conductor to ground while the rod remains nearby;
4. remove the ground connection;
5. finally remove the charged rod.

The conductor may then be left with a net charge opposite to that of the inducing rod.

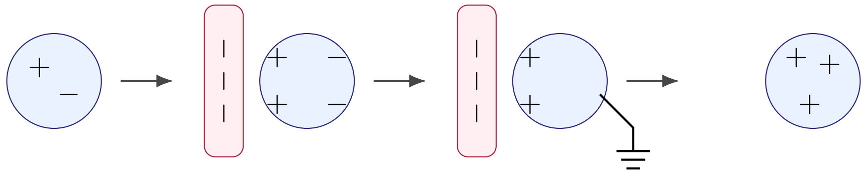

Figure 1.4: A schematic induction process using a negatively charged rod. The rod polarises the neutral conductor. Grounding allows electrons to leave. After the ground is removed and then the rod is removed, the conductor is left positively charged.

1.5.3 Grounding

Grounding means connecting an object to a very large conducting reservoir, usually Earth or a grounded conductor. Ground can supply or remove electrons with negligible change in its own potential state for introductory purposes. Grounding is not a mathematical trick; it is a physical conducting path.

Method

Reasoning through induction problems.

1. Identify the sign of the external charged object.
2. Decide which mobile charges in the conductor move and in which direction.
3. Mark the near and far regions of the conductor.
4. If grounding occurs, decide whether electrons enter or leave the conductor.
5. Remove the ground before removing the inducing object if the usual induction-charging process is intended.
6. Check total charge only for the isolated system. A grounded conductor is not isolated.

Guided checks

Check 1. A negatively charged rod is brought near a neutral metal sphere. Which side of the sphere becomes positive?
Answer. The side near the rod becomes positive because mobile electrons are repelled toward the far side.

Check 2. During grounding, is the conductor still an isolated system?
Answer. No. Charge can enter or leave through the ground connection.

Check 3. Why does the order of removing the ground and removing the rod matter?
Answer. Removing the ground first traps the net charge on the conductor. If the rod is removed first while the ground remains, charge can flow back and neutralise the conductor.

1.6 Conservation of electric charge

Electric charge is conserved. In an isolated system, the total charge does not change:

$$Q_{\text{total}} = \sum_i q_i = \text{constant}. \tag{1.1}$$

This law does not say that charge cannot move. It says that the algebraic sum of charge in an isolated system is unchanged. Charge can be transferred from one part of the system to another. Positive and negative particles can be created in pairs in high-energy processes. Particles and antiparticles can annihilate into neutral radiation. In each case the total charge is conserved.

For ordinary electrostatic charging, conservation is often simple bookkeeping. If a neutral object loses electrons to another neutral object, one becomes positive and the other becomes negative by equal amounts. The total remains zero if the pair is isolated.

Key message

Charge conservation is an algebraic statement. Positive and negative charges must be added with their signs. A system with $+3\,\mu\text{C}$ on one object and $-3\,\mu\text{C}$ on another has total charge zero, not $6\,\mu\text{C}$.

Example 1.1: Charge bookkeeping during electron transfer

A neutral plastic bead transfers 2.5×10^{10} electrons to a neutral metal bead during contact and separation. Find the final charge of each bead.
Model and assumptions. The two beads form an isolated pair during the transfer. Only electrons are transferred. The elementary charge is $e = 1.602 \times 10^{-19}\,\text{C}$.
Solution. The charge magnitude transferred is

$$|\Delta q| = Ne = (2.5 \times 10^{10})(1.602 \times 10^{-19}\,\text{C}) = 4.01 \times 10^{-9}\,\text{C}. \tag{1.2}$$

The plastic bead loses electrons, so it loses negative charge and becomes positively charged:

$$q_{\text{plastic}} = +4.01\,\text{nC}. \tag{1.3}$$

The metal bead gains electrons and becomes negatively charged:

$$q_{\text{metal}} = -4.01\,\text{nC}. \tag{1.4}$$

The total final charge is zero, as required.
Reasonableness check. A nanocoulomb is a small macroscopic charge but corresponds to billions of elementary charges, so the number of transferred electrons is plausible for a static-charging situation.

1.7 Quantization of charge

Electric charge is quantized. Observable free charges in ordinary introductory contexts occur in integer multiples of the elementary charge:

$$q = ne, \qquad n \in \mathbb{Z}, \tag{1.5}$$

where

$$e = 1.602\,176\,634 \times 10^{-19}\,\text{C} \tag{1.6}$$

is the elementary charge. In numerical examples we shall often use $e \simeq 1.602 \times 10^{-19}\,\text{C}$.

The electron has charge $-e$, and the proton has charge $+e$. The neutron has zero net charge. At a more advanced level, protons and neutrons are composed of quarks with fractional charges, but isolated quarks are not observed as free particles in ordinary conditions. For this introductory chapter, the relevant free charge unit is e.

Caution

Charge quantization is microscopic, but it is often hidden macroscopically. A charge of $1\,\text{nC}$ already corresponds to about 6.2×10^{9} elementary charges. Such a large integer can make charge appear continuous in many engineering measurements.

Example 1.2: How many electrons correspond to a microcoulomb?

A small object has excess charge $q = -0.80\,\mu\text{C}$. Estimate the number of excess electrons.
Solution. The number of elementary charges is

$$n = \frac{|q|}{e} = \frac{0.80 \times 10^{-6}\,\text{C}}{1.602 \times 10^{-19}\,\text{C}} = 4.99 \times 10^{12}. \tag{1.7}$$

Because the charge is negative, these are excess electrons. Thus the object has approximately

$$5.0 \times 10^{12} \tag{1.8}$$

extra electrons.
Interpretation. A microcoulomb is a small amount of charge in many laboratory settings, yet it corresponds to trillions of electrons.

1.7.1 Electric current as a bridge concept

Current will be developed in detail later. It is useful here only as a reminder that charge can move. Electric current is the rate at which charge passes through a chosen surface or point in a circuit:

$$I = \frac{\mathrm{d}q}{\mathrm{d}t}. \tag{1.9}$$

The SI unit is the ampere, where $1\,\text{A} = 1\,\text{C}\,\text{s}^{-1}$. In this chapter, current is not yet a circuit variable. It is simply a rate of charge transfer.

Pedagogical boundary

Current and voltage are not fully introduced in this chapter. The equation $I = \mathrm{d}q/\mathrm{d}t$ is only a bridge from charge conservation to later circuit physics. The microscopic and macroscopic theory of current, resistance and power begins in the current-and-resistance chapter.

1.8 Coulomb's law in magnitude form

The electrostatic force between two point charges at rest is described by Coulomb's law. If two point charges q_1 and q_2 are separated by distance r, the magnitude of the force on either charge due to the other is

$$F = k_e \frac{|q_1 q_2|}{r^2}, \qquad k_e = \frac{1}{4\pi\varepsilon_0}. \tag{1.10}$$

The vacuum permittivity is

$$\varepsilon_0 \simeq 8.85 \times 10^{-12}\,\text{C}^2/(\text{N}\,\text{m}^2), \tag{1.11}$$

and hence

$$k_e \simeq 8.99 \times 10^9\,\text{N}\,\text{m}^2/\text{C}^2. \tag{1.12}$$

The magnitude form tells how large the force is, but it does not by itself specify the vector direction. The direction is along the line connecting the charges. Like charges repel; unlike charges attract. For numerical problems it is often safest to compute the magnitude using absolute values, then determine the direction from a diagram.

Note

The factor $1/r^2$ means that if the separation is doubled, the force magnitude becomes four times smaller. If the separation is halved, the force magnitude becomes four times larger. This inverse-square structure also appears in Newton's law of gravitation, but electric charge can have two signs whereas ordinary mass in Newtonian gravity is positive.

Example 1.3: Force between two point charges on a line

Two small charged beads are fixed on a horizontal insulating rail. Bead 1 has charge $q_1 = +3.0\,\mu\text{C}$ and bead 2 has charge $q_2 = -2.0\,\mu\text{C}$. Their separation is $r = 0.15\,\text{m}$. Find the magnitude and direction of the force on bead 1 due to bead 2.
Assumptions. The beads are small compared with their separation, so they are treated as point charges. The charges are at rest.
Magnitude.

$$
\begin{aligned}
F_{12} &= k_e \frac{|q_1 q_2|}{r^2} && (1.13)\\
&= (8.99 \times 10^9)\frac{(3.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.15)^2}\,\text{N} && (1.14)\\
&= 2.40\,\text{N}. && (1.15)
\end{aligned}
$$

Direction. The charges have opposite signs, so they attract. Therefore the force on bead 1 points toward bead 2.
Interpretation. A few microcoulombs separated by centimetres can produce a force of order newtons, which is easily measurable in a laboratory.

1.9 Coulomb's law in vector form

For systematic work in two or three dimensions, the vector form of Coulomb's law is better than the magnitude form. Let charge q_1 be at position $\boldsymbol{r}_1$ and charge q_2 at position $\boldsymbol{r}_2$. Define

$$\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2, \qquad r_{12} = |\boldsymbol{r}_{12}|, \qquad \hat{\boldsymbol{r}}_{12} = \frac{\boldsymbol{r}_{12}}{r_{12}}. \tag{1.16}$$

Then the force on charge 1 due to charge 2 is

$$\boldsymbol{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\boldsymbol{r}}_{12}. \tag{1.17}$$

Equivalently,

$$\boldsymbol{F}_{12} = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^3}. \tag{1.18}$$

The sign of $q_1 q_2$ is now part of the vector equation. If $q_1 q_2 > 0$, the force is in the direction of $\hat{\boldsymbol{r}}_{12}$, away from charge 2. If $q_1 q_2 < 0$, the force is opposite to $\hat{\boldsymbol{r}}_{12}$, toward charge 2.

Method

Using Coulomb's law in vector form.

1. Assign coordinates to every charge.
2. Choose the charge on which the force is required.
3. For each source charge, compute $\boldsymbol{r}_{\text{target}} - \boldsymbol{r}_{\text{source}}$.

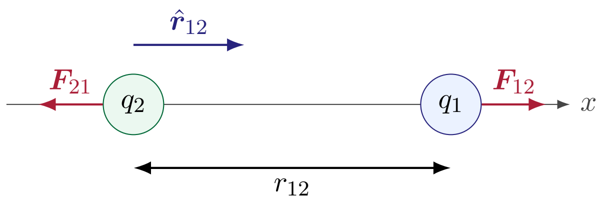

Figure 1.5: Vector convention for Coulomb's law. The vector $\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ points from charge 2 to charge 1. For like charges, $\boldsymbol{F}_{12}$ points in the same direction as $\hat{\boldsymbol{r}}_{12}$, while $\boldsymbol{F}_{21} = -\boldsymbol{F}_{12}$.

4. Insert the signed charges in the vector formula.
5. Add vector contributions component by component.
6. Check whether the final direction agrees with attraction/repulsion reasoning.

1.10 Free-body diagrams and direction conventions

A free-body diagram is a diagram of forces acting on one chosen object. This rule is simple and often violated. If the question asks for the force on charge 1, draw all force vectors with their tails on charge 1. Do not draw the force on charge 2 and call it the force on charge 1. Do not draw a force floating between the charges.

For two charges, the forces form a Newton's-third-law pair:

$$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}. \tag{1.19}$$

They have equal magnitude and opposite direction, but they act on different objects. They do not cancel when analysing one object because they are not forces acting on the same object.

Caution

Equal-and-opposite interaction forces do not cancel in a free-body diagram for one object. The force on charge 1 due to charge 2 acts on charge 1. The force on charge 2 due to charge 1 acts on charge 2. They belong to different free-body diagrams.

Guided checks

Check 1. Two positive charges repel. Where should the tail of $\boldsymbol{F}_{12}$ be drawn?
Answer. On charge 1, because $\boldsymbol{F}_{12}$ is the force on charge 1.

Check 2. If q_1 is positive and q_2 is negative, is $\boldsymbol{F}_{12}$ toward or away from q_2?
Answer. Toward q_2, because opposite charges attract.

Check 3. Can $\boldsymbol{F}_{12}$ and $\boldsymbol{F}_{21}$ cancel when considering the motion of charge 1 alone?
Answer. No. Only $\boldsymbol{F}_{12}$ acts on charge 1.

1.11 Superposition of electrostatic forces

If several charges act on a chosen charge, the net force is the vector sum of the individual forces:

$$\boldsymbol{F}_{\text{net}} = \sum_i \boldsymbol{F}_i. \tag{1.20}$$

For a charge q_0 at position $\boldsymbol{r}_0$ acted on by source charges q_i at positions $\boldsymbol{r}_i$,

$$\boldsymbol{F}_0 = \frac{1}{4\pi\varepsilon_0}\sum_i q_0 q_i \frac{\boldsymbol{r}_0 - \boldsymbol{r}_i}{|\boldsymbol{r}_0 - \boldsymbol{r}_i|^3}. \tag{1.21}$$

This equation is compact, but its practical meaning is the same as in mechanics: calculate each force vector on the chosen object and then add the vectors.

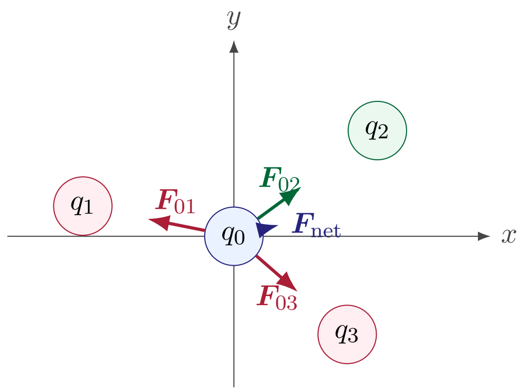

Figure 1.6: Superposition of electrostatic forces. The net force on the chosen charge q_0 is found by drawing all individual forces on q_0 and adding them as vectors.

Caution

Do not add Coulomb-force magnitudes unless all force vectors are along the same line and the directions have been accounted for. In two dimensions, forces must be resolved into components or added using vector geometry.

Example 1.4: Three charges on a line

Three point charges lie on the x axis. Charge $q_0 = +2.0\,\mu$C is at the origin. Charge $q_1 = +4.0\,\mu$C is at $x = -0.30$ m, and charge $q_2 = -3.0\,\mu$C is at $x = +0.20$ m. Find the net force on q_0.
Direction reasoning. Charge q_1 is positive and repels q_0, pushing it in the $+x$ direction. Charge q_2 is negative and attracts q_0, also pulling it in the $+x$ direction. Thus the force components add in this special one-dimensional case.
Magnitudes.

$$F_{01} = k_e\frac{|q_0q_1|}{(0.30)^2} = (8.99\times10^9)\frac{(2.0\times10^{-6})(4.0\times10^{-6})}{0.090}\,\text{N} = 0.799\,\text{N}, \tag{1.22}$$

$$F_{02} = k_e\frac{|q_0q_2|}{(0.20)^2} = (8.99\times10^9)\frac{(2.0\times10^{-6})(3.0\times10^{-6})}{0.040}\,\text{N} = 1.35\,\text{N}. \tag{1.23}$$

Net force.

$$\boldsymbol{F}_{\text{net}} = (0.799 + 1.35)\hat{\boldsymbol{\imath}}\,\text{N} = 2.15\hat{\boldsymbol{\imath}}\,\text{N}. \tag{1.24}$$

Interpretation. Both source charges push or pull the chosen charge toward $+x$ in this arrangement.

Example 1.5: Three charges in two dimensions

A target charge $q_0 = +1.0\,\mu$C is at the origin. A charge $q_1 = +2.0\,\mu$C is at $(0.30\,\text{m}, 0)$, and a charge $q_2 = -2.0\,\mu$C is at $(0, 0.40\,\text{m})$. Find the net force on q_0.

Force from q_1. Since q_0 and q_1 are both positive, q_1 repels q_0 in the $-x$ direction:

$$F_{01} = k_e \frac{(1.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.30)^2} = 0.200\,\mathrm{N}, \tag{1.25}$$

so

$$\boldsymbol{F}_{01} = -0.200\hat{\boldsymbol{\imath}}\,\mathrm{N}. \tag{1.26}$$

Force from q_2. Since q_0 is positive and q_2 is negative, q_2 attracts q_0 in the $+y$ direction:

$$F_{02} = k_e \frac{(1.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.40)^2} = 0.112\,\mathrm{N}, \tag{1.27}$$

so

$$\boldsymbol{F}_{02} = +0.112\hat{\boldsymbol{\jmath}}\,\mathrm{N}. \tag{1.28}$$

Net force.

$$\boldsymbol{F}_{\mathrm{net}} = (-0.200\hat{\boldsymbol{\imath}} + 0.112\hat{\boldsymbol{\jmath}})\,\mathrm{N}. \tag{1.29}$$

Its magnitude is

$$F_{\mathrm{net}} = \sqrt{0.200^2 + 0.112^2}\,\mathrm{N} = 0.229\,\mathrm{N}. \tag{1.30}$$

The direction is in the second quadrant, at an angle

$$\theta = \tan^{-1}\left(\frac{0.112}{0.200}\right) = 29.3^\circ \tag{1.31}$$

above the negative x direction.

Common pitfall. The answer is not $0.200 + 0.112 = 0.312\,\mathrm{N}$ because the two forces are perpendicular.

1.12 Equilibrium of charged particles

A charged particle is in translational equilibrium if the net force on it is zero:

$$\sum_i \boldsymbol{F}_i = \boldsymbol{0}. \tag{1.32}$$

This is a vector equation. In one dimension it becomes one scalar equation. In two dimensions it becomes two scalar equations:

$$\sum_i F_{x,i} = 0, \qquad \sum_i F_{y,i} = 0. \tag{1.33}$$

Equilibrium problems are excellent tests of whether the force directions are understood.

Example 1.6: Equilibrium position on a line

Two fixed charges lie on the x axis. Charge $q_1 = +9.0\,\mu\mathrm{C}$ is at $x = 0$, and charge $q_2 = +1.0\,\mu\mathrm{C}$ is at $x = 0.40\,\mathrm{m}$. Where between them can a positive test charge be placed so that the net electrostatic force on it is zero?

Reasoning. Between the two positive source charges, the test charge is repelled by both. The force from q_1 points to the right; the force from q_2 points to the left. The forces can cancel there.

Let the test charge be at distance x from q_1, so its distance from q_2 is $0.40 - x$. Equal magnitudes give

$$k_e \frac{q_0 q_1}{x^2} = k_e \frac{q_0 q_2}{(0.40 - x)^2}. \tag{1.34}$$

Cancel k_e and q_0:

$$\frac{9.0}{x^2} = \frac{1.0}{(0.40 - x)^2}. \tag{1.35}$$

Taking the positive square root,

$$\frac{3.0}{x} = \frac{1.0}{0.40 - x}. \tag{1.36}$$

Therefore

$$3.0(0.40 - x) = x, \tag{1.37}$$

so

$$x = 0.30\,\text{m}. \tag{1.38}$$

Interpretation. The equilibrium point is closer to the smaller charge. This makes sense because the smaller charge must be closer to produce a force as large as the larger charge.

Guided checks

Check 1. Why is the equilibrium point in Example 1.6 not halfway between the charges?
Answer. The charges have unequal magnitudes. The smaller charge must be closer to produce the same force.

Check 2. If one source charge were positive and the other negative, could a positive test charge be in equilibrium between them?
Answer. No. Between opposite source charges, the test charge is repelled by the positive charge and attracted by the negative charge in the same direction.

1.13 Charge sharing between conducting spheres

When two identical conducting spheres touch, charge can move between them until symmetry requires equal final charges. If the initial charges are q_A and q_B, then after contact and separation,

$$q'_A = q'_B = \frac{q_A + q_B}{2}. \tag{1.39}$$

This simple formula relies on identical spheres and isolation from external charge reservoirs. If the spheres are different sizes or nearby objects strongly distort the charge distribution, a more advanced analysis is needed.

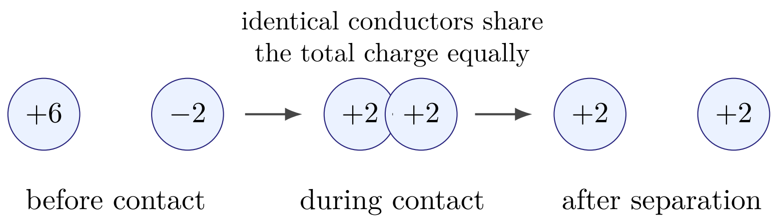

Figure 1.7: Charge sharing between identical conducting spheres. The numbers represent charge in an arbitrary common unit. The total charge +4 is conserved, so each identical sphere ends with +2.

Example 1.7: Two identical conducting spheres

Two identical small conducting spheres are far apart. Sphere A has charge +12 nC and sphere B has charge −4 nC. They are touched together and separated again. They are then placed 0.25 m apart. Find the final charge on each sphere and the magnitude of the force between

them.
Charge sharing. The total charge is

$$Q_{\text{total}} = +12\,\text{nC} - 4\,\text{nC} = +8\,\text{nC}. \tag{1.40}$$

Because the spheres are identical,

$$q'_A = q'_B = +4\,\text{nC}. \tag{1.41}$$

Force after separation. Treating the separated small spheres as point charges,

$$F = k_e \frac{|q'_A q'_B|}{r^2} \tag{1.42}$$
$$= (8.99 \times 10^9) \frac{(4.0 \times 10^{-9})(4.0 \times 10^{-9})}{(0.25)^2}\,\text{N} \tag{1.43}$$
$$= 2.30 \times 10^{-6}\,\text{N}. \tag{1.44}$$

The force is repulsive because both final charges are positive.

1.14 Comparison with Newtonian gravitation

Coulomb's law and Newton's gravitational law have similar mathematical structures:

$$F_e = k_e \frac{|q_1 q_2|}{r^2}, \qquad F_g = G \frac{m_1 m_2}{r^2}. \tag{1.45}$$

Both are inverse-square laws. Both act along the line joining the interacting particles. Both satisfy superposition in the introductory classical model. Yet they differ profoundly. Electric charge has two signs; ordinary mass in Newtonian gravity has one sign. Electrostatic forces can attract or repel, while gravitational forces between ordinary masses are attractive. At the scale of elementary charged particles, the electrostatic force is enormously stronger than gravity.

Example 1.8: Electric and gravitational forces between electron and proton

Estimate the ratio of the electrostatic force to the gravitational force between an electron and a proton separated by the same distance.
Model. Use magnitudes. The distance cancels because both laws have the same $1/r^2$ dependence.

$$\frac{F_e}{F_g} = \frac{k_e e^2}{G m_e m_p}. \tag{1.46}$$

Using $k_e = 8.99 \times 10^9\,\text{N}\,\text{m}^2/\text{C}^2$, $e = 1.602 \times 10^{-19}\,\text{C}$, $G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2$, $m_e = 9.11 \times 10^{-31}\,\text{kg}$ and $m_p = 1.67 \times 10^{-27}\,\text{kg}$,

$$\frac{F_e}{F_g} \simeq \frac{(8.99 \times 10^9)(1.602 \times 10^{-19})^2}{(6.67 \times 10^{-11})(9.11 \times 10^{-31})(1.67 \times 10^{-27})} \simeq 2.3 \times 10^{39}. \tag{1.47}$$

Interpretation. At the electron–proton scale, electrostatic attraction overwhelms gravitational attraction. Gravity dominates planets and stars not because it is stronger microscopically, but because bulk matter is nearly electrically neutral while mass contributions add with the same sign.

1.15 Engineering and applied-science connections

Electrostatics is technically important precisely because small charge imbalances can produce large effects. Some applications use electrostatic forces deliberately; others try to suppress them.

1.15.1 Electrostatic discharge in electronics

A human body can acquire excess charge by contact and separation with insulating materials. Touching a conductor can then produce a rapid discharge. Sensitive semiconductor devices can be damaged by electrostatic discharge even when the spark is too small for a person to notice. Engineering controls include grounded wrist straps, dissipative mats, conductive packaging, humidity control and careful handling protocols.

1.15.2 Electrostatic attraction in printing and material handling

Laser printers and photocopiers use controlled charge distributions to attract toner particles. Electrostatic precipitators use electric forces to remove particles from gas streams. Powder coating uses electrostatic attraction to help charged powder adhere to a workpiece. In each case the same basic idea appears: charged or polarised particles experience forces that can move them.

1.15.3 Materials and ionic bonding

The attraction between positive and negative ions is one component of ionic bonding. A simplified pair of ions can be modelled with Coulomb's law at a first level. A real crystal requires many-ion interactions, quantum mechanics and short-range repulsion, but Coulomb's law explains why charge signs and separation matter so strongly.

Example 1.9: Estimate of force between neighbouring ions

Estimate the magnitude of the electrostatic force between a singly charged positive ion and a singly charged negative ion separated by 0.28 nm.
Solution. Treat the ions as point charges with $|q_1| = |q_2| = e$ and $r = 0.28 \times 10^{-9}$ m:

$$F = k_e \frac{e^2}{r^2} \tag{1.48}$$

$$= (8.99 \times 10^9) \frac{(1.602 \times 10^{-19})^2}{(0.28 \times 10^{-9})^2} \text{ N} \tag{1.49}$$

$$= 2.9 \times 10^{-9} \text{ N}. \tag{1.50}$$

Interpretation. A nanonewton is tiny macroscopically but enormous for an atomic-scale ion. Real crystals are not only two-ion systems, so this is a scale estimate rather than a full model of binding.

1.16 Laboratory connection: electrostatics observations

Electrostatic experiments are often visually simple but experimentally delicate. Humidity, surface contamination, dust, grounding paths and previous charge history can strongly affect results. That is a useful lesson: electrostatic measurements require control of environment and boundary conditions.

1.16.1 Suggested electrostatics laboratory

A basic laboratory can include charged rods, cloths, an electroscope, a metal can, pith balls, insulating supports and a grounding wire. Students can observe attraction of neutral objects, repulsion of similarly charged objects, discharge by grounding and charging by induction.

Method

Suggested laboratory workflow.

1. Record room conditions, especially humidity if available.
2. Charge a plastic or glass rod by rubbing with an appropriate cloth.
3. Test attraction of small neutral paper pieces.
4. Bring the charged rod near an electroscope without contact and observe induction.
5. Touch the electroscope with the charged rod and observe charging by contact.
6. Ground the electroscope and observe discharge.
7. Repeat with a neutral metal can to distinguish attraction by induction from attraction of opposite net charge.
8. Discuss repeatability and environmental sensitivity.

1.16.2 What is measured and what is inferred

In many introductory electrostatics demonstrations, students do not directly measure charge. They observe motion, leaf separation, attraction, repulsion or discharge. Charge sign and redistribution are inferred from a model. This is normal in physics: observations become meaningful through a controlled interpretation.

Guided checks

Check 1. Why might an electrostatics demonstration work well on one day and poorly on another?
Answer. Humidity and surface contamination can allow charge to leak away.

Check 2. Does attraction of a neutral metal can prove that the can had net charge?
Answer. No. It may be neutral but polarised by induction.

Check 3. Why is an insulating handle useful?
Answer. It prevents charge from flowing easily through the experimenter to ground.

1.17 Computational connection: many-charge force summation

Electrostatic force superposition is naturally computational. Suppose we know the charges q_i and positions $\boldsymbol{r}_i$ of many source charges. The force on a target charge q_0 at $\boldsymbol{r}_0$ is given by Eq. (1.21). A computer implementation follows the mathematics closely.

Method

Pseudocode for Coulomb-force summation.

1. Store the target charge q_0 and position $\boldsymbol{r}_0$.
2. Store arrays of source charges q_i and source positions $\boldsymbol{r}_i$.
3. Initialize $\boldsymbol{F}_{\text{net}} = \boldsymbol{0}$.
4. For each source charge, compute $\Delta\boldsymbol{r} = \boldsymbol{r}_0 - \boldsymbol{r}_i$.
5. Compute $r = |\Delta\boldsymbol{r}|$ and stop or regularise if $r = 0$.
6. Add
$$\Delta\boldsymbol{F} = k_e q_0 q_i \frac{\Delta\boldsymbol{r}}{r^3}$$
to the net force.

7. Return the vector components and optionally plot an arrow.

Example 1.10: Numerical superposition from tabulated coordinates

A target charge $q_0 = +1.0\,\mu\text{C}$ is at the origin. Three source charges are placed at

source	charge	position
1	$+1.0\,\mu\text{C}$	$(0.20, 0)$ m
2	$-1.0\,\mu\text{C}$	$(0, 0.20)$ m
3	$+2.0\,\mu\text{C}$	$(-0.20, 0)$ m

Find the net force on q_0.

Solution. Each source is 0.20 m from the target. The force scale for $1.0\,\mu\text{C}$ and $1.0\,\mu\text{C}$ at this distance is

$$F_* = (8.99 \times 10^9)\frac{(1.0 \times 10^{-6})^2}{(0.20)^2} = 0.225\,\text{N}. \tag{1.51}$$

Source 1 is positive and lies to the right, so it repels the target to the left:

$$\boldsymbol{F}_{01} = -0.225\hat{\boldsymbol{\imath}}\,\text{N}. \tag{1.52}$$

Source 2 is negative and lies above, so it attracts the target upward:

$$\boldsymbol{F}_{02} = +0.225\hat{\boldsymbol{\jmath}}\,\text{N}. \tag{1.53}$$

Source 3 is positive, twice as large, and lies to the left, so it repels the target to the right with twice the scale:

$$\boldsymbol{F}_{03} = +0.450\hat{\boldsymbol{\imath}}\,\text{N}. \tag{1.54}$$

Therefore

$$\boldsymbol{F}_{\text{net}} = (0.225\hat{\boldsymbol{\imath}} + 0.225\hat{\boldsymbol{\jmath}})\,\text{N}. \tag{1.55}$$

The magnitude is 0.318 N and the direction is 45° above the positive x axis.

1.18 Common mistakes and how to avoid them

Caution

Mistake 1: treating charge as a visible fluid. Charge is a property of particles. Speaking of charge transfer is convenient, but what usually moves in a solid conductor is electrons.

Caution

Mistake 2: thinking neutral means empty of charge. Neutral matter contains positive and negative charge whose algebraic sum is zero.

Caution

Mistake 3: assuming attraction always means opposite net charges. A neutral conductor can be attracted to a charged object because its charges rearrange.

Caution

Mistake 4: drawing force vectors between objects instead of on objects. A force vector in a free-body diagram must have its tail on the object experiencing the force.

Caution

Mistake 5: adding force magnitudes in two dimensions. Use vector components unless all forces are collinear and directions have been handled carefully.

Caution

Mistake 6: confusing $\boldsymbol{F}_{12}$ and $\boldsymbol{F}_{21}$. The first subscript names the object feeling the force. The two forces are equal in magnitude and opposite in direction but act on different objects.

Caution

Mistake 7: using Coulomb's law outside its model. The simple formula applies to point charges or objects that can be approximated as point charges or suitable spherical charge distributions. Irregular extended objects require more advanced methods.

Caution

Mistake 8: treating grounding as a symbol instead of a physical path. A grounded conductor can exchange charge with a large reservoir. It is not isolated during grounding.

Caution

Mistake 9: forgetting units. Charge in microcoulombs must be converted to coulombs before using k_e in SI units.

Caution

Mistake 10: confusing k_e with a spring constant. The Coulomb constant has units $\mathrm{N\,m^2/C^2}$ and belongs to electrostatics, not Hooke's law.

1.19 Guided checks for the main chapter

Guided checks

1. **Is an electrically neutral metal sphere charge-free?** No. It contains positive ion cores and electrons; its total charge is zero.
2. **Why can a charged rod attract a neutral conductor?** The rod causes charge separation in the conductor. The nearer induced charge is opposite in sign and closer, so attraction dominates.
3. **Does charge conservation forbid charge transfer?** No. It forbids change of total charge in an isolated system. Charge can move within the system.
4. **What determines whether two point charges attract or repel?** The sign of the product q_1q_2: positive gives repulsion, negative gives attraction.
5. **Where should a force vector be drawn?** On the object experiencing the force.

6. **Why is Coulomb's law called an inverse-square law?** The magnitude varies as $1/r^2$ with separation.
7. **Can $F = k_e|q_1q_2|/r^2$ give the sign of a vector component by itself?** No. It gives a magnitude. Direction must be determined separately or by vector form.
8. **Why does a small macroscopic charge correspond to many electrons?** Because e is extremely small in coulombs.
9. **Why is the electrostatic force more important than gravity inside atoms?** For elementary charged particles, the electric force is enormously stronger than gravitational attraction.
10. **What is postponed to the next chapter?** The electric field as a local description of electrostatic interaction.

1.20 Chapter summary

Summary

- Electric charge is an intrinsic property of matter and comes in positive and negative signs.
- Neutral objects contain charge; their total positive and negative charges cancel.
- Conductors contain mobile charge carriers, while insulators do not allow charge to move freely over macroscopic distances under ordinary conditions.
- Charging usually means transferring charge, not creating charge.
- A neutral conductor can be attracted to a charged object because of induced charge separation.
- Electric charge is conserved in isolated systems.
- Observable free charge is quantized in integer multiples of the elementary charge e.
- Coulomb's law gives the force between point charges at rest or moving slowly relative to one another.
- Electrostatic force is a vector; direction matters as much as magnitude.
- The net force on a charge due to several other charges is the vector sum of individual Coulomb forces.
- Equilibrium requires the vector net force to vanish.
- The simple point-charge model has validity limits; extended charge distributions require later field methods.

Formula summary

Charge quantization

$$q = ne, \qquad n \in \mathbb{Z}, \qquad e = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C}. \tag{1.56}$$

Charge conservation for an isolated system

$$Q_{\text{total}} = \sum_i q_i = \text{constant}. \tag{1.57}$$

Current as charge flow rate

$$I = \frac{\mathrm{d}q}{\mathrm{d}t}. \tag{1.58}$$

Coulomb's law, magnitude form

$$F = k_e \frac{|q_1 q_2|}{r^2}, \qquad k_e = \frac{1}{4\pi\varepsilon_0}. \tag{1.59}$$

Coulomb's law, vector form

$$\boldsymbol{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{\boldsymbol{r}}_{12}. \tag{1.60}$$

Superposition and equilibrium

$$\boldsymbol{F}_{\text{net}} = \sum_i \boldsymbol{F}_i, \qquad \sum_i \boldsymbol{F}_i = \boldsymbol{0} \quad \text{in equilibrium.} \tag{1.61}$$

Identical conducting spheres after contact

$$q'_A = q'_B = \frac{q_A + q_B}{2}. \tag{1.62}$$

1.21 Original practice problems

The following problems are original and are designed for conceptual understanding, calculation practice, laboratory interpretation, computational modelling and technical extensions. Unless stated otherwise, use $k_e = 8.99 \times 10^9\,\mathrm{N\,m^2/C^2}$ and $e = 1.602 \times 10^{-19}\,\mathrm{C}$.

A. Conceptual questions

1. Explain why a neutral copper sphere is not charge-free.
2. A negatively charged rod attracts a neutral metal can. Explain the attraction without saying that the can has net positive charge.
3. Why is charge conservation not the same as saying that charge cannot move?
4. In a metal, what usually moves when the metal is charged or discharged: electrons or protons?
5. Why can a spark occur through air even though air is normally an insulator?
6. Two positive charges repel. Do the forces on the two charges cancel each other when analysing only one charge?
7. Why is a free-body diagram necessary before using signs in Coulomb's law?
8. Explain why $F = k_e|q_1q_2|/r^2$ is not a vector equation.
9. What assumption is made when two small charged beads are treated as point charges?
10. Why is humidity relevant in electrostatics demonstrations?
11. What does it mean to ground a conductor?
12. Why should k_e not be confused with the spring constant k?

B. Charge quantization and charge conservation

1. How many excess electrons correspond to a charge of $-5.0\,\mathrm{nC}$?
2. How many electrons must be removed from a neutral object to leave charge $+2.0\,\mathrm{nC}$?

3. Object A has charge +8 nC and object B has charge −3 nC. What is the total charge of the two-object system?

4. A neutral object receives 4.0×10^{11} electrons. What is its final charge?

5. A small bead with charge −12 nC loses half of its excess electrons. What is its final charge?

6. A pair-production event creates an electron and a positron from neutral radiation. Check charge conservation.

7. An electron and a positron annihilate into neutral radiation. Check charge conservation.

8. Two objects in an isolated system have final charges +3 nC and −7 nC. What was the total initial charge?

C. Coulomb-force magnitudes

1. Find the force magnitude between charges $+1.0\,\mu$C and $+2.0\,\mu$C separated by 0.50 m.

2. Find the force magnitude between charges $-3.0\,\mu$C and $+4.0\,\mu$C separated by 0.20 m.

3. By what factor does the force change if the separation between two point charges is tripled?

4. By what factor does the force change if both charges are doubled and the separation is unchanged?

5. What separation is required for two charges of magnitude $1.0\,\mu$C to repel with force 1.0 N?

6. Two equal charges repel with force 0.20 N at separation 0.30 m. Find the charge magnitude.

D. Vector-force problems

1. Charges $q_0 = +2.0\,\mu$C and $q_1 = +3.0\,\mu$C lie on the x axis with q_1 at $x = 0.40$ m and q_0 at the origin. Write the force on q_0 in unit-vector notation.

2. Repeat the previous problem if $q_1 = -3.0\,\mu$C.

3. A charge $q_0 = +1.0\,\mu$C is at the origin. A charge $+1.0\,\mu$C is at $(0.30, 0)$ m and another charge $+1.0\,\mu$C is at $(0, 0.30)$ m. Find the net force on q_0.

4. A charge $q_0 = -2.0\,\mu$C is at the origin. A charge $+3.0\,\mu$C is at $(0.20, 0)$ m and a charge $-3.0\,\mu$C is at $(-0.20, 0)$ m. Find the net force on q_0.

5. Three identical positive charges are placed at three corners of a square. Determine qualitatively the direction of the force on a positive charge placed at the fourth corner.

6. A target charge is acted on by two equal-magnitude perpendicular forces. Explain why the net force magnitude is not twice the magnitude of one force.

E. Equilibrium and charge sharing

1. Charges $+4q$ and $+q$ are separated by distance L. Find the point between them where a positive test charge feels zero net force.

2. Charges $+Q$ and $-4Q$ are separated by distance L. Explain qualitatively why no equilibrium point for a positive test charge lies between them.

3. Two identical conducting spheres have charges +10 nC and +2 nC. They touch and separate. Find the final charge on each.

4. Two identical conducting spheres have charges $+10\,\mathrm{nC}$ and $-6\,\mathrm{nC}$. They touch and separate. Find the final charge on each.

5. After two identical spheres touch and separate, each has charge $-3\,\mathrm{nC}$. If one initial charge was $+2\,\mathrm{nC}$, what was the other initial charge?

6. Three identical conducting spheres A, B and C have initial charges $+6\,\mathrm{nC}$, 0 and $-2\,\mathrm{nC}$. Sphere A touches B and separates. Then B touches C and separates. Find the final charges.

F. Engineering and laboratory applications

1. A device accumulates $-20\,\mathrm{nC}$ of excess charge. Estimate the number of excess electrons.

2. Explain why grounded wrist straps are used when handling sensitive electronic components.

3. In a dry room, a charged plastic rod attracts paper pieces more strongly than in a humid room. Explain why.

4. A metal can rolls toward a charged rod. Describe the charge distribution on the can qualitatively.

5. A student touches a charged electroscope and its leaves collapse. Explain the role of grounding through the student's body.

6. A simplified powder-coating particle has charge $+5.0\,\mathrm{pC}$ and experiences an electrostatic force of $2.0 \times 10^{-6}\,\mathrm{N}$. If the force is due to a single opposite charge at a fixed position, explain what additional information would be required to find that source charge.

G. Computational and challenge problems

1. Write pseudocode to compute the net Coulomb force on one target charge due to N source charges.

2. Why must a computational Coulomb-force algorithm check for zero separation?

3. For four charges at the corners of a square, outline how symmetry can reduce the number of explicit force calculations.

4. Consider N source charges. How does the computational cost of direct force summation scale with N for one target charge?

5. Challenge: derive the coordinate form of Coulomb's law from the unit-vector form.

6. Challenge: four identical positive charges are fixed at the corners of a square. Determine the direction of the net force on an additional positive charge placed at the centre.

7. Challenge: compare the electrostatic and gravitational forces between two protons. Explain why the strong nuclear interaction is needed in nuclei.

8. Challenge: two equal positive charges are fixed at $(\pm a, 0)$. A third charge is placed on the y axis. Determine the direction of the net force for positive and negative third charge.

1.22 Selected solutions to representative practice problems

Example 1.11: Selected solution: excess electrons for nanocoulomb charge

For $q = -5.0\,\text{nC}$,

$$N = \frac{|q|}{e} = \frac{5.0 \times 10^{-9}}{1.602 \times 10^{-19}} = 3.1 \times 10^{10}. \tag{1.63}$$

The negative sign means these are excess electrons.

Example 1.12: Selected solution: force magnitude

For $q_1 = +1.0\,\mu\text{C}$, $q_2 = +2.0\,\mu\text{C}$ and $r = 0.50\,\text{m}$,

$$F = (8.99 \times 10^9)\frac{(1.0 \times 10^{-6})(2.0 \times 10^{-6})}{0.50^2} = 7.2 \times 10^{-2}\,\text{N}. \tag{1.64}$$

The force is repulsive.

Example 1.13: Selected solution: equal charge sharing

For identical spheres initially at $+10\,\text{nC}$ and $-6\,\text{nC}$, the total charge is $+4\,\text{nC}$. After contact and separation,

$$q_1' = q_2' = \frac{+4\,\text{nC}}{2} = +2\,\text{nC}. \tag{1.65}$$

Chapter 2

Advanced Topics for Technical and Engineering Students

This section is intended for very strong students, honours tutorials, engineering extensions and computational projects. It deepens the same ideas without replacing later chapters on electric fields, Gauss's law or electric potential.

2.0.1 Coordinate-vector formulation of Coulomb's law

The compact coordinate form,

$$\boldsymbol{F}_{12} = k_e q_1 q_2 \frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^3}, \tag{2.1}$$

contains both magnitude and direction. To see this, write

$$\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2, \qquad \hat{\boldsymbol{r}}_{12} = \frac{\boldsymbol{r}_{12}}{|\boldsymbol{r}_{12}|}. \tag{2.2}$$

Substituting this unit vector into the unit-vector form gives

$$\boldsymbol{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \frac{\boldsymbol{r}_{12}}{r_{12}} = k_e q_1 q_2 \frac{\boldsymbol{r}_{12}}{r_{12}^3}. \tag{2.3}$$

The denominator r_{12}^3 is not a new inverse-cube force. It appears because a displacement vector of length r_{12} is divided by r_{12}^3, leaving an overall magnitude proportional to $1/r_{12}^2$.

Caution

The coordinate form is undefined at $\boldsymbol{r}_1 = \boldsymbol{r}_2$. Point charges placed at exactly the same mathematical point produce a singular expression. In real matter, finite size, quantum physics and non-electrostatic interactions become relevant before such a classical singularity can be treated literally.

Example 2.1: Advanced example: vector force from coordinates

Charge $q_1 = +2.0\,\mu\text{C}$ is at $\boldsymbol{r}_1 = (0.30\hat{\boldsymbol{\imath}} + 0.40\hat{\boldsymbol{\jmath}})\,\text{m}$. Charge $q_2 = -1.0\,\mu\text{C}$ is at the origin. Find $\boldsymbol{F}_{12}$.
Solution. The displacement from 2 to 1 is

$$\boldsymbol{r}_{12} = 0.30\hat{\boldsymbol{\imath}} + 0.40\hat{\boldsymbol{\jmath}}\,\text{m}, \tag{2.4}$$

with magnitude

$$r_{12} = \sqrt{0.30^2 + 0.40^2}\,\text{m} = 0.50\,\text{m}. \tag{2.5}$$

The vector force is

$$\boldsymbol{F}_{12} = k_e q_1 q_2 \frac{\boldsymbol{r}_{12}}{r_{12}^3} \tag{2.6}$$

$$= (8.99 \times 10^9)(2.0 \times 10^{-6})(-1.0 \times 10^{-6}) \frac{0.30\hat{\boldsymbol{\imath}} + 0.40\hat{\boldsymbol{\jmath}}}{(0.50)^3}. \tag{2.7}$$

The coefficient is

$$\frac{(8.99 \times 10^9)(-2.0 \times 10^{-12})}{0.125} = -0.1438\,\mathrm{N\,m^{-1}}. \tag{2.8}$$

Therefore

$$\boldsymbol{F}_{12} = (-0.043\hat{\boldsymbol{\imath}} - 0.058\hat{\boldsymbol{\jmath}})\,\mathrm{N}. \tag{2.9}$$

The force points from charge 1 toward the negative charge at the origin, as expected for attraction.

2.0.2 Many-charge systems and computational scaling

For one target charge and N source charges, direct summation requires N pair-force evaluations. For all pairwise forces in a system of N charges, a direct algorithm requires a number of pair interactions proportional to $N(N-1)/2$. This scaling becomes important in large simulations. Advanced computational physics uses tree methods, particle-mesh methods or fast multipole methods to reduce cost in large systems, but the direct sum is the correct conceptual starting point.

Theoretical point 2.1: Direct pairwise electrostatic summation

For N point charges at positions $\boldsymbol{r}_i$, the net force on charge i due to all others is

$$\boldsymbol{F}_i = k_e \sum_{j \neq i} q_i q_j \frac{\boldsymbol{r}_i - \boldsymbol{r}_j}{|\boldsymbol{r}_i - \boldsymbol{r}_j|^3}. \tag{2.10}$$

The exclusion $j \neq i$ is essential: a point charge does not exert a Coulomb force on itself in this classical particle model.

2.0.3 Symmetry and cancellation

Symmetry can turn a long vector calculation into a short argument. If equal charges are arranged symmetrically around a central charge, pairs of force contributions may cancel. Symmetry arguments must still identify the object of interest and the direction of each force.

Example 2.2: Advanced example: charge at the centre of a square

Four identical positive charges $+Q$ are fixed at the corners of a square. A positive test charge $+q$ is placed at the centre. Find the net force on the test charge.
Solution. Each corner charge repels the central charge along a diagonal. The force from the top-right corner is cancelled by the force from the bottom-left corner. The force from the top-left corner is cancelled by the force from the bottom-right corner. Therefore

$$\boldsymbol{F}_{\text{net}} = \boldsymbol{0}. \tag{2.11}$$

Important note. Zero net force at the centre does not mean that no forces act. Four nonzero forces act, but their vector sum is zero.

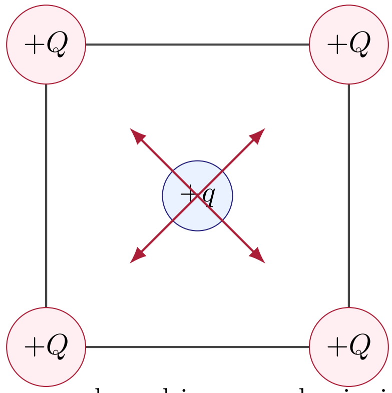

Figure 2.1: Symmetry cancellation for a test charge at the centre of a square of four identical charges. Each force has a partner with equal magnitude and opposite direction.

2.0.4 Scaling estimates: atomic, laboratory and macroscopic scales

Coulomb's law is useful for order-of-magnitude reasoning. At atomic separations, even elementary charges exert large accelerations on particles because particle masses are tiny. At laboratory separations, microcoulomb charges can produce measurable forces. At macroscopic scales, ordinary objects are usually nearly neutral, so large electric forces are often hidden unless charge separation occurs.

Consider two charges of magnitude q separated by distance r:

$$F \sim 9 \times 10^9 \frac{q^2}{r^2}. \tag{2.12}$$

If $q = 1\,\mathrm{nC}$ and $r = 1\,\mathrm{cm}$, then

$$F \sim 9 \times 10^9 \frac{10^{-18}}{10^{-4}}\,\mathrm{N} = 9 \times 10^{-5}\,\mathrm{N}. \tag{2.13}$$

If $q = 1\,\mu\mathrm{C}$ at the same separation, the force is 10^6 times larger, about $90\,\mathrm{N}$. This illustrates why apparently small charges can be significant.

2.0.5 Electrostatic discharge engineering

Electrostatic discharge engineering is the art of preventing uncontrolled charge accumulation and rapid discharge in sensitive environments. The physical ideas are simple: charge can accumulate on insulating surfaces, conductors can discharge rapidly, and semiconductors can be damaged by large transient electric stresses. The engineering implementation is detailed, involving material choice, grounding, humidity control, packaging, personnel procedures and device-level protection.

The introductory physics message is that grounding provides a controlled path for charge redistribution, while insulation can either protect or create risk depending on context. An insulating shoe sole may help a body accumulate charge by reducing leakage to ground. A dissipative mat provides a slow, controlled path to ground. A grounded wrist strap keeps the person and work surface close to the same electrical state so that sudden discharge through a device is less likely.

Method

Qualitative ESD risk analysis.

1. Identify insulating surfaces that can accumulate charge.
2. Identify conductors that can deliver a rapid discharge.

3. Identify sensitive devices or measurement inputs.
4. Provide controlled grounding paths where appropriate.
5. Avoid isolated conductors near sensitive electronics.
6. Control handling procedures rather than relying on intuition about whether a spark is visible.

2.0.6 Charge quantization and particle-physics preview

At the level of atoms, electrons carry charge $-e$ and protons carry charge $+e$. At a deeper level, protons and neutrons are composite particles built from quarks. The up quark has charge $+2e/3$ and the down quark has charge $-e/3$. A proton has quark content uud, giving

$$\frac{2e}{3} + \frac{2e}{3} - \frac{e}{3} = +e. \tag{2.14}$$

A neutron has quark content udd, giving

$$\frac{2e}{3} - \frac{e}{3} - \frac{e}{3} = 0. \tag{2.15}$$

This does not contradict the introductory statement that free observable charges are integer multiples of e in ordinary conditions. Quarks are not isolated as free particles in normal matter. The detailed explanation belongs to particle physics and quantum chromodynamics, far beyond this chapter.

Pedagogical boundary

The quark discussion is a preview only. It is included to show that charge conservation and charge bookkeeping survive into modern physics. It is not a substitute for the later chapter on particles and fundamental interactions.

2.0.7 Charge conservation in simple reactions

Charge conservation is often easiest to check in symbolic reactions. For example, in beta-minus decay at a schematic level,

$$n \to p + e^- + \bar{\nu}, \tag{2.16}$$

the initial charge is zero. The final charges are $+e$, $-e$ and 0, whose sum is zero. Thus charge is conserved. In electron–positron annihilation,

$$e^- + e^+ \to \gamma + \gamma, \tag{2.17}$$

the initial charge is $-e + e = 0$, and photons have zero charge, so the final charge is also zero.

2.0.8 From inverse-square force to flux intuition

The inverse-square form of Coulomb's law suggests a geometric idea that will become central in Gauss's law. The surface area of a sphere grows as $4\pi r^2$. A quantity that spreads uniformly over spherical surfaces from a point source naturally decreases like $1/r^2$. This is not yet Gauss's law, because we have not defined electric field or flux. It is only a preview of why inverse-square laws and spherical geometry are connected.

Note

The next chapters will replace force on a particular test charge by electric field in space. The inverse-square force law will become an inverse-square electric field for a point charge, and the geometry of closed surfaces will lead to Gauss's law.

2.0.9 Multipole intuition without the full field formalism

A single isolated net charge is called a monopole distribution at the introductory level. A pair of equal and opposite charges separated by a small distance is called an electric dipole. Even without calculating fields, one can understand that a dipole has zero total charge but can still interact electrically because its positive and negative parts are separated. This helps explain why neutral molecules and neutral pieces of matter can respond to nearby charges.

A full treatment of dipole fields and torques belongs to later sections. For now the important lesson is conceptual: net charge is not the only possible measure of electrical structure. Spatial separation of positive and negative charge matters.

2.0.10 Advanced computational mini-project

A useful computational project is to build a two-dimensional Coulomb-force visualizer. The program stores a list of source charges and positions, then computes the force on a movable target charge. Students can vary charge signs, positions and magnitudes, and observe how the force arrow changes.

Method

Project specification.

1. Input: a list of charges q_i and positions (x_i, y_i).
2. Input: target charge q_0 and position (x_0, y_0).
3. Output: force components F_x, F_y and magnitude F.
4. Visualization: plot source charges and the force arrow on the target.
5. Test cases: one source charge, two symmetric equal charges, two opposite charges, square symmetry.
6. Error handling: avoid source and target at identical coordinates.

Example 2.3: Advanced design example: direct-sum algorithm test

A code is written to compute the force on $q_0 = +1\,\mu\text{C}$ at the origin. Two equal positive source charges are placed at $(+a, 0)$ and $(-a, 0)$. What should the code return?
Reasoning. The charge at $+a$ repels the target toward $-x$. The charge at $-a$ repels the target toward $+x$. The magnitudes are equal because the charges and distances are equal. Therefore the net force is

$$\boldsymbol{F}_{\text{net}} = \mathbf{0}. \tag{2.18}$$

This is an excellent test of sign conventions and coordinate differences in a program.

2.0.11 Extended advanced worked example: regular polygon of charges

Suppose N identical positive charges are equally spaced on a circle and a positive test charge is placed at the centre. By symmetry, for every charge at angle θ there is a charge or a combination of charges that cancels its force contribution. Therefore the net force at the centre is zero. This is true for $N \geq 2$ with a symmetric arrangement.

If one charge is removed, the cancellation is broken. The net force on the central test charge is equal in magnitude and opposite in direction to the force that the missing charge would have contributed.

This is a powerful symmetry trick: instead of summing $N-1$ forces, imagine the full symmetric sum is zero and subtract the missing contribution.

Example 2.4: Advanced example: one missing charge from a symmetric ring

Six identical positive charges $+Q$ would form a regular hexagon of radius R around a positive test charge $+q$ at the centre, but the charge at angle 0 is missing. Find the direction and magnitude of the net force on the central test charge.
Solution. With all six charges present, the net force at the centre would be zero by symmetry. The missing charge at angle 0 would have repelled the central positive charge in the negative x direction with magnitude

$$F_* = k_e \frac{qQ}{R^2}. \tag{2.19}$$

Removing that negative-x contribution leaves a net force in the positive x direction:

$$\boldsymbol{F}_{\text{net}} = +k_e \frac{qQ}{R^2}\hat{\boldsymbol{i}}. \tag{2.20}$$

2.0.12 Extended advanced worked example: suspended charged balls

Two identical small conducting balls of mass m hang from insulating threads of length L from the same support point. If both carry equal charge q, they repel and the threads make equal angles with the vertical. This is a classic example because it combines electrostatic force with mechanics.

For each ball in equilibrium, the forces are weight mg downward, tension T along the thread, and electrostatic repulsion F_e horizontally. The equilibrium equations are

$$T\cos\theta = mg, \qquad T\sin\theta = F_e. \tag{2.21}$$

Therefore

$$\tan\theta = \frac{F_e}{mg}. \tag{2.22}$$

If the separation between the balls is $r = 2L\sin\theta$, then

$$F_e = k_e \frac{q^2}{(2L\sin\theta)^2}. \tag{2.23}$$

This equation can be solved for q if m, L and θ are measured:

$$q = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k_e}}. \tag{2.24}$$

Take-home message

Advanced electrostatics often begins by combining Coulomb's law with mechanics. The electrostatic force is one term in a free-body diagram, and equilibrium or motion follows from Newton's laws.

2.0.13 Advanced guided checks

Guided checks

1. **Why does the coordinate form of Coulomb's law contain r^3 in the denominator?**
   Because the displacement vector in the numerator has magnitude r, leaving an overall $1/r^2$

force magnitude.

2. **Why is $j \neq i$ required in the many-charge sum?** A point charge is not included as a source of force on itself in this model.
3. **How can a zero net force arise from several nonzero forces?** Vector cancellation.
4. **Why does gravity dominate astronomical structures despite being weaker microscopically?** Bulk matter is nearly electrically neutral, while mass contributions add with the same sign.
5. **Why is a missing-charge symmetry trick valid?** The full symmetric sum is zero; the incomplete sum equals the negative of the removed contribution.
6. **What later concept is foreshadowed by the inverse-square law and spherical area?** Electric flux and Gauss's law.

2.0.14 Extended tutorial: from qualitative charging to quantitative modelling

Introductory electrostatics often begins with observations: a rod attracts paper, an electroscope leaf opens, a metal can rolls, or two suspended balls repel. The scientific task is to translate such observations into a model. This translation has several layers. First, one decides which objects can exchange charge. Second, one decides whether the objects can be approximated as point charges, conducting spheres, insulating bodies with fixed charge, or neutral bodies that polarise. Third, one decides whether a quantitative force calculation is justified or whether the observation should remain qualitative.

A rubbed rod attracting paper is not a clean point-charge experiment. The paper is extended, neutral at first, and polarised by the nearby rod. Coulomb's law still underlies the interaction at the microscopic level, but the simple two-point-charge equation is not directly applicable to the whole rod-paper system. By contrast, two small charged beads suspended far apart compared with their radii are much closer to the point-charge idealization. A careful physicist does not ask only "which formula applies?" but also "which model makes the formula applicable?"

Method

Model selection checklist for elementary electrostatics.

1. Is the object of interest a particle-like charged body, an extended conductor, or an insulator?
2. Is the object neutral, charged, or polarised by another object?
3. Is the charge distribution fixed, mobile, or unknown?
4. Is the separation large compared with object size?
5. Is the system isolated, grounded, or in contact with other objects?
6. Is the goal qualitative direction, force magnitude, charge estimate, or conceptual explanation?

A good model is not necessarily the most detailed model. A good introductory model includes the physical features needed for the question and excludes features that would obscure the main idea. For example, a pith-ball repulsion experiment can be modelled with two equal point charges if one wants a first estimate of charge. The same experiment would need a more refined model if the balls are close enough that induced charge distributions on their surfaces matter significantly.

2.0.15 Extended worked example: estimating charge from suspended pith balls

A classic electrostatic measurement uses two identical lightweight balls suspended by insulating threads. When charged equally, they repel and come to equilibrium at a measurable angle. This experiment is valuable because it connects electrostatics directly to mechanics.

Example 2.5: Estimating charge from a small-angle pith-ball experiment

Two identical small balls of mass $m = 0.20\,\mathrm{g}$ hang from insulating threads of length $L = 0.50\,\mathrm{m}$ attached to the same point. After being given equal charge, each thread makes an angle $\theta = 6.0^\circ$ with the vertical. Estimate the charge on each ball. Treat the balls as point charges and use $g = 9.81\,\mathrm{m\,s^{-2}}$.

Free-body model. On each ball there are three forces: weight mg downward, tension T along the thread, and electrostatic repulsion F_e horizontally outward. Static equilibrium gives

$$T\cos\theta = mg, \qquad T\sin\theta = F_e. \tag{2.25}$$

Dividing the equations,

$$F_e = mg\tan\theta. \tag{2.26}$$

The separation between the balls is

$$r = 2L\sin\theta. \tag{2.27}$$

Coulomb's law gives

$$F_e = k_e\frac{q^2}{r^2}. \tag{2.28}$$

Thus

$$q = r\sqrt{\frac{F_e}{k_e}} = 2L\sin\theta\sqrt{\frac{mg\tan\theta}{k_e}}. \tag{2.29}$$

Numerical calculation. Convert the mass:

$$m = 0.20\,\mathrm{g} = 2.0\times10^{-4}\,\mathrm{kg}. \tag{2.30}$$

Then

$$F_e = (2.0\times10^{-4})(9.81)\tan 6.0^\circ = 2.06\times10^{-4}\,\mathrm{N}. \tag{2.31}$$

The separation is

$$r = 2(0.50)\sin 6.0^\circ = 0.105\,\mathrm{m}. \tag{2.32}$$

Therefore

$$q = 0.105\sqrt{\frac{2.06\times10^{-4}}{8.99\times10^{9}}}\,\mathrm{C} = 1.6\times10^{-8}\,\mathrm{C}. \tag{2.33}$$

Interpretation. The charge is about $16\,\mathrm{nC}$ on each ball. This corresponds to roughly 10^{11} elementary charges. The estimate is sensitive to the angle measurement, and the point-charge approximation is best when the ball radius is much smaller than the separation.

This example illustrates a recurring theme in technical physics: a measurement is interpreted through a model. The measured angle is not itself a charge. The charge is inferred using equilibrium, geometry and Coulomb's law. If the threads are not identical, if the balls leak charge, if air currents move the balls, or if the balls are too close to act as point charges, the inferred charge becomes less reliable.

2.0.16 Extended tutorial: dimensional analysis of Coulomb's constant

The dimensions of k_e follow from Coulomb's law:

$$F = k_e\frac{q_1 q_2}{r^2}. \tag{2.34}$$

Solving for k_e dimensionally gives

$$[k_e] = \frac{[F][r]^2}{[q]^2} = \mathrm{N\,m^2\,C^{-2}}. \tag{2.35}$$

This unit is not optional. If charge is entered in microcoulombs while k_e is used in SI form, the result is wrong by factors of 10^6 or 10^{12}.

The vacuum permittivity ε_0 has reciprocal dimensions up to the factor 4π:

$$[\varepsilon_0] = \mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2}. \tag{2.36}$$

The name "permittivity" becomes more physically meaningful later when electric fields in matter and dielectrics are introduced. In this first chapter, ε_0 mainly fixes the numerical strength of Coulomb's law in SI units.

Guided checks

1. If q is in microcoulombs and r is in centimetres, can one insert those numbers directly into $F = k_e|q_1q_2|/r^2$ with $k_e = 8.99 \times 10^9$? No. Convert to coulombs and metres.
2. What units must $k_eq_1q_2/r^2$ have? Newtons.
3. Why does a dimensional check not guarantee a correct sign? Dimensions check units, not direction conventions.

2.0.17 Extended worked example: diagnosing a unit error

Example 2.6: Finding a unit mistake in a Coulomb-force calculation

A student calculates the force between $q_1 = 2.0\,\mu\mathrm{C}$ and $q_2 = 3.0\,\mu\mathrm{C}$ separated by 4.0 cm and writes

$$F = (8.99 \times 10^9)\frac{(2.0)(3.0)}{(4.0)^2} = 3.37 \times 10^9\,\mathrm{N}. \tag{2.37}$$

Identify the error and compute the correct magnitude.
Diagnosis. The student used microcoulombs as if they were coulombs and centimetres as if they were metres. The SI form of k_e requires

$$q_1 = 2.0 \times 10^{-6}\,\mathrm{C}, \qquad q_2 = 3.0 \times 10^{-6}\,\mathrm{C}, \qquad r = 4.0 \times 10^{-2}\,\mathrm{m}. \tag{2.38}$$

Correct calculation.

$$F = (8.99 \times 10^9)\frac{(2.0 \times 10^{-6})(3.0 \times 10^{-6})}{(4.0 \times 10^{-2})^2}\,\mathrm{N} \tag{2.39}$$

$$= 33.7\,\mathrm{N}. \tag{2.40}$$

Interpretation. The correct force is still large, but not astronomically large. Unit conversion changed the numerical result by a factor of 10^8.

2.0.18 Extended tutorial: charge sharing beyond one contact

Sequential charge sharing is a useful way to test conservation reasoning. The important point is that after each contact, the new charges become the initial charges for the next step. Students often try to average all objects at once even when only two touch at a time. That is wrong unless all conductors are connected simultaneously and are identical.

Example 2.7: Sequential contact of three identical spheres

Three identical conducting spheres A, B and C have initial charges

$$q_A = +12\,\mathrm{nC}, \qquad q_B = 0, \qquad q_C = -6\,\mathrm{nC}. \tag{2.41}$$

Sphere A touches B and separates. Then B touches C and separates. Find the final charges.
First contact: A with B. The total charge on A and B is +12 nC, so after contact

$$q'_A = q'_B = +6\,\text{nC}. \tag{2.42}$$

Sphere C is still −6 nC.
Second contact: B with C. Now B has +6 nC and C has −6 nC. Their total is zero, so after they touch,

$$q''_B = q''_C = 0. \tag{2.43}$$

Sphere A remains +6 nC.
Final result.

$$q_A = +6\,\text{nC}, \qquad q_B = 0, \qquad q_C = 0. \tag{2.44}$$

The total final charge is +6 nC, equal to the total initial charge.

2.0.19 Extended tutorial: equilibrium and stability

An equilibrium point is a point where the net force is zero. Stability is a separate question. If a small displacement from equilibrium produces a force back toward equilibrium, the equilibrium is stable along that displacement. If the force pushes the object farther away, the equilibrium is unstable. In electrostatic point-charge problems, many simple equilibrium points are unstable in at least one direction. A complete stability analysis is beyond this chapter, but the distinction matters conceptually.

For one-dimensional equilibrium between two like charges, a test charge placed at the cancellation point may be restored for small displacements along the line if the force points back toward the point; but transverse displacements can behave differently. Later field and potential methods provide a cleaner language for stability.

Caution

Do not confuse zero net force at one point with automatic stable trapping. Equilibrium asks whether $\boldsymbol{F} = \mathbf{0}$. Stability asks what happens after a small displacement.

2.0.20 Extended worked example: unequal charges and force cancellation outside the interval

Example 2.8: Equilibrium outside two opposite charges

Charge $q_1 = +4Q$ is fixed at $x = 0$ and charge $q_2 = -Q$ is fixed at $x = L$. Find where a positive test charge can be placed so that the net force on it is zero.
Region analysis. Between the charges, the test charge is repelled by $+4Q$ and attracted toward $-Q$, both toward $+x$. No cancellation is possible there. To the left of q_1, the forces are opposite, but the larger charge $+4Q$ is closer than the smaller charge $-Q$ for any such point, so cancellation is not possible there. To the right of q_2, the forces are opposite and cancellation may occur.
Let the test charge be at coordinate $x > L$. Its distances from the two charges are x and $x - L$. Equal magnitudes require

$$k_e \frac{q_0(4Q)}{x^2} = k_e \frac{q_0 Q}{(x-L)^2}. \tag{2.45}$$

Cancel common factors:

$$\frac{4}{x^2} = \frac{1}{(x-L)^2}. \tag{2.46}$$

Taking positive square roots,

$$\frac{2}{x} = \frac{1}{x-L}. \tag{2.47}$$

Thus

$$2(x-L) = x, \tag{2.48}$$

so

$$x = 2L. \tag{2.49}$$

Interpretation. The cancellation point lies outside the interval, on the side of the smaller-magnitude charge.

2.0.21 Extended tutorial: why neutral matter is not usually explosive

The electrostatic force between elementary charges is enormous compared with gravity. One might then wonder why everyday objects do not violently repel or attract each other. The answer is electrical neutrality and microscopic binding. Ordinary matter contains nearly equal positive and negative charge. At distances large compared with atomic dimensions, the effects of positive and negative charges almost cancel. Small residual charge imbalances can still produce noticeable electrostatic effects, especially on dry insulating surfaces.

This cancellation is not perfect in every situation. Surfaces can acquire charge by contact and separation. Materials can polarise. Conductors can redistribute charge. Ionic crystals contain strong local attractions and repulsions arranged in a stable structure only because short-range quantum and many-body effects also matter. The simple pairwise Coulomb law is a foundation, not a complete theory of matter.

Note

A macroscopic object that had even a tiny fractional imbalance between all its positive and negative charges would experience enormous electrostatic effects. The ordinary quietness of matter is evidence of extremely good charge balance, not evidence that charge is unimportant.

2.0.22 Extended worked example: hypothetical charge imbalance of a small object

Example 2.9: How much charge imbalance is needed for a visible force?

Two identical small objects are separated by 0.50 m. What equal charge magnitude on each object would produce a repulsive force of 0.10 N?
Solution. For equal charges q,

$$F = k_e \frac{q^2}{r^2}. \tag{2.50}$$

Thus

$$q = r\sqrt{\frac{F}{k_e}} = (0.50)\sqrt{\frac{0.10}{8.99\times 10^9}}\,\text{C} = 1.7\times 10^{-6}\,\text{C}. \tag{2.51}$$

This is 1.7 μC.

Number of elementary charges.

$$N = \frac{1.7 \times 10^{-6}}{1.602 \times 10^{-19}} \simeq 1.1 \times 10^{13}. \tag{2.52}$$

Interpretation. A visible tenth-newton force at half a metre requires only a microcoulomb-scale imbalance, still a tiny fraction of the total positive or negative charge in a macroscopic object.

2.0.23 Extended tutorial: reading signs from the vector formula

The vector formula can be trusted only if its geometry is defined consistently. Suppose q_1 is the target and q_2 is the source. The vector $\hat{\boldsymbol{r}}_{12}$ points from source to target. If the charges have the same sign, $q_1 q_2 > 0$, and the force on the target is along $\hat{\boldsymbol{r}}_{12}$: away from the source. If the charges have opposite signs, $q_1 q_2 < 0$, and the force is opposite $\hat{\boldsymbol{r}}_{12}$: toward the source.

This is why notation matters. If instead one defines $\boldsymbol{r}_{21} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, then the formula must be written for the correct force using that vector. A common sign error is to change the displacement vector convention without changing the formula.

Method

Sign audit for vector Coulomb calculations.

1. Write in words which force is required: "force on 1 due to 2."
2. Write the displacement vector from source to target.
3. Check whether like charges should give a force away from the source.
4. Check whether opposite charges should give a force toward the source.
5. Compare the final vector direction with a simple attraction/repulsion sketch.

2.0.24 Extended technical note: electrostatics and measurement instruments

Many electrostatic instruments are indirect. An electroscope does not display charge in coulombs directly; it displays mechanical separation caused by electrostatic effects. A Coulomb balance infers force from torsion. A modern electrometer infers charge or current through calibrated electronics. In each case the measurement chain matters. Charge is not usually counted one elementary charge at a time in macroscopic laboratory electrostatics.

For a teaching laboratory, the most valuable measurement lesson is not high precision but model awareness. A charged rod might discharge during the measurement. The experimenter's hand might provide a leakage path. Humidity can reduce charge lifetime. Nearby conducting objects can alter charge distributions. These effects are not nuisances to ignore; they are physical reminders that electrostatics is sensitive to boundaries and materials.

2.0.25 Extended tutorial: qualitative electrostatic safety

High voltage and stored charge can be dangerous, but the danger depends on energy, current path, duration, source impedance and physiology, not only on voltage or charge separately. This chapter does not develop electrical safety quantitatively. However, it does establish the physical basis for safe practice: charge can accumulate, conductors can discharge rapidly, and grounding can create or remove hazardous paths depending on context.

In teaching laboratories, electrostatic demonstrations should use low stored energy and suitable insulating handles. In electronics laboratories, the goal is often the opposite: prevent charge accumulation by using controlled grounding and dissipative materials. In both settings, the correct physical question is not simply "is there charge?" but "where can charge move, how quickly, through what path, and into what object?"

Pedagogical boundary

Electrical safety calculations involving human current thresholds, capacitor discharge energy and power systems require circuit concepts introduced later. This chapter gives only the charge and force foundations needed to understand why uncontrolled discharge matters.

2.0.26 Additional advanced exercises

1. Derive the expression for the charge on each suspended ball in the pith-ball experiment in terms of m, L, g and θ.
2. In the small-angle limit, use $\sin\theta \simeq \tan\theta \simeq \theta$ to show how the pith-ball charge scales with angle.
3. Three identical positive charges are placed at the vertices of an equilateral triangle. Find the direction of the net force on one charge due to the other two.
4. Four charges $+Q, +Q, -Q, -Q$ are placed alternately around a square. Determine qualitatively the direction of the force on a positive charge at the centre.
5. A direct Coulomb-force program gives a nonzero net force at the centre of a symmetric square of identical charges. List three possible coding errors.
6. Two identical conducting spheres touch and separate. One is then grounded and separated from ground. Describe the charge state if the other sphere is kept far away.
7. Estimate the charge required on two small objects separated by $1.0\,\mathrm{m}$ to produce a force equal to the weight of a $1.0\,\mathrm{g}$ mass.
8. Compare the electrostatic force between two protons at $1.0\,\mathrm{fm}$ separation with their gravitational attraction.
9. Explain why an electroscope can respond when a charged rod is nearby even without contact.
10. Design a qualitative experiment to distinguish charging by contact from charging by induction.
11. A ring of eight identical positive charges has one charge removed. Use symmetry to find the direction of the force on a positive charge at the centre.
12. In a numerical direct-sum calculation, all distances are scaled by a factor s while charges are unchanged. How do the force magnitudes scale?

2.0.27 Additional selected advanced solutions

Example 2.10: Selected advanced solution: equilateral triangle

Three identical positive charges $+Q$ occupy the vertices of an equilateral triangle of side a. Consider the force on the charge at the top vertex. The two forces from the lower charges have equal magnitude

$$F_* = k_e \frac{Q^2}{a^2}. \tag{2.53}$$

Their horizontal components cancel by symmetry. Their vertical components add. Each force makes an angle 30° from the vertical, so

$$F_{\text{net}} = 2F_* \cos 30^\circ = \sqrt{3}\, k_e \frac{Q^2}{a^2}, \tag{2.54}$$

pointing upward along the symmetry axis.

Example 2.11: Selected advanced solution: scaling with distance

If all distances in a charge configuration are scaled by a factor s while all charges are unchanged, each Coulomb-force magnitude scales as

$$F' = k_e \frac{|q_1 q_2|}{(sr)^2} = \frac{1}{s^2} F. \tag{2.55}$$

Thus doubling all distances reduces every pair-force magnitude by a factor of four. Directions may remain geometrically similar if the whole configuration is scaled uniformly.

Example 2.12: Selected advanced solution: charge for gram-weight force

The weight of a 1.0 g mass is

$$mg = (1.0 \times 10^{-3})(9.81) = 9.81 \times 10^{-3}\,\text{N}. \tag{2.56}$$

For two equal charges at $r = 1.0\,\text{m}$ producing this force,

$$q = r\sqrt{\frac{F}{k_e}} = 1.0\sqrt{\frac{9.81 \times 10^{-3}}{8.99 \times 10^{9}}}\,\text{C} = 1.04 \times 10^{-6}\,\text{C}. \tag{2.57}$$

A microcoulomb-scale charge can therefore produce a gram-weight-scale force at metre distance in this idealized two-charge model.

2.0.28 Supplementary tutorial: writing professional electrostatic solutions

A professional solution to an electrostatic problem should be readable even before the arithmetic is checked. The reader should be able to identify the system, the object of interest, the coordinate convention, the assumptions and the physical reason for each force direction. This standard is especially important in technical fields, where numerical answers may later be used in design decisions, laboratory notes or simulation validation.

A minimal written solution should include the following ingredients. First, state the model: point charges, isolated conductors, identical spheres, negligible leakage, electrostatic conditions, or other assumptions. Second, define the notation: which charge is q_0, which force is $\boldsymbol{F}_{01}$, and which direction is positive. Third, draw or describe the force directions. Fourth, calculate magnitudes with SI units. Fifth, assemble vector components. Finally, interpret the result physically.

Method

Rubric for a complete Coulomb-law solution.

1. **Model:** point charges or justified approximation stated.
2. **Diagram:** source charges and target charge identified.

3. **Coordinates:** axes, distances and signs defined.
4. **Forces:** each force vector acts on the target object.
5. **Units:** charges in C, distances in m, forces in N.
6. **Vector sum:** components or geometry used correctly.
7. **Interpretation:** direction and magnitude checked against physical intuition.

This rubric may look longer than the calculation itself, but it prevents the most common errors. A one-line answer such as "$F = 0.27\,\text{N}$" is incomplete if the force direction is not specified. A vector answer with a wrong sign can be worse than no answer in engineering use because it points a design conclusion in the wrong direction.

2.0.29 Case study: electroscope interpretation without overclaiming

An electroscope is a sensitive qualitative detector of electrostatic effects. If a charged object is brought near the terminal of a neutral electroscope, charges inside the conductor redistribute and the leaves may separate. If the charged object touches the terminal, charge can be transferred and the electroscope may remain charged after the object is removed. If the electroscope is grounded, it can discharge or be charged by induction depending on the sequence of steps.

The important pedagogical point is that leaf separation alone does not automatically reveal the sign of charge. It indicates that like charges are present on the two leaves or that charge redistribution has occurred. To determine sign, the electroscope must be calibrated with a known charge sign, or the sequence of induction and contact steps must be analysed carefully.

Guided checks

1. If the leaves of an electroscope separate when a charged rod is nearby but not touching, must net charge have been transferred to the electroscope? No. Induction can separate charge without net transfer.
2. If the leaves remain separated after the rod touches the terminal and is removed, what is the likely explanation? Charge has been transferred to the electroscope.
3. Why should the sign of charge not be inferred from leaf separation alone? Both positive and negative net charge can make leaves repel each other.

2.0.30 Case study: Coulomb force as a design-scale estimate

In engineering practice, early calculations are often scale estimates rather than final designs. Suppose small insulating particles in a device acquire nanocoulomb charges. Coulomb's law can estimate whether electrostatic forces are comparable to weight, adhesion, air drag or mechanical vibration. Even if the actual geometry is more complicated than two point charges, the estimate can decide whether electrostatics deserves attention.

For example, a particle with mass $10^{-6}\,\text{kg}$ has weight about $10^{-5}\,\text{N}$. Two particles each carrying $1\,\text{nC}$ at a distance of $1\,\text{cm}$ repel with force about $9 \times 10^{-5}\,\text{N}$, almost an order of magnitude larger than the particle's weight. This does not prove that the point-charge model is exact, but it tells the engineer that charge control may matter.

Example 2.13: Design estimate: electrostatic force versus weight

A small insulating grain has mass $m = 2.0 \times 10^{-7}\,\text{kg}$ and excess charge $q = 0.50\,\text{nC}$. A second identical grain with the same sign of charge is $5.0\,\text{mm}$ away. Compare the electrostatic repulsion

with the weight of one grain.
Electrostatic force.

$$F_e = k_e \frac{q^2}{r^2} \tag{2.58}$$
$$= (8.99 \times 10^9) \frac{(0.50 \times 10^{-9})^2}{(5.0 \times 10^{-3})^2} \,\text{N} \tag{2.59}$$
$$= 9.0 \times 10^{-5} \,\text{N}. \tag{2.60}$$

Weight.

$$mg = (2.0 \times 10^{-7})(9.81) = 2.0 \times 10^{-6} \,\text{N}. \tag{2.61}$$

Comparison.

$$\frac{F_e}{mg} \simeq 45. \tag{2.62}$$

Interpretation. In this idealized estimate, electrostatic repulsion is much larger than weight. A powder-handling or contamination-control system cannot ignore charge accumulation.

2.0.31 Supplementary derivation: force components from geometry

Many students understand the magnitude of Coulomb's law but hesitate when a charge is not located on an axis. The component method is systematic. Suppose a source charge lies at displacement $\Delta \boldsymbol{r} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$ from the target to the source. The distance is $r = \sqrt{x^2 + y^2}$. The direction of attraction or repulsion is along the line connecting the charges. The unit vector along the displacement has components x/r and y/r.

If one uses the source-to-target vector convention, the displacement from source to target is $-x\hat{\boldsymbol{\imath}} - y\hat{\boldsymbol{\jmath}}$. The vector formula then automatically handles signs. If one uses magnitude and direction, one must decide separately whether the force points toward or away from the source. Both approaches are valid if used consistently.

Example 2.14: Component method for an off-axis source charge

A target charge $q_0 = +3.0\,\mu\text{C}$ is at the origin. A source charge $q_s = -2.0\,\mu\text{C}$ is at $(0.30, 0.40)$ m. Find the force on the target.
Geometry. The source is 0.50 m from the target. Because the charges are opposite in sign, the force on the target points toward the source. The unit vector from the target toward the source is

$$\hat{\boldsymbol{u}} = \frac{0.30\hat{\boldsymbol{\imath}} + 0.40\hat{\boldsymbol{\jmath}}}{0.50} = 0.60\hat{\boldsymbol{\imath}} + 0.80\hat{\boldsymbol{\jmath}}. \tag{2.63}$$

Magnitude.

$$F = k_e \frac{|q_0 q_s|}{r^2} = (8.99 \times 10^9) \frac{(3.0 \times 10^{-6})(2.0 \times 10^{-6})}{0.50^2} = 0.216 \,\text{N}. \tag{2.64}$$

Vector force.

$$\boldsymbol{F} = F\hat{\boldsymbol{u}} = (0.130\hat{\boldsymbol{\imath}} + 0.173\hat{\boldsymbol{\jmath}}) \,\text{N}. \tag{2.65}$$

2.0.32 Supplementary derivation: force on a charge near a symmetric pair

A common arrangement consists of two equal charges placed symmetrically about an axis, with a target charge on the axis. The transverse force components cancel and the axial components add.

This is the same mathematical idea that later appears in field calculations for rings, rods and disks.

Example 2.15: Symmetric pair of source charges

Two identical positive charges $+Q$ are fixed at $(0,+a)$ and $(0,-a)$. A positive target charge $+q$ is placed at $(x,0)$ with $x>0$. Find the direction and magnitude of the net force on the target.
Geometry. Each source charge is at distance

$$r=\sqrt{x^2+a^2} \tag{2.66}$$

from the target. Each repels the target. The vertical components cancel by symmetry. The horizontal components add in the $+x$ direction.
The magnitude of each force is

$$F_*=k_e\frac{qQ}{x^2+a^2}. \tag{2.67}$$

The cosine of the angle between each force and the $+x$ axis is

$$\cos\theta=\frac{x}{\sqrt{x^2+a^2}}. \tag{2.68}$$

Therefore

$$F_{\text{net,x}}=2F_*\cos\theta=2k_e\frac{qQx}{(x^2+a^2)^{3/2}}. \tag{2.69}$$

Thus

$$\boldsymbol{F}_{\text{net}}=2k_e\frac{qQx}{(x^2+a^2)^{3/2}}\hat{\boldsymbol{\imath}}. \tag{2.70}$$

Note

This symmetric-pair result foreshadows later field calculations. When many small charge elements are arranged symmetrically, perpendicular components cancel and only selected components survive. The calculation here is still force-based, but the geometry is the same.

2.0.33 Supplementary case study: when the point-charge approximation fails

The point-charge approximation requires that the size of a charged object be small compared with the distance to other objects and that the details of charge distribution not matter for the question. It fails when objects are close, extended, irregular, conducting in the presence of other charges, or polarised. In such cases Coulomb's law still applies at the microscopic level between charge elements, but the whole-object force requires integration or field methods.

Examples of point-charge failure include a charged rod attracting a nearby metal sphere, two charged plates separated by a small gap, a charged comb attracting paper, and charge distribution on a conductor near a sharp tip. These are not failures of electrostatics; they are failures of an oversimplified model. The later chapters provide the language of fields, potentials and boundary conditions needed for such systems.

Caution

Do not use the point-charge formula simply because charge is present. Ask whether the charged object is small compared with the separation and whether its charge distribution can reasonably be ignored.

2.0.34 Supplementary mini-project: validating a Coulomb-force code

A computational implementation should be tested on problems with known answers. Four useful tests are:

1. one source charge on the positive x axis;
2. two equal source charges symmetrically placed about an axis;
3. four equal charges at the corners of a square with target at the centre;
4. a configuration scaled by a known distance factor.

The first test checks direction and sign. The second and third check symmetry cancellation. The fourth checks inverse-square scaling. These tests are more useful than comparing only random numerical outputs, because they target common conceptual and programming errors.

Method

Validation tests for a direct Coulomb-force program.

- *Single positive source to the right of positive target:* force should point left.
- *Single negative source to the right of positive target:* force should point right.
- *Equal positive sources left and right:* net force should be zero at the midpoint.
- *All distances doubled:* force magnitudes should be four times smaller.
- *All charges multiplied by two:* force magnitudes should be four times larger.

2.0.35 Supplementary selected solutions for computational checks

Example 2.16: Code test: two equal sources on opposite sides

A positive target charge is at the origin. Two equal positive source charges are at $x = +a$ and $x = -a$. The source at $+a$ repels the target toward $-x$. The source at $-a$ repels the target toward $+x$. The distances and charge magnitudes are equal, so the two force magnitudes are equal. The net force is therefore zero. A code returning a nonzero value has a sign, indexing or round-off issue. In exact arithmetic with symmetric input it should return exactly zero; in floating-point arithmetic it should return a value consistent with numerical round-off.

Example 2.17: Code test: distance scaling

Suppose a configuration gives force magnitude F on a target charge. If every coordinate is multiplied by 3 and all charges are unchanged, every separation becomes three times larger. Therefore every pair-force magnitude becomes

$$F'_{ij} = \frac{1}{3^2} F_{ij} = \frac{F_{ij}}{9}. \tag{2.71}$$

The net force vector also scales by $1/9$ if the geometry is uniformly scaled about the same origin. This is a simple numerical validation test for inverse-square behaviour.

2.0.36 Supplementary conceptual synthesis

The chapter began with simple observations and ends with a modelling framework. Charge is conserved and quantized. Materials determine how charge can move or rearrange. Coulomb's law gives pairwise forces under appropriate assumptions. Newton's laws and vector addition tell how those forces combine. Grounding and induction show that system boundaries matter. Computational summation shows that the same law can be applied systematically to many-charge configurations.

This conceptual synthesis should make the next chapter feel necessary rather than abrupt. Calculating force directly between every pair of charges is possible in principle for discrete point charges, but it is not the most powerful language for extended distributions or for electromagnetic waves. The electric field provides a local description, and potential provides an energy description. Coulomb's law is the gateway to both.

2.0.37 Extended selected solutions for tutorial and self-study

The following extended solutions are included to model the level of reasoning expected in written work. They are not merely answer keys. Each solution identifies the physical model, the direction logic and the unit conversion that make the numerical result meaningful.

Example 2.18: Extended solution: force direction for four sign combinations

Two point charges lie on a horizontal line. Charge 2 is to the right of charge 1. Determine the direction of the force on charge 1 for the four sign combinations $(q_1, q_2) = (+,+)$, $(+,-)$, $(-,+)$ and $(-,-)$.
Solution. If the signs are the same, the charges repel. If charge 2 is to the right of charge 1, repulsion pushes charge 1 to the left. Therefore for $(+,+)$ and $(-,-)$, $\boldsymbol{F}_{12}$ points left. If the signs are opposite, the charges attract. Charge 1 is attracted toward charge 2, which is to the right. Therefore for $(+,-)$ and $(-,+)$, $\boldsymbol{F}_{12}$ points right.
Lesson. The direction depends on relative sign and geometry. It does not depend on whether the target charge alone is positive or negative.

Example 2.19: Extended solution: interpreting a neutral conductor near a negative rod

A negatively charged rod is held near the left side of an isolated neutral metal sphere without touching it. Describe the charge distribution on the sphere and the direction of the force on the sphere.
Solution. Mobile electrons in the metal are repelled by the negative rod. They move toward the right side of the sphere. The left side is left with a deficit of electrons and is therefore positively charged relative to neutrality. The right side has an excess of electrons and is negative. The sphere as a whole remains neutral because no charge has entered or left it.
The positive induced charge on the left side is closer to the rod than the negative induced charge on the right side. The attraction of the near positive side is stronger than the repulsion of the far negative side, so the net force on the sphere is toward the rod.
Lesson. Neutral conductors can be attracted by charged objects because induced charges are separated in space.

Example 2.20: Extended solution: grounding sequence with a positive rod

A positively charged rod is brought near a neutral metal sphere. While the rod is held nearby, the sphere is grounded. The ground connection is removed first, and then the rod is removed. What is the final sign of the sphere?
Solution. The positive rod attracts mobile electrons toward the near side of the sphere. When the sphere is grounded, electrons can flow from ground onto the sphere because they are attracted by the positive rod. Removing the ground traps this extra negative charge on the

sphere. When the rod is later removed, the excess electrons redistribute over the sphere. The final sphere is negatively charged.
Lesson. Induction charging by a positive rod leaves the conductor with negative net charge if the ground is removed before the rod.

Example 2.21: Extended solution: two-dimensional vector sum with unequal distances

A target charge $q_0 = +2.0\,\mu\text{C}$ is at the origin. A charge $q_1 = +3.0\,\mu\text{C}$ is at $(0.50, 0)$ m, and a charge $q_2 = -1.0\,\mu\text{C}$ is at $(0, 0.25)$ m. Find the net force on q_0.
Force from q_1. The charges q_0 and q_1 are both positive, so q_1 repels q_0 in the negative x direction. The magnitude is

$$F_{01} = (8.99 \times 10^9)\frac{(2.0 \times 10^{-6})(3.0 \times 10^{-6})}{(0.50)^2} = 0.216\,\text{N}. \tag{2.72}$$

Thus $\boldsymbol{F}_{01} = -0.216\hat{\boldsymbol{i}}\,\text{N}$.
Force from q_2. The charges q_0 and q_2 have opposite signs, so q_2 attracts q_0 upward. The magnitude is

$$F_{02} = (8.99 \times 10^9)\frac{(2.0 \times 10^{-6})(1.0 \times 10^{-6})}{(0.25)^2} = 0.288\,\text{N}. \tag{2.73}$$

Thus $\boldsymbol{F}_{02} = +0.288\hat{\boldsymbol{j}}\,\text{N}$.
Net force.

$$\boldsymbol{F}_{\text{net}} = (-0.216\hat{\boldsymbol{i}} + 0.288\hat{\boldsymbol{j}})\,\text{N}. \tag{2.74}$$

The magnitude is

$$F_{\text{net}} = \sqrt{0.216^2 + 0.288^2} = 0.360\,\text{N}. \tag{2.75}$$

The direction is in the second quadrant, with

$$\tan\alpha = \frac{0.288}{0.216} = 1.333, \tag{2.76}$$

so $\alpha = 53.1^\circ$ above the negative x axis.

Example 2.22: Extended solution: unknown charge from a measured force

A small bead with charge $q_1 = +4.0\,\mu\text{C}$ is separated by 0.30 m from a second bead. The force magnitude is 0.80 N and the interaction is attractive. Find the charge of the second bead.
Magnitude. From Coulomb's law,

$$|q_2| = \frac{Fr^2}{k_e|q_1|}. \tag{2.77}$$

Substitute SI values:

$$|q_2| = \frac{(0.80)(0.30)^2}{(8.99 \times 10^9)(4.0 \times 10^{-6})} = 2.0 \times 10^{-6}\,\text{C}. \tag{2.78}$$

Sign. The force is attractive and q_1 is positive, so q_2 must be negative:

$$q_2 = -2.0\,\mu\text{C}. \tag{2.79}$$

Example 2.23: Extended solution: charge sharing followed by force comparison

Two identical spheres initially carry charges $+9\,\text{nC}$ and $-1\,\text{nC}$ and are separated by $0.20\,\text{m}$. They are first far enough apart to approximate as point charges. Find the force magnitude before contact. Then they touch, separate and are again placed $0.20\,\text{m}$ apart. Find the new force magnitude and state whether the force is attractive or repulsive.

Before contact.

$$F_{\text{before}} = k_e \frac{(9\times10^{-9})(1\times10^{-9})}{0.20^2} = 2.0\times10^{-6}\,\text{N}. \tag{2.80}$$

The force is attractive because the charges have opposite signs.

After contact. The total charge is $+8\,\text{nC}$, so identical spheres share it equally:

$$q_1' = q_2' = +4\,\text{nC}. \tag{2.81}$$

The new force magnitude is

$$F_{\text{after}} = k_e \frac{(4\times10^{-9})(4\times10^{-9})}{0.20^2} = 3.6\times10^{-6}\,\text{N}. \tag{2.82}$$

The force is now repulsive because both final charges are positive.

Example 2.24: Extended solution: locating an equilibrium point between equal like charges

Two equal positive charges $+Q$ are fixed at $x=-a$ and $x=+a$. Where can a positive test charge be placed on the x axis so that the net force on it is zero?

Symmetry solution. At $x=0$, the test charge is exactly halfway between the two source charges. The left charge repels it to the right with the same magnitude as the right charge repels it to the left. The forces cancel. Therefore $x=0$ is an equilibrium point.

Uniqueness on the interval. If the test charge moves closer to the right charge, the repulsion from the right charge becomes stronger and the force no longer cancels. Similarly on the left. Along the line between the charges, the only cancellation point is the midpoint.

Example 2.25: Extended solution: square corner force

Three identical positive charges $+Q$ occupy three corners of a square of side a. A fourth identical positive charge is placed at the remaining corner. Find the magnitude and direction of the net force on the fourth charge.

Model. Let the target charge be at the origin. Put source charges at $(a,0)$, $(0,a)$ and (a,a). The charge at $(a,0)$ repels the target in the $-x$ direction with magnitude $F_* = k_eQ^2/a^2$. The charge at $(0,a)$ repels it in the $-y$ direction with the same magnitude. The diagonal charge at (a,a) is at distance $a\sqrt{2}$, so its force magnitude is $F_*/2$ and it points along $(-\hat{\boldsymbol{\imath}}-\hat{\boldsymbol{\jmath}})/\sqrt{2}$.

Components.

$$F_x = -F_* - \frac{F_*}{2\sqrt{2}}, \qquad F_y = -F_* - \frac{F_*}{2\sqrt{2}}. \tag{2.83}$$

The components are equal and negative, so the net force points along the diagonal away from the occupied square, in the direction $-\hat{\boldsymbol{\imath}}-\hat{\boldsymbol{\jmath}}$.

Magnitude.

$$F_{\text{net}} = \sqrt{2}\left(F_* + \frac{F_*}{2\sqrt{2}}\right) = \left(\sqrt{2}+\frac{1}{2}\right)\frac{k_eQ^2}{a^2}. \tag{2.84}$$

Example 2.26: Extended solution: direct computation of elementary charges

A laboratory object has charge $+25\,\mathrm{nC}$. How many electrons have been removed from it compared with neutrality?
Solution. Positive charge means a deficit of electrons. The number missing is

$$N = \frac{25 \times 10^{-9}\,\mathrm{C}}{1.602 \times 10^{-19}\,\mathrm{C}} = 1.56 \times 10^{11}. \tag{2.85}$$

Thus about 1.6×10^{11} electrons have been removed.
Scale. This is a large number of electrons but a tiny fraction of the electrons in a macroscopic object.

Example 2.27: Extended solution: proton-proton electrostatic repulsion at nuclear scale

Estimate the electrostatic repulsion between two protons separated by $1.5\,\mathrm{fm}$, where $1\,\mathrm{fm} = 10^{-15}\,\mathrm{m}$.
Solution.

$$F = k_e \frac{e^2}{r^2} = (8.99 \times 10^9) \frac{(1.602 \times 10^{-19})^2}{(1.5 \times 10^{-15})^2}\,\mathrm{N}. \tag{2.86}$$

This gives

$$F \simeq 1.0 \times 10^2\,\mathrm{N}. \tag{2.87}$$

Interpretation. A force of order $100\,\mathrm{N}$ acting on a proton is enormous. Stable nuclei require a strong attractive nuclear interaction at short distances to overcome proton-proton electrostatic repulsion. The nuclear force is not developed in this chapter.

2.0.38 Additional conceptual checkpoints for exam preparation

Guided checks

1. If two charges are both negative, do they attract because one is "more negative"? No. Same signs repel, regardless of whether the sign is positive or negative.
2. If an object has charge $+1\,\mathrm{C}$, does that mean it contains no electrons? No. It means it has a net deficit of electrons compared with its positive charge content.
3. Can the net force on a charge be zero if every individual force on it is nonzero? Yes, by vector cancellation.
4. Can an object be polarised but have zero net charge? Yes. Polarisation separates positive and negative charge without necessarily changing the total.
5. Why is the direction of $\boldsymbol{F}_{12}$ not necessarily the same as the direction of $\boldsymbol{r}_{21}$? The displacement convention and the sign of $q_1 q_2$ both matter.
6. In a charge-sharing problem, why are signs included in the average? Charge is algebraic; $+Q$ and $-Q$ can cancel.
7. Why is a grounded conductor not isolated? It can exchange charge with ground.
8. What happens to the force if one charge changes sign but magnitudes and positions remain fixed? The force magnitude is unchanged, but the direction reverses for that pair interaction.
9. Why is the point-charge approximation better at larger separations? Details of size and charge distribution become less important relative to the separation.
10. Why are field methods needed later? Direct force summation becomes inefficient or conceptually awkward for continuous or extended charge distributions.

2.0.39 Extended synthesis table

Concept	Correct interpretation	Common wrong interpretation
Neutral object	total charge is zero	object contains no charged particles
Positive charge on metal	electron deficit	mobile protons added to metal
Grounding	conducting connection to charge reservoir	symbol that magically removes charge
Induction	charge separation due to nearby charge	charge created inside object
Coulomb magnitude	non-negative force size	signed force component
Vector force	magnitude plus direction on one object	line drawn between two objects
Superposition	vector sum of forces on same target	arithmetic sum of all magnitudes
Equilibrium	net vector force zero	no interactions present
Quantization	charge occurs in elementary units	macroscopic charge must visibly jump
Point-charge model	approximation valid when size is negligible	universal formula for any charged object

2.0.40 Advanced mini-project: laboratory report structure

Students carrying out an electrostatics laboratory should be encouraged to write a report that separates observation from interpretation. A concise professional report can use the following structure.

1. **Aim:** identify which charging process or force effect is being investigated.
2. **Apparatus:** list rods, cloths, electroscope, conductors, insulating supports and grounding wire.
3. **Environmental conditions:** note humidity, visible surface contamination and repeated handling.
4. **Procedure:** describe contact, separation, grounding and observation sequence.
5. **Observations:** record attraction, repulsion, leaf separation, discharge or absence of effect.
6. **Interpretation:** explain observations using charge transfer, induction or grounding.
7. **Uncertainty and limitations:** discuss leakage, humidity, object geometry and qualitative nature of measurement.
8. **Conclusion:** state what physical idea was supported and what was not directly measured.

This structure prevents a common mistake in laboratory writing: treating an inferred charge sign as if it had been directly observed. The motion of an object is observed. Charge sign and distribution are inferred through a model.

2.0.41 Advanced closing perspective

Coulomb's law is both simple and deep. It is simple because the ideal point-charge force depends only on two charges and their separation. It is deep because it points toward nearly every later idea in electromagnetism. The $1/r^2$ dependence foreshadows flux and Gauss's law. The vector nature of the force foreshadows electric fields. The work done by or against electrostatic forces foreshadows potential. Charge storage foreshadows capacitance. Charge motion foreshadows current and magnetism. Charge conservation foreshadows circuit laws and particle reactions.

For this reason, Chapter 1 should not be rushed. Students who learn to reason carefully here will find later chapters more coherent. Students who memorize only the scalar formula will struggle when fields, potentials, circuits and magnetic forces demand precise signs, directions and assumptions.

2.0.42 Additional engineering case study: charged particles in manufacturing environments

In manufacturing environments, small particles can acquire charge through contact, friction, separation, spraying, grinding or transport through tubes. Once charged, they may adhere to surfaces, repel one another, contaminate optical components or respond strongly to nearby conductors. A full industrial model involves fluid flow, humidity, material science and surface physics. Nevertheless, Coulomb's law gives the first scale estimate: if charged particles are close together, electrostatic forces may exceed weight by large factors.

Consider a simplified production line in which insulating particles of radius much smaller than their separation acquire charges of order 0.1 to 1 nC. If two such particles approach within a millimetre, the ideal pair force can be estimated before more detailed modelling. At $q = 0.1\,\text{nC}$ and $r = 1\,\text{mm}$,

$$F \sim (9 \times 10^9)\frac{(10^{-10})^2}{(10^{-3})^2}\,\text{N} = 9 \times 10^{-5}\,\text{N}. \tag{2.88}$$

This is already comparable to or larger than the weights of many small particles. The estimate tells the engineer that charge neutralization, humidity control or grounded surfaces may be relevant.

Example 2.28: Manufacturing estimate: charged dust near a grounded tool

A dust particle of mass $5.0 \times 10^{-9}\,\text{kg}$ carries charge $-0.20\,\text{nC}$. A nearby small charged contaminant carries $+0.30\,\text{nC}$ and is 2.0 mm away. Estimate the electrostatic attraction and compare it with the dust particle's weight.

Electrostatic attraction.

$$F_e = k_e\frac{|q_1 q_2|}{r^2} \tag{2.89}$$

$$= (8.99 \times 10^9)\frac{(0.20 \times 10^{-9})(0.30 \times 10^{-9})}{(2.0 \times 10^{-3})^2}\,\text{N} \tag{2.90}$$

$$= 1.35 \times 10^{-4}\,\text{N}. \tag{2.91}$$

Weight.

$$mg = (5.0 \times 10^{-9})(9.81) = 4.9 \times 10^{-8}\,\text{N}. \tag{2.92}$$

Interpretation. In this simplified pair model, the electrostatic force is thousands of times larger than the particle weight. Real geometry, image charges, air flow and surface adhesion may matter, but the scale estimate clearly warns that electrostatics cannot be ignored.

2.0.43 Additional mathematical note: algebraic charge versus absolute value

The magnitude form of Coulomb's law uses $|q_1 q_2|$ because magnitudes are non-negative. The vector form uses $q_1 q_2$ with its sign because the sign carries direction information relative to the chosen displacement vector. Mixing these two forms is a common source of errors. A safe rule is:

- if calculating a force magnitude, use absolute values and decide direction from a diagram;

- if calculating a vector directly, use signed charges and a clearly defined displacement vector.

Never use signed charges in the magnitude formula and then also reverse the direction by hand; that double-counts the sign.

Example 2.29: Sign bookkeeping in magnitude and vector methods

Let $q_1 = +1.0\,\mu\text{C}$ be at $x = 0$ and $q_2 = -2.0\,\mu\text{C}$ be at $x = 0.10\,\text{m}$. Find the force on q_1.
Magnitude method. The magnitude is

$$F = k_e \frac{|q_1 q_2|}{r^2} = (8.99 \times 10^9)\frac{(1.0 \times 10^{-6})(2.0 \times 10^{-6})}{0.10^2} = 1.80\,\text{N}. \tag{2.93}$$

The charges are opposite, so q_1 is attracted toward q_2, i.e. in the $+x$ direction:

$$\boldsymbol{F}_{12} = +1.80\hat{\boldsymbol{\imath}}\,\text{N}. \tag{2.94}$$

Vector method. Here $\boldsymbol{r}_1 = 0$ and $\boldsymbol{r}_2 = 0.10\hat{\boldsymbol{\imath}}\,\text{m}$, so

$$\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2 = -0.10\hat{\boldsymbol{\imath}}\,\text{m}. \tag{2.95}$$

The product $q_1 q_2$ is negative. In the coordinate formula, a negative charge product times a negative displacement direction gives a positive x force. The result is again

$$\boldsymbol{F}_{12} = +1.80\hat{\boldsymbol{\imath}}\,\text{N}. \tag{2.96}$$

2.0.44 Additional student-writing checklist

Before submitting a solution to a Coulomb-law problem, a student should check:

1. Have all microcoulombs, nanocoulombs and centimetres been converted to SI units?
2. Is the answer a vector if the problem asks for a force rather than a force magnitude?
3. Is the object experiencing the force clearly identified?
4. Are force directions consistent with attraction for opposite signs and repulsion for like signs?
5. If more than one force acts, were components added rather than magnitudes?
6. Are the assumptions of the point-charge model stated or at least reasonable?
7. Does the numerical size make sense compared with the charges and separations?
8. Is charge conservation applied with signs rather than magnitudes?
9. In a grounding problem, is the system isolated or connected to a reservoir at each step?
10. Is the final interpretation written in words, not only as a number?

Take-home message

A correct electrostatic calculation is not only arithmetic. It is a chain of modelling decisions: charge bookkeeping, material behaviour, geometry, vector direction, unit conversion and physical interpretation.

2.0.45 Final review tutorial: connecting representations

A single electrostatic situation can be represented in several ways: a verbal description, a physical sketch, a charge-sign diagram, a free-body diagram, a component table, an algebraic equation and a numerical answer. Students often jump directly from the verbal description to an equation. That shortcut is risky because Coulomb's law is geometrical and vectorial. The intermediate representations carry the physical meaning.

For example, the statement "a positive charge is above a negative charge" should immediately produce a sketch. The sketch should show which charge is the target. The free-body diagram should show whether the target is attracted upward or downward. Only then should the scalar distance and the charge magnitudes be inserted into Coulomb's law. The final answer should return to words: "the force on the positive charge is downward, toward the negative charge."

Representation	What it checks
Verbal statement	identifies objects, charge signs and requested quantity
Sketch	displays geometry and relative positions
Free-body diagram	ensures forces are drawn on the correct object
Component table	separates x and y contributions systematically
Algebraic formula	encodes Coulomb magnitude or vector law
Unit conversion	makes the SI calculation meaningful
Numerical answer	gives size, direction and physical interpretation

Example 2.30: Representation chain for a simple off-axis problem

A charge $q_0 = +1.0\,\mu\text{C}$ is at the origin and a source charge $q_s = +1.0\,\mu\text{C}$ is at $(0.30, 0.40)\,\text{m}$. Give the representation chain leading to the force on q_0.

Sketch and direction. The source is up and to the right of the target. The charges have the same sign, so the source repels the target down and to the left.

Distance and unit vector. The distance is $r = 0.50\,\text{m}$. The unit vector from the target toward the source is $0.60\hat{\boldsymbol{\imath}} + 0.80\hat{\boldsymbol{\jmath}}$. Since the force on the target is away from the source, its direction is $-0.60\hat{\boldsymbol{\imath}} - 0.80\hat{\boldsymbol{\jmath}}$.

Magnitude.

$$F = (8.99 \times 10^9)\frac{(1.0 \times 10^{-6})(1.0 \times 10^{-6})}{0.50^2} = 0.0360\,\text{N}. \tag{2.97}$$

Vector answer.

$$\boldsymbol{F} = 0.0360(-0.60\hat{\boldsymbol{\imath}} - 0.80\hat{\boldsymbol{\jmath}})\,\text{N} = (-0.0216\hat{\boldsymbol{\imath}} - 0.0288\hat{\boldsymbol{\jmath}})\,\text{N}. \tag{2.98}$$

2.0.46 Final conceptual review before moving to fields

The most important preparation for the next chapter is to separate three ideas that are often mixed together. First, a source charge is the charge responsible for an interaction. Second, a target or test charge is the charge whose force or motion is being discussed. Third, the space between them will soon be described by a field. In this chapter the field is not yet needed for calculation, but the distinction between source and target is already essential.

If students do not distinguish source and target charges, the definition of electric field in the next chapter becomes confusing. The field at a point is not the force on the source charge. It is a property of space associated with what a positive test charge would experience there. The force-based notation $\boldsymbol{F}_{12}$ in this chapter is therefore not just bookkeeping; it prepares the conceptual discipline needed for field notation.

Guided checks

1. Which charge is the target in $\boldsymbol{F}_{12}$? Charge 1.
2. Which charge is the source in $\boldsymbol{F}_{12}$? Charge 2.
3. If charge 1 is removed, does charge 2 still exist? Yes. Later we shall say that charge 2 still creates an electric field in the surrounding space.
4. Why is this distinction useful? It prevents confusing the object producing an interaction with the object experiencing a force.

2.0.47 Instructor and self-study checklist for Chapter 1

The following checklist can be used at the end of a tutorial, before a quiz, or while revising the chapter independently. It is deliberately written as a diagnostic checklist rather than as a list of formulas. A student who can answer these questions in words is ready to use the equations responsibly.

Summary

Conceptual readiness checklist.

1. I can explain why a neutral object still contains charged particles.
2. I can identify which charges are mobile in an ordinary metal.
3. I can distinguish charging by contact from charging by induction.
4. I can explain what grounding does physically.
5. I can use charge conservation with algebraic signs.
6. I can convert elementary-charge counts into coulombs and back.
7. I can state when the point-charge approximation is reasonable.
8. I can compute a Coulomb-force magnitude with SI units.
9. I can determine attraction or repulsion from charge signs.
10. I can draw a force vector on the object that experiences the force.
11. I can add electrostatic forces as vectors, not only as magnitudes.
12. I can recognise when symmetry makes a net force vanish.
13. I can explain why electrostatic forces dominate gravity at atomic scales.
14. I can describe why direct force language motivates the electric-field language of the next chapter.

If any item on this checklist feels uncertain, the best repair is not to memorize more formulas. The best repair is to draw two or three simple charge configurations and explain, in words and arrows, which forces act on which object and why. Electrostatics becomes much easier when the diagram, the signs and the vector notation tell the same story.

2.1 Bridge to Chapter 2

Coulomb's law tells us the force between point charges, but it leaves a conceptual question: how does one charge influence another without contact? The next chapter introduces the electric field, a vector field in space. The field language allows us to say that source charges create an electric field, and a charge placed in that field experiences a force. This is not merely a new notation. It is the beginning of the local field picture that leads to Maxwell's equations and electromagnetic waves.

Take-home message

The central result of this chapter is not only a formula. It is a disciplined modelling pattern: identify charge, state the material response, use conservation and quantization when needed, compute Coulomb forces with correct directions, and add them as vectors. This pattern will become the foundation for electric fields in the next chapter.

Chapter A

Notation dictionary

Notation	Meaning
q, Q	electric charge
e	elementary charge
n	integer charge number in $q = ne$
ε_0	vacuum permittivity
k_e	Coulomb constant, $1/(4\pi\varepsilon_0)$
$\boldsymbol{F}_{12}$	force on charge 1 due to charge 2
$\boldsymbol{r}_i$	position vector of charge i
$\boldsymbol{r}_{12}$	displacement from charge 2 to charge 1
r_{12}	magnitude of $\boldsymbol{r}_{12}$
$\hat{\boldsymbol{r}}_{12}$	unit vector from charge 2 to charge 1
I	electric current, introduced as charge flow rate
G	Newtonian gravitational constant

Chapter B

Compact formula sheet

Formula summary

Charge

$$q = ne, \qquad e = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C}, \tag{B.1}$$

$$Q_{\text{total}} = \sum_i q_i = \text{constant} \quad \text{(isolated system)}. \tag{B.2}$$

Current bridge

$$I = \frac{\mathrm{d}q}{\mathrm{d}t}. \tag{B.3}$$

Coulomb law

$$F = k_e \frac{|q_1 q_2|}{r^2}, \qquad k_e = \frac{1}{4\pi\varepsilon_0}, \tag{B.4}$$

$$\boldsymbol{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2} \hat{\boldsymbol{r}}_{12}, \tag{B.5}$$

$$\boldsymbol{F}_{12} = k_e q_1 q_2 \frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^3}. \tag{B.6}$$

Superposition and equilibrium

$$\boldsymbol{F}_{\text{net}} = \sum_i \boldsymbol{F}_i, \qquad \sum_i \boldsymbol{F}_i = \boldsymbol{0}. \tag{B.7}$$

Identical conducting spheres

$$q_A' = q_B' = \frac{q_A + q_B}{2}. \tag{B.8}$$

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