

Chapter 24

Particles, Fundamental  
Interactions, and the Big Bang  
First Edition

**Hamzeh Khanpour**

[hamzeh.khanpour@agh.edu.pl](mailto:hamzeh.khanpour@agh.edu.pl)

Faculty of Physics and Applied Computer Science

**AGH University of Krakow**

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Quantum Physics, and Nuclear Physics

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# Preface

This chapter closes *Physics for Technical and Engineering Students - Volume II* by connecting microscopic particles to the hot early universe. The main text develops quarks and leptons, reactions and conservation rules, accelerators and detectors, and then expansion, thermal history, the cosmic microwave background, dark matter, and dark energy. Its quantitative tools are relativistic energy and momentum, basic quantum ideas, and the thermal and electromagnetic physics already studied in Physics 2.

The emphasis is on what can be measured and what a model allows us to infer. A track is measured; its momentum is inferred using the magnetic field. A redshift is measured; a cosmological distance generally requires a model. Current particle data and cosmological status are checked against the 2026 Particle Data Group reviews. Approximate numbers in examples are teaching inputs, with their assumptions stated. Explanations, exercises, and diagrams are developed for this chapter; the references guide scope and scientific checks.

## The Guideline

Before calculating, identify the objects, frame, conserved quantities, units, and approximation. After calculating, check a limiting case and ask which measurement could test the result. A model that passes a bookkeeping check still needs a physical interaction and an experimentally testable prediction.

## Pedagogical boundary

The main chapter assumes introductory calculus, special relativity, and basic quantum and nuclear physics. The optional enrichment modules introduce four-vectors, detector performance, statistical inference, neutrino mixing, and Friedmann cosmology. Detailed quantum field theory and general relativity are future courses, rather than prerequisites for this chapter.

# Conventions and notation

We use SI units when they make physical scales transparent, and electronvolts for microscopic energies. The elementary positive charge is denoted by  $e$ ; an electron has electric charge  $-e$ . Particle masses are frequently quoted through their rest-energy equivalent  $mc^2$  in MeV or GeV. Unless explicitly stated otherwise, factors of  $c$  are retained in the main chapter. Natural units with  $c = \hbar = 1$  are introduced only in the advanced enrichment section.

The relativistic energy and three-momentum of a particle are denoted by  $E$  and  $\mathbf{p}$ . The four-momentum is introduced later as  $p^\mu = (E/c, \mathbf{p})$ . A particle and its antiparticle are distinguished by a bar where convenient, for example  $p$  and  $\bar{p}$ ,  $q$  and  $\bar{q}$ , or  $\nu$  and  $\bar{\nu}$ . The symbols  $u, d, s, c, b, t$  denote the six quark flavours. Charged leptons are  $e^-, \mu^-, \tau^-$  and their antiparticles; the corresponding neutrinos are  $\nu_e, \nu_\mu, \nu_\tau$ .

For cosmology,  $a(t)$  denotes the scale factor,  $z$  the cosmological redshift,  $H(t) = \dot{a}/a$  the Hubble parameter, and  $H_0$  its present value. Distances may be expressed in parsecs, with  $1 \text{ pc} \simeq 3.086 \times 10^{16} \text{ m}$  and  $1 \text{ Mpc} = 10^6 \text{ pc}$ . The low-redshift form of the Hubble relation is written  $v \simeq H_0 d$  and must not be extrapolated naively to arbitrarily large redshift.

| Symbol                | Meaning                                 | Typical unit              |
|-----------------------|-----------------------------------------|---------------------------|
| $E$                   | relativistic energy                     | eV, MeV, GeV, J           |
| $\mathbf{p}$          | relativistic momentum                   | MeV/ $c$ , GeV/ $c$       |
| $m$                   | rest mass                               | kg or MeV/ $c^2$          |
| $Q$                   | electric charge                         | $e$ or C                  |
| $B$                   | baryon number in reaction bookkeeping   | dimensionless             |
| $L$                   | lepton number in elementary bookkeeping | dimensionless             |
| $\sigma$              | cross section                           | barn, pb, fb              |
| $\mathcal{L}$         | instantaneous luminosity                | area $^{-1}$ time $^{-1}$ |
| $\int \mathcal{L} dt$ | integrated luminosity                   | fb $^{-1}$                |
| $z$                   | cosmological redshift                   | dimensionless             |
| $a(t)$                | cosmological scale factor               | dimensionless             |
| $H_0$                 | present Hubble parameter                | km s $^{-1}$ Mpc $^{-1}$  |
| $T$                   | temperature                             | K                         |

## Note

The word *mass* in particle physics can be a source of unit confusion. A statement such as “the proton mass is about 938 MeV/ $c^2$ ” is equivalent to saying that the proton rest energy is about 938 MeV. The distinction is maintained explicitly in this chapter whenever it helps dimensional reasoning.

# Chapter 24

## Particles, Fundamental Interactions, and the Big Bang

### 24.1 Motivation and roadmap

A century of experiments has repeatedly changed the meaning of the word *elementary*. Atoms were once treated as indivisible. The discovery of the electron and the atomic nucleus showed that they are composite. Protons and neutrons were then treated as basic nuclear constituents, but deep-inelastic scattering and the successful quark model revealed substructure inside nucleons. The present Standard Model describes ordinary matter using quarks and leptons, together with quantum fields responsible for their interactions. No experimentally established internal structure of quarks or leptons is known at present.

The same physics reappears when we look backward in cosmic time. A hot early universe naturally contained energetic radiation and particle-antiparticle pairs. As expansion proceeded, the average energy per particle decreased, unstable species disappeared, nuclei formed, electrons eventually combined with nuclei into neutral atoms, and radiation decoupled from matter. The cooled remnant of that radiation is observed today as the cosmic microwave background (CMB).

Figure 24.1 summarizes the chapter. The left side develops the microscopic language of particles, interactions, reactions, accelerators, and detectors. The right side develops the macroscopic language of expansion, redshift, thermal history, and cosmological evidence. The two meet in the early universe, where temperature sets the characteristic microscopic energy scale.

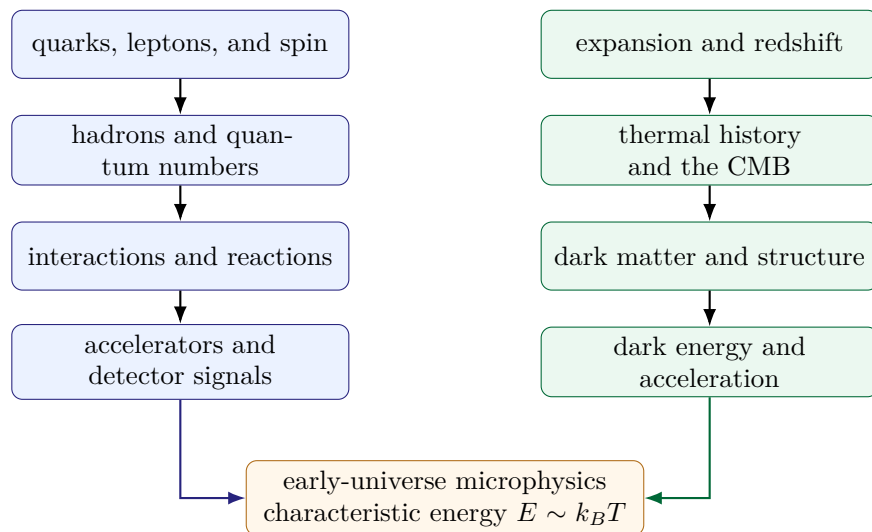


**Figure 24.1:** Chapter roadmap. The two branches use different measurements and meet through the microphysics of the hot early universe.

**Learning goals**

After studying the main chapter, the student should be able to:

- distinguish leptons, quarks, hadrons, baryons, mesons, gauge bosons, and antiparticles;
- use quark electric charges to construct the charges of simple hadrons;
- apply electric-charge, energy-momentum, baryon-number, and lepton-number bookkeeping to elementary reactions, while recognizing the status and limitations of the latter two rules;
- explain qualitatively the strong, electromagnetic, weak, and gravitational interactions and identify the Standard Model gauge bosons;
- use  $E^2 = p^2c^2 + m^2c^4$  and simple threshold reasoning for particle production;
- explain why colliders can be more energy-efficient than fixed-target experiments for producing massive states;
- relate track curvature in a magnetic field to charge sign and momentum at introductory level;
- explain what calorimeters, tracking detectors, and muon systems measure;
- distinguish expansion of space from motion through a fixed background and use the low-redshift relation  $v \simeq H_0 d$  appropriately;
- interpret cosmological redshift, the CMB, galaxy rotation evidence, and accelerated expansion at a qualitative-to-semiquantitative level;
- outline the thermal history from an early hot plasma to nuclei, atoms, and large-scale structure;
- distinguish ordinary matter, dark matter, and dark energy, including the observational nature of the evidence;
- carry out simple computational checks of particle reactions, tracks, and Hubble diagrams.

**Key message**

Modern particle physics is not mainly a catalogue of exotic particles. Its central achievement is a compact organizing framework: matter fields, interaction fields, symmetries, conservation principles, and a small set of parameters. Modern cosmology is similarly not just a chronology; it is a quantitative framework that connects expansion, thermal physics, gravity, and observations.

**24.2 From atoms to elementary constituents**

A useful starting point is to organize matter by length scale. Ordinary solids and liquids are built from atoms at scales near  $10^{-10}$  m. Atomic nuclei occupy scales of order  $10^{-15}$  m. Protons and neutrons are not pointlike on that scale: they have internal structure associated with quarks and gluons. Electrons, quarks, and the other Standard Model fermions are treated as elementary in current experiments, meaning that no smaller constituent structure has been experimentally resolved.

The word *elementary* is therefore operational. It does not mean that physics has proved a particle to be structureless at every imaginable distance. It means that within the resolution reached by experiments, the particle behaves as a pointlike excitation with specified quantum numbers. Increasing collision energy improves access to shorter distance scales, roughly reflecting the de Broglie relation  $\lambda \sim h/p$ .

The Standard Model contains two broad classes of elementary fermions: *quarks* and *leptons*. They appear in three generations. Each generation contains two quark flavours and two leptons. The first generation forms almost all stable ordinary matter: up and down quarks build protons and

neutrons, while electrons surround nuclei. Heavier generations are produced in energetic processes and then decay.

| Generation | Quarks | Charged lepton | Neutrino   |
|------------|--------|----------------|------------|
| 1          | $u, d$ | $e^-$          | $\nu_e$    |
| 2          | $c, s$ | $\mu^-$        | $\nu_\mu$  |
| 3          | $t, b$ | $\tau^-$       | $\nu_\tau$ |

### Guided checks

1. Why is a proton no longer called elementary in the modern classification?
2. Why does “no observed substructure” not logically mean “proved to have no substructure at any scale”?
3. Which first-generation particles are needed to describe electrically neutral hydrogen at the constituent level?

## Spin and the matter–mediator distinction

All elementary quarks and leptons have spin  $\hbar/2$  and obey Fermi–Dirac statistics. Two identical fermions cannot occupy the same complete quantum state: the Pauli exclusion principle helps explain atomic structure and the stability of ordinary matter. Gauge bosons have spin  $\hbar$ , and the Higgs boson has spin zero; these particles obey Bose–Einstein statistics. Here the dimensionless terms *spin-1/2*, *spin-1*, and *spin-0* mean angular momentum in units of  $\hbar$ .

The classification also applies to composites: conventional baryons have half-integer spin and are fermions, while mesons have integer spin and are bosons. Spin statistics classifies allowed quantum states; it does not mean that a fermion is a spinning classical ball or that every boson is an interaction mediator.

## 24.3 Relativistic energy, momentum, and microscopic units

Particle reactions often convert kinetic energy into rest energy and rest energy back into kinetic energy. Newtonian expressions therefore become inadequate once particle speeds approach  $c$  or once particles are created and destroyed. The central one-particle relation is

$$E^2 = p^2 c^2 + m^2 c^4. \quad (24.1)$$

For a particle at rest,  $p = 0$  and  $E = mc^2$ . For a photon,  $m = 0$  and therefore  $E = pc$ . These are not separate laws but limiting cases of the same relativistic relation.

Microscopic energies are conveniently measured in electronvolts:

$$1 \text{ eV} = 1.602176634 \times 10^{-19} \text{ J}. \quad (24.2)$$

Typical atomic transitions are of order electronvolts, nuclear energies of order MeV, and high-energy particle reactions commonly involve GeV or TeV. The conversion hierarchy is

$$1 \text{ GeV} = 10^3 \text{ MeV} = 10^9 \text{ eV}, \quad 1 \text{ TeV} = 10^3 \text{ GeV}. \quad (24.3)$$

### Method

For relativistic one-particle problems, resist the temptation to compute a speed first. If  $E$ ,  $p$ , and  $m$  are given or requested, start from Eq. (24.1). It is algebraically direct and avoids unnecessary Lorentz-factor manipulations.

**Example 24.1: Relativistic momentum from total energy**

A proton has total energy  $E = 5.00$  GeV. Take its rest energy as  $m_p c^2 = 0.938$  GeV. Find the magnitude of its momentum.

From Eq. (24.1),

$$pc = \sqrt{E^2 - m_p^2 c^4} = \sqrt{(5.00)^2 - (0.938)^2} \text{ GeV} \simeq 4.91 \text{ GeV}.$$

Therefore

$$p \simeq 4.91 \text{ GeV}/c.$$

The result is close to  $E/c$  because the proton is highly relativistic: its total energy is much larger than its rest energy.

**Caution**

Do not write  $E = pc + mc^2$ . Relativistic energy does not split into a linear sum of  $pc$  and  $mc^2$ . The correct relation is quadratic, Eq. (24.1). Only in appropriate limits can it be approximated by simpler expressions.

**24.4 Leptons and neutrinos**

Leptons are elementary fermions that do not carry the strong-interaction color charge. The charged leptons  $e^-$ ,  $\mu^-$ , and  $\tau^-$  all have electric charge  $-e$ . Their antiparticles have charge  $+e$ . The three neutrinos are electrically neutral and interact through the weak interaction and gravity; their tiny masses also make neutrino kinematics and oscillations especially interesting.

The electron is stable as far as current observations show. The muon and tau are heavier and unstable. A typical muon decay is

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu. \quad (24.4)$$

This process illustrates an important feature of weak interactions: one particle species may transform into another while electric charge and energy-momentum are conserved.

The statement that there are three neutrino flavours needs a quantum-mechanical qualification. Neutrinos created in weak interactions have a flavour label, but propagation is governed by states of definite mass. Because the flavour states are superpositions of mass eigenstates, the probability of detecting a different flavour can vary with distance and energy. Oscillation measurements establish two independent mass-squared splittings and thus at least two nonzero neutrino masses. They do not by themselves determine the lightest mass or establish violation of total lepton number [5].

**Key message**

A neutrino flavour label such as  $\nu_\mu$  describes how the neutrino participates in a weak interaction. It is not, in general, the same as a state of definite mass. This distinction is a concrete example of quantum superposition appearing in particle physics.

**Example 24.2: Bookkeeping in muon decay**

Check electric charge and simple lepton-family bookkeeping for

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu.$$

Electric charge gives  $-1$  on the left and  $-1 + 0 + 0 = -1$  on the right. For the traditional

family bookkeeping used at introductory level, assign  $L_\mu = +1$  to  $\mu^-$  and  $\nu_\mu$ , and  $L_e = +1$  to  $e^-$  and  $\nu_e$ , with opposite signs for antiparticles. Initially  $(L_e, L_\mu) = (0, 1)$ . Finally the electron contributes  $(1, 0)$ , the electron antineutrino  $(-1, 0)$ , and the muon neutrino  $(0, 1)$ , again giving  $(0, 1)$ .

This bookkeeping is useful for ordinary weak decays, but individual flavour lepton numbers are not exact symmetries of nature because neutrino oscillations change flavour during propagation. The example is therefore a useful reaction check, not a statement that each flavour number is a fundamental exact law in all circumstances.

### Guided checks

1. Which interaction distinguishes quarks from leptons most directly?
2. Why does neutrino oscillation imply that a flavour eigenstate cannot also be a single mass eigenstate if the masses differ?
3. In Eq. (24.4), which quantities must be conserved event by event even if individual flavour labels are not exact global symmetries?

## 24.5 Quarks, flavour, and color

The six quark flavours are  $u, d, s, c, b, t$ . Their electric charges occur in two values:

$$Q_u = Q_c = Q_t = +\frac{2}{3}e, \quad Q_d = Q_s = Q_b = -\frac{1}{3}e. \quad (24.5)$$

Antiquarks carry the opposite electric charges. Quarks also carry *color charge*, the charge associated with the strong interaction in quantum chromodynamics (QCD). The label “color” has no connection with visible optical color; it is a name for a three-valued internal quantum degree of freedom.

An isolated free quark is not observed under ordinary conditions. At accessible low energies, quarks and gluons appear in color-neutral composite states called hadrons. The phenomenon that prevents isolated color charge from appearing as a free low-energy particle is called *confinement*. When energetic quarks or gluons are produced in a collider, they do not emerge as individual tracks. Instead they initiate cascades that hadronize into collimated sprays of particles called jets.

Figure 24.2 gives a compact visual classification. It is intentionally a particle-content chart, not a complete statement of the Standard Model Lagrangian.

### Caution

The conventional red/green/blue language of QCD color is a mathematical label. It is not electromagnetic color, not a wavelength of light, and not a pigment property. Likewise, a color-neutral hadron need not contain visually colored constituents.

## 24.6 Hadrons: baryons, mesons, and quark bookkeeping

Hadrons are strongly interacting composite particles. The simplest and most familiar classes are baryons and mesons. In the basic quark model, a baryon contains three quarks and an antibaryon three antiquarks, while a meson contains a quark-antiquark pair. Modern QCD also allows more complicated color-neutral combinations, and experimentally established exotic hadrons include tetraquark and pentaquark candidates. For this course, the three-quark and quark-antiquark patterns provide the essential foundation.

| Generation 1                             | Generation 2                               | Generation 3                                | bosons                                                         |
|------------------------------------------|--------------------------------------------|---------------------------------------------|----------------------------------------------------------------|
| $\begin{matrix} u \\ +2e/3 \end{matrix}$ | $\begin{matrix} c \\ +2e/3 \end{matrix}$   | $\begin{matrix} t \\ +2e/3 \end{matrix}$    | $\begin{matrix} g \\ \text{strong} \end{matrix}$               |
| $\begin{matrix} d \\ -e/3 \end{matrix}$  | $\begin{matrix} s \\ -e/3 \end{matrix}$    | $\begin{matrix} b \\ -e/3 \end{matrix}$     | $\begin{matrix} \gamma \\ \text{electromagnetic} \end{matrix}$ |
| $\begin{matrix} e^- \\ -e \end{matrix}$  | $\begin{matrix} \mu^- \\ -e \end{matrix}$  | $\begin{matrix} \tau^- \\ -e \end{matrix}$  | $\begin{matrix} W^\pm, Z \\ \text{weak} \end{matrix}$          |
| $\begin{matrix} \nu_e \\ 0 \end{matrix}$ | $\begin{matrix} \nu_\mu \\ 0 \end{matrix}$ | $\begin{matrix} \nu_\tau \\ 0 \end{matrix}$ | $\begin{matrix} H \\ \text{Higgs boson} \end{matrix}$          |

**Figure 24.2:** Compact Standard Model particle-content chart. Quarks and leptons have spin 1/2; the eight gluons, photon, and  $W$  and  $Z$  have spin 1. Antiparticles are omitted from this chart. The Higgs boson is a scalar excitation of the Higgs field. Gravity is not included in the Standard Model.

The proton and neutron have quark contents

$$p = (uud), \quad n = (udd). \quad (24.6)$$

Using Eq. (24.5),

$$Q_p = \frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +e, \quad (24.7)$$

$$Q_n = \frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0. \quad (24.8)$$

The neutron is electrically neutral, but it is not empty of charged constituents. This fact already hints why a neutron can possess internal electromagnetic structure despite zero net charge.

Common pions have

$$\pi^+ = (u\bar{d}), \quad \pi^- = (d\bar{u}), \quad (24.9)$$

while the neutral pion is a quantum superposition involving  $u\bar{u}$  and  $d\bar{d}$  components. The details of flavour mixing are beyond the main chapter, but the charged pion examples are enough to practice constituent charge addition.

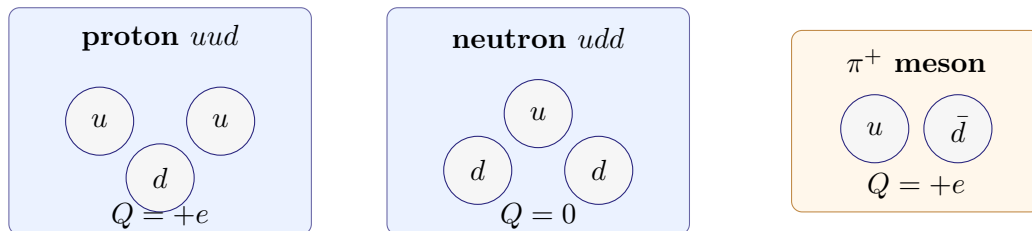


**Figure 24.3:** Constituent charge bookkeeping for familiar hadrons. Dots record valence flavour and charge, with no fixed positions or flavour-colour association implied. Sea quarks and gluons are not drawn.

#### Example 24.3: Proton and neutron charges from quark content

Use the quark charges to verify the proton and neutron charges.

For  $p = (uud)$ ,

$$Q_p = 2 \left( +\frac{2}{3}e \right) + \left( -\frac{1}{3}e \right) = +e.$$

For  $n = (udd)$ ,

$$Q_n = \left( +\frac{2}{3}e \right) + 2 \left( -\frac{1}{3}e \right) = 0.$$

The calculation is simple, but the interpretation is important: integer hadron charges emerge from fractionally charged quark constituents.

#### Example 24.4: Charge of a kaon from its quark content

A  $K^+$  meson has quark content  $u\bar{s}$ . The strange quark has charge  $-e/3$ , so the antistrange quark has  $+e/3$ . Therefore

$$Q_{K^+} = Q_u + Q_{\bar{s}} = \frac{2}{3}e + \frac{1}{3}e = +e.$$

This is a useful reminder that antiparticles reverse additive quantum numbers such as electric charge.

#### Guided checks

1. What is the net electric charge of a hypothetical baryon  $udd$ ? Which familiar particle has this composition?
2. Why is the statement “a meson contains two quarks” imprecise?
3. Does a neutron’s zero net charge imply that every constituent is electrically neutral?

### Strangeness: a useful flavour quantum number

Besides electric charge and baryon number, a reaction may conserve quark flavour. Strong and electromagnetic interactions conserve the net number of each quark flavour. A traditional and useful label is

$$S = N_{\bar{s}} - N_s. \quad (24.10)$$

A strange quark has  $S = -1$ , an antistrange quark  $S = +1$ , and the other quarks have  $S = 0$ . Thus  $K^+ = (u\bar{s})$  and  $K^0 = (d\bar{s})$  have  $S = +1$ , whereas  $\Lambda^0 = (uds)$  has  $S = -1$ . Weak interactions can change strangeness. This explains why strange particles can be produced rapidly in strong reactions yet decay much more slowly by weak processes.

#### Example 24.5: Associated production and a weak strange decay

For  $\pi^- + p \rightarrow K^0 + \Lambda^0$ , the initial totals are  $Q = 0$ ,  $B = 1$ , and  $S = 0$ . The final totals are also  $Q = 0$ ,  $B = 1$ , and  $S = (+1) + (-1) = 0$ . At quark level, creating an  $s\bar{s}$  pair preserves net flavour. The reaction can proceed through the strong interaction if the invariant energy exceeds the final mass threshold.

For  $K^+ \rightarrow \mu^+ + \nu_\mu$ , charge and total lepton bookkeeping are consistent, but strangeness changes from  $+1$  to zero. The decay is weak. The quark assignments give a selection rule, not the numerical rate; matrix elements and phase space still determine the probability.

## 24.7 Antiparticles, annihilation, and pair production

Every Standard Model matter particle has an antiparticle. Particle and antiparticle have the same mass and spin, while additive charges reverse sign. The electron antiparticle is the positron  $e^+$ . The proton antiparticle is the antiproton  $\bar{p}$ . A neutrino has a corresponding antineutrino, although the deeper question of whether neutrinos are Dirac or Majorana particles lies beyond this course.

Particle-antiparticle annihilation does not mean that energy disappears. It means that the initial rest energy and kinetic energy are converted into allowed final-state particles while energy, momentum, angular momentum, electric charge, and other applicable quantum numbers remain consistent. Conversely, pair production converts energy into particle rest energy and kinetic energy.

A basic threshold condition is

$$E_{\text{available}} \geq \sum_i m_i c^2, \quad (24.11)$$

but this statement must be used with care. Momentum conservation can require additional kinetic energy in the final state. The most efficient frame for threshold reasoning is the center-of-momentum frame, where the total three-momentum vanishes.

### Example 24.6: Threshold for muon-pair production in an electron-positron collider

Consider

$$e^- + e^+ \rightarrow \mu^- + \mu^+.$$

In the center-of-momentum frame, the minimum collision energy occurs when the outgoing muons have arbitrarily small momentum. Neglecting the electron rest energies relative to the beam energy, the threshold center-of-mass energy is therefore

$$\sqrt{s_{\text{th}}} = 2m_\mu c^2.$$

With  $m_\mu c^2 \simeq 105.7 \text{ MeV}$ ,

$$\sqrt{s_{\text{th}}} \simeq 211.4 \text{ MeV}.$$

The result is a center-of-mass energy requirement. It is not automatically equal to the energy of one beam unless the beam configuration is specified.

### Caution

A high-energy photon cannot create an electron-positron pair in completely empty space by itself while conserving both energy and momentum. A nearby nucleus, another photon, or some other participant is required to take recoil momentum. Threshold questions are always energy-and-momentum questions, not energy-only questions.

## 24.8 Conservation laws and reaction bookkeeping

Conservation laws are among the fastest tools for testing whether a proposed reaction can occur. Some are exact within the physical framework being used: electric charge and energy-momentum conservation are indispensable. Angular momentum must also be conserved. Other familiar particle-physics bookkeeping rules have a more nuanced status. Baryon number and lepton number are extraordinarily useful at introductory level and are conserved in all ordinary reactions observed so far in the ways relevant here, but they are not all fundamental exact gauge charges of the Standard Model. Individual lepton-flavour numbers are explicitly not conserved during neutrino oscillations.

For elementary bookkeeping, assign

$$B = +\frac{1}{3} \quad \text{to a quark,} \quad B = -\frac{1}{3} \quad \text{to an antiquark.} \quad (24.12)$$

A conventional baryon such as the proton then has  $B = +1$ ; an antibaryon has  $B = -1$ ; a meson has  $B = 0$ . Similarly, one often assigns total lepton number  $L = +1$  to a lepton and  $L = -1$  to an antilepton for simple reactions.

The neutron beta decay

$$n \rightarrow p + e^- + \bar{\nu}_e \quad (24.13)$$

passes the basic checks. Electric charge is  $0 = +1 - 1 + 0$ . Baryon number is  $1 = 1 + 0 + 0$ . Total lepton number is  $0 = 0 + 1 - 1$ . At quark level, one of the neutron's down quarks changes flavour through a weak interaction, schematically  $d \rightarrow u + W^{*-}$ , followed by  $W^{*-} \rightarrow e^- + \bar{\nu}_e$ . The star denotes a virtual internal  $W$ : the small beta-decay energy cannot produce a real  $W$  boson. Four-momentum is conserved throughout.

### Example 24.7: Testing a proposed decay

Could a free proton decay as

$$p \rightarrow e^+ + \pi^0$$

within the ordinary Standard Model processes studied here?

Electric charge is conserved:  $+1 = (+1) + 0$ . Energy conservation alone does not forbid the reaction because the proton is more massive than the electron plus neutral pion. However, the initial baryon number is  $B = 1$ , whereas the positron and pion each have  $B = 0$ . The reaction would violate baryon number by one unit.

No proton decay has been established experimentally. Many theories beyond the Standard Model predict baryon-number-violating proton decay at extremely long lifetimes, so such a process is scientifically important as a search channel. For introductory reaction bookkeeping, however, it is classified as forbidden by the usual baryon-number rule.

### Method

A robust reaction checklist is:

1. electric charge;
2. total energy and momentum;
3. angular momentum and, when relevant, parity or other quantum numbers;
4. baryon and lepton bookkeeping;
5. whether the proposed interaction can actually connect the initial and final particle types.

Passing the first four checks does not guarantee that a reaction has a large probability; it only removes obvious impossibilities.

### Guided checks

For each proposed process, identify the first obvious problem, if any:

1.  $n \rightarrow p + e^-$  with no neutrino;
2.  $\mu^- \rightarrow e^- + \gamma$ ;
3.  $e^- + e^+ \rightarrow \gamma + \gamma$ ;
4.  $p \rightarrow n + \pi^+$  for a free proton at rest.

Do not rely on a single bookkeeping number: energy-momentum conservation and interaction structure matter too.

## 24.9 The fundamental interactions

At the level of established fundamental physics, four interactions are distinguished: strong, electromagnetic, weak, and gravitational. The Standard Model gives a quantum-field description of the first three. General relativity describes gravity classically as spacetime geometry; a complete experimentally established quantum theory of gravity is not part of the Standard Model.

| Interaction     | Acts on / associated charge | Standard Model mediator          | Introductory physical role                              |
|-----------------|-----------------------------|----------------------------------|---------------------------------------------------------|
| Strong          | color charge                | eight gluons $g$                 | binds quarks into hadrons; residual nuclear force       |
| Electromagnetic | electric charge             | photon $\gamma$                  | atoms, molecules, electricity, magnetism, light         |
| Weak            | electroweak quantum numbers | $W^\pm, Z$                       | beta decay, neutrino interactions, flavour change       |
| Gravity         | energy-momentum             | not a Standard Model gauge boson | dominant long-range interaction for astronomical masses |

Comparing interactions by a single universal “strength number” can be misleading because the effective importance depends on energy scale, distance, particle species, and the observable. A safer introductory statement is qualitative: the strong interaction dominates hadronic binding at nuclear scales; electromagnetism is long-range and dominates atomic and molecular structure; the weak interaction enables processes such as beta decay; gravity is extraordinarily weak between individual elementary particles but accumulates coherently for macroscopic astronomical bodies.

The language of *mediator particles* is shorthand for a quantum-field description. In perturbative calculations, interactions are represented through terms involving gauge fields, and diagrams may contain internal lines often called virtual particles. These should not be imagined as classical little balls secretly travelling along measurable trajectories between particles.

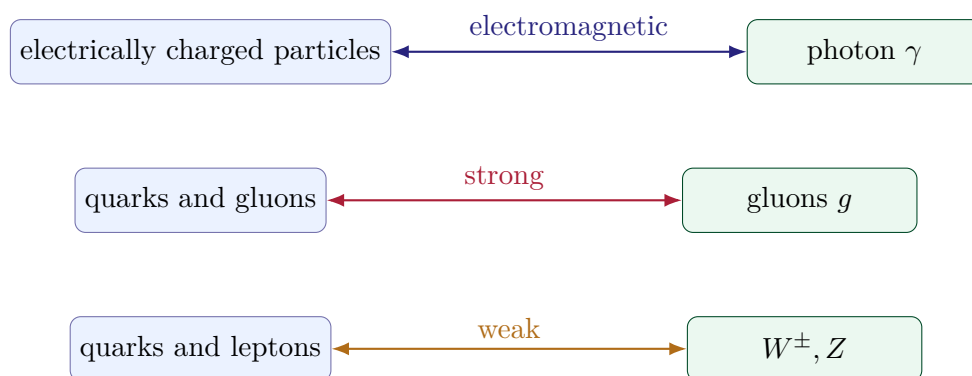


**Figure 24.4:** Qualitative interaction map for the three Standard Model gauge interactions. The arrows summarize coupling structure; they are not literal trajectories of mediator particles.

### Caution

The often-used phrase “particles exchange a force carrier” is a useful mnemonic but not a complete physical picture. Virtual internal lines in a perturbative calculation are not directly detected particles, and a Feynman diagram is an element of a calculation, not a microscopic

movie.

### Guided checks

1. Why can a neutrino participate in the weak interaction but not ordinary electromagnetic scattering at tree level?
2. Why does gravity dominate planetary motion even though it is negligible in most laboratory particle collisions?
3. What distinguishes the role of the photon from the role of the Higgs boson in the Standard Model?

## 24.10 The Higgs field and the origin of particle masses

The Standard Model contains a scalar field, the Higgs field, whose nonzero vacuum value is essential to electroweak symmetry breaking. Through their couplings to this field, the  $W$  and  $Z$  bosons acquire mass while the photon remains massless. The charged-fermion masses are generated through Yukawa couplings to the Higgs field. Neutrinos are massless in the minimal Standard Model; their observed mixing and mass splittings require an extension. The particle observed near 125 GeV in 2012 is interpreted as the Higgs boson, a quantum excitation of this field.

A common popular statement says that the Higgs field “gives everything its mass.” That is too broad. The rest masses of elementary charged leptons and quarks are related to their Higgs couplings, and the electroweak gauge-boson masses arise from the Higgs mechanism. But most of the mass of ordinary protons and neutrons is not the sum of the light-quark mass contributions. It is predominantly associated with the strongly interacting QCD system: quark and gluon dynamics and the energy stored in the confined hadron. Therefore the mass of an everyday object is, to a large extent, a manifestation of strong-interaction energy through  $E = mc^2$ .

At introductory level we do not derive the Higgs potential or electroweak gauge structure. The conceptual points are sufficient:

- the Higgs field is present throughout space in the Standard Model vacuum;
- the Higgs boson is not the field itself, but an excitation of it;
- the couplings of particles to the Higgs sector are connected to their masses;
- the Higgs mechanism does not explain the numerical pattern of all fermion masses and mixing parameters; those values are inputs to the Standard Model.

### Key message

The Higgs mechanism solves a structural problem: how the electroweak theory can contain massive  $W$  and  $Z$  bosons and massive fermions while retaining the gauge-theory framework. It does not explain why the electron, muon, tau, and quarks have the particular masses observed in nature.

## 24.11 Feynman diagrams as a calculation language

Particle reactions are often represented using Feynman diagrams. A diagram specifies external particles, interaction vertices, and internal propagators. In a full quantum-field calculation, each allowed diagram contributes a complex amplitude. Probabilities arise only after amplitudes are

combined and squared in the appropriate way. Thus a diagram is more like a term in an algebraic expansion than a photograph of a unique history.

Consider the electromagnetic process

$$e^- + e^+ \rightarrow \mu^- + \mu^+. \quad (24.14)$$

At leading order one contribution proceeds through a virtual photon. The schematic drawing in Fig. 24.5 is useful for checking electric charge and identifying the interaction, but it should not be overinterpreted as a classical spacetime trajectory.

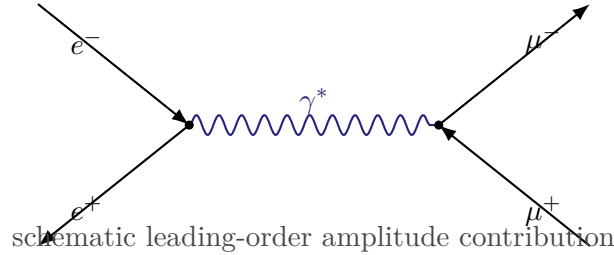


**Figure 24.5:** A schematic Feynman diagram for  $e^-e^+ \rightarrow \mu^-\mu^+$  through a virtual photon. It represents a quantum-amplitude contribution. Fermion arrows indicate fermion-number flow; the positron arrows do not indicate motion backward along a detected track.

The same external reaction may receive several quantum contributions. At sufficiently high energy, for example,  $Z$ -boson exchange can contribute to  $e^-e^+ \rightarrow \mu^-\mu^+$ . Interference between amplitudes matters. This is one reason why particle physics cannot be reduced to a billiard-ball picture.

#### Pedagogical boundary

This chapter uses Feynman diagrams only as qualitative bookkeeping. We do not introduce Feynman rules, propagator integrals, loop diagrams, renormalization, gauge fixing, or perturbative cross-section calculations. Those belong to quantum field theory and advanced particle-physics courses.

## 24.12 Particle production and threshold energy

Creating a new massive particle requires sufficient invariant collision energy. The phrase “beam energy” by itself is not enough, because two beams with the same laboratory energy can have very different available center-of-momentum energies depending on geometry and target configuration.

For a system with total energy  $E_{\text{tot}}$  and total momentum  $\mathbf{P}_{\text{tot}}$ , the invariant mass energy satisfies

$$E_{\text{CM}}^2 = E_{\text{tot}}^2 - P_{\text{tot}}^2 c^2. \quad (24.15)$$

In the center-of-momentum frame,  $\mathbf{P}_{\text{tot}} = 0$ , so all of the system energy is center-of-mass energy. At the threshold for producing a specified final state, the final particles have no relative kinetic energy in the center-of-momentum frame, so

$$E_{\text{CM,th}} = \sum_i m_i c^2. \quad (24.16)$$

This compact statement is one of the most useful principles in introductory collider physics.

A fixed-target experiment is different. If a fast projectile hits a target particle initially at rest, the initial total momentum is not zero. Momentum conservation requires the final state to carry that momentum, so not all projectile energy can become rest mass. Colliders arrange two opposing beams so that their momenta cancel approximately in the laboratory, making the laboratory nearly the center-of-momentum frame.



some energy must remain as bulk momentum, a larger fraction available for new rest mass

**Figure 24.6:** Why colliders are efficient for producing massive states. Opposing beam momenta largely cancel, maximizing center-of-mass energy for a given beam-energy scale.

### Example 24.8: Fixed-target versus collider scaling

Suppose a proton of total energy  $E$  hits a proton at rest. Let the proton rest energy be  $mc^2$ . The invariant energy of the two-proton system is

$$s \equiv E_{\text{tot}}^2 - c^2 |\mathbf{P}_{\text{tot}}|^2 = (E + mc^2)^2 - p^2 c^2.$$

Substituting  $E^2 - p^2 c^2 = m^2 c^4$  gives

$$s = 2m^2 c^4 + 2mc^2 E.$$

At  $E \gg mc^2$ ,

$$\sqrt{s} \approx \sqrt{2mc^2 E}.$$

Thus center-of-mass energy grows only as  $\sqrt{E}$  in a fixed-target configuration.

For two equal, ultrarelativistic counter-propagating proton beams of energy  $E_b$ , by contrast,

$$\sqrt{s} = 2E_b.$$

The scaling is linear. This is the central energy-efficiency advantage of colliders.

### Guided checks

1. Why is the sum of beam energies not always equal to the energy available for producing rest mass?
2. At threshold in the center-of-momentum frame, do final particles necessarily have zero momentum in the laboratory frame?
3. Which configuration is preferable if the goal is maximum invariant collision energy for a given accelerator energy: fixed target or symmetric collider?

## 24.13 Unstable particles, lifetime, decay width, and decay length

Most known particles are unstable. If an ensemble of identical unstable particles has a constant decay probability per unit proper time, the surviving number follows the exponential law already encountered in radioactive decay:

$$N(t) = N_0 e^{-t/\tau}, \quad (24.17)$$

where  $t$  is elapsed proper time and  $\tau$  is the mean proper lifetime. The half-life is  $t_{1/2} = \tau \ln 2$ . In particle physics it is common to describe instability using a decay width  $\Gamma$ , related approximately by

$$\tau = \frac{\hbar}{\Gamma}. \quad (24.18)$$

A short lifetime corresponds to a broad energy width; a long lifetime corresponds to a narrow width.

Relativistic time dilation strongly affects laboratory decay distances. If the proper lifetime is  $\tau$ , then the average laboratory lifetime is  $\gamma\tau$ , and the mean flight distance is

$$\ell = \beta\gamma c\tau. \quad (24.19)$$

This relation is central to detector design. Some unstable particles leave measurable displaced decay vertices because their boosted decay lengths are millimetres, centimetres, or more.

#### Example 24.9: Mean flight distance of a relativistic muon

A muon has proper mean lifetime  $\tau \simeq 2.20 \mu\text{s}$  and travels with  $\gamma = 10$ . Its speed is close to  $c$ ; more precisely  $\beta = \sqrt{1 - 1/\gamma^2} \simeq 0.995$ . The mean laboratory flight distance is

$$\ell = \beta\gamma c\tau \simeq (0.995)(10)(3.00 \times 10^8 \text{ m/s})(2.20 \times 10^{-6} \text{ s}).$$

Thus

$$\ell \simeq 6.57 \times 10^3 \text{ m}.$$

Without relativistic time dilation the corresponding distance would be about 660 m. The result illustrates why cosmic-ray muons created high in the atmosphere can reach detectors near Earth's surface in substantial numbers.

#### Note

The lifetime-width relation is quantum mechanical. It is not caused by an experimental apparatus making a poor energy measurement. An unstable state does not have a perfectly sharp energy in the same sense as an eternal stationary state.

### From a mean decay length to detector acceptance

For a monoenergetic sample travelling at constant speed, the laboratory survival probability is

$$P_{\text{survive}}(t_{\text{lab}}) = \exp\left(-\frac{t_{\text{lab}}}{\gamma\tau}\right), \quad P_{\text{survive}}(L) = \exp(-L/\ell). \quad (24.20)$$

The mean length  $\ell$  is an average of an exponential distribution; individual particles do not all decay at that distance. For a path segment from  $L_{\text{min}}$  to  $L_{\text{max}}$ , the probability of decay inside it is

$$P_{\text{segment}} = \exp(-L_{\text{min}}/\ell) - \exp(-L_{\text{max}}/\ell). \quad (24.21)$$

Angular coverage, reconstruction, and trigger efficiency are additional factors. A realistic beam with a momentum distribution requires an average over boosts.

#### Example 24.10: A displaced-decay design estimate

A neutral unstable species has proper decay length  $c\tau = 2.0 \text{ cm}$  and  $\beta\gamma = 5.0$ . For trajectories that cross a detector region between path lengths 2.0 and 20 cm,

$$\ell = 10 \text{ cm}, \quad P_{\text{segment}} = e^{-0.2} - e^{-2.0} \simeq 0.683.$$

About 68% of these trajectories decay in the region before applying detection efficiency. Increasing the boost lengthens flight distances but can move decays beyond the instrument.

## 24.14 Accelerators: creating controlled high-energy collisions

An accelerator controls charged particles using electric and magnetic fields. Electric fields do work and change particle energy. Magnetic fields bend trajectories and focus beams. These ideas

connect directly to earlier Physics 2 chapters: Lorentz force, circular motion in magnetic fields, radio-frequency electromagnetic fields, resonance, and vacuum technology all reappear in accelerator engineering.

A circular accelerator must satisfy the transverse magnetic-force condition

$$|q|vB = \frac{\gamma m v^2}{R}, \quad (24.22)$$

which may be written in momentum form as

$$p = |q|BR \quad (24.23)$$

for motion perpendicular to a uniform magnetic field. In practical high-energy units,

$$p[\text{GeV}/c] \simeq 0.2998 |q/e| B[\text{T}] R[\text{m}]. \quad (24.24)$$

The same relation underlies momentum measurements in tracking detectors.

Linear accelerators avoid synchrotron-radiation losses associated with curved trajectories but require long structures to reach very high energies. Circular proton colliders can reuse accelerating structures over many turns because heavy particles radiate much less synchrotron energy than electrons of the same energy. Electron-positron colliders, however, provide especially clean initial states. Accelerator design is therefore a trade-off involving particle mass, energy, luminosity, ring size, magnet strength, radiation, beam stability, and scientific goals.

### Key message

The accelerator and the detector solve different problems. The accelerator prepares a controlled initial state with high energy and high collision rate. The detector reconstructs the final state. Particle physics comes from comparing those measured final states with theoretical predictions.

## 24.15 Particle detectors and event reconstruction

A modern collider detector does not “see” quarks, weak bosons, or most unstable particles directly. It records signals left by sufficiently long-lived final-state particles as they pass through matter. Reconstruction software then combines those signals into tracks, energy clusters, vertices, jets, and missing-momentum observables.

A typical general-purpose detector uses concentric subsystems:

- an inner tracking detector measures charged-particle trajectories and interaction vertices;
- an electromagnetic calorimeter measures showers initiated mainly by electrons and photons;
- a hadronic calorimeter measures hadronic showers and contributes to jet-energy reconstruction;
- an outer muon system identifies penetrating muons;
- a magnetic field bends charged tracks, allowing charge-sign and momentum measurement.

Neutrinos usually leave the detector without interacting. Their presence is inferred indirectly from an imbalance of reconstructed transverse momentum when the initial transverse momentum is known to be approximately zero.

Track curvature is one of the most direct applications of the Lorentz force. A positively and a negatively charged particle curve in opposite directions for the same magnetic field. For transverse motion,

$$p_T[\text{GeV}/c] \simeq 0.3 |q/e| B[\text{T}] R[\text{m}]. \quad (24.25)$$

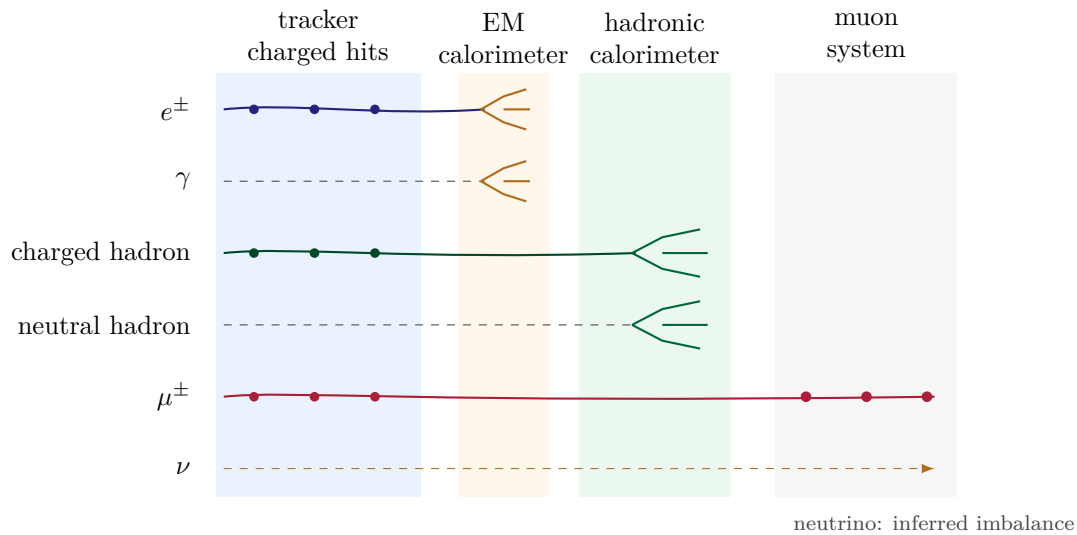


**Figure 24.7:** Typical detector responses, arranged as a layer schematic. Electrons and photons shower predominantly in the electromagnetic calorimeter; hadronic showers extend into the hadronic calorimeter. Muons penetrate and leave outer hits. Dashed neutral-particle paths are explanatory guides, not measured tracks; neutrinos are inferred from momentum imbalance. Actual field and layer arrangements vary between experiments.

Large-radius tracks correspond to high momentum and are therefore harder to measure precisely from curvature alone.

#### Example 24.11: Momentum from track curvature

A singly charged particle follows a circular arc of radius  $R = 2.0 \text{ m}$  in a  $3.8 \text{ T}$  solenoidal magnetic field. Estimate its transverse momentum.

$$p_T \simeq 0.3BR = (0.3)(3.8)(2.0) \text{ GeV}/c \simeq 2.28 \text{ GeV}/c.$$

If the curvature direction is reversed while  $B$  is unchanged, the sign of the particle charge is reversed. The magnitude of the curvature alone gives  $|q|/p_T$ , not a unique particle identity.

#### Method

Detector identification is multi-observable reasoning. A curved track tells you charge sign and momentum-to-charge ratio. Calorimeter response tells you deposited energy and shower type. Penetration into an outer system suggests a muon. A displaced vertex suggests a long-lived parent. No single subsystem should be treated as an all-purpose particle label.

#### Guided checks

1. Why can a tracker measure the sign of a charged particle but not directly distinguish a pion from a muon at the same momentum?
2. Why are photons visible in an electromagnetic calorimeter despite leaving no charged track before conversion or showering?
3. Why is a neutrino often represented by missing transverse momentum rather than a visible detector object?

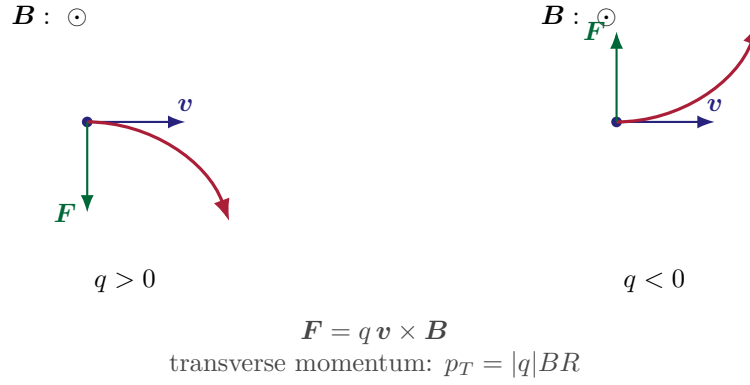


**Figure 24.8:** Charge sign from curvature for an initial velocity to the right and magnetic field out of the page ( $\odot$ ). A positive particle bends down and a negative particle bends up. The field does no work: it changes direction, with constant speed in the ideal uniform-field model.

## 24.16 Jets, missing momentum, and indirect evidence

Because isolated quarks and gluons are not observed, a high-energy quark or gluon produced in a collision manifests as a jet: a collimated spray of hadrons. A jet is therefore not a single fundamental particle. It is a reconstructed experimental object intended to approximate the direction and energy flow associated with an underlying parton.

The same theme – indirect inference – appears throughout particle physics. A  $Z$  boson is often identified as a resonance in the invariant mass of its decay products. A Higgs boson is reconstructed statistically from specific final states. Neutrinos are inferred from missing momentum. Heavy-flavour hadrons can be tagged using displaced decay vertices. The scientific object is therefore often a pattern across many detector measurements rather than a visible “dot” corresponding to the original unstable particle.

At a hadron collider, the longitudinal momentum of the initial interacting partons is not known event by event. The transverse momentum before the collision, however, is approximately balanced. This motivates the missing-transverse-momentum vector

$$\mathbf{p}_T^{\text{miss}} = - \sum_{\text{reconstructed}} \mathbf{p}_{T,i}. \quad (24.26)$$

A significant imbalance can indicate neutrinos or other invisible particles, but detector inefficiency and energy mismeasurement can also create apparent missing momentum. Therefore missing momentum is an observable requiring calibration and background understanding, not automatic proof of a new invisible particle.

### Caution

“Missing energy” is often used informally, but at a hadron collider the most robust event-level quantity is usually missing *transverse* momentum. The unknown longitudinal parton momenta prevent a simple event-by-event longitudinal energy balance of the same kind.

## 24.17 Neutrino oscillations: a qualitative quantum bridge

Neutrino oscillation is one of the clearest examples showing that the simplest minimal Standard Model picture must be extended to account for observed neutrino masses. A neutrino produced together with an electron, muon, or tau is associated with a flavour state  $|\nu_\alpha\rangle$ . The propagation states  $|\nu_i\rangle$  have definite masses  $m_i$ . Quantum mechanics relates the two bases through a mixing matrix.

In a two-flavour approximation, the conversion probability may be written

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta) \sin^2\left(1.267 \frac{\Delta m^2 [\text{eV}^2/c^4] L [\text{km}]}{E [\text{GeV}]}\right), \quad (24.27)$$

Here  $\theta$  is a mixing angle and  $\Delta m^2$  is a squared-mass difference in  $\text{eV}^2/c^4$ . The familiar shorthand  $\Delta m^2$  in  $\text{eV}^2$  assumes  $c = 1$ . The numerical factor 1.267 uses the displayed units; the sine argument is dimensionless. For ultrarelativistic coherent propagation in vacuum, the same phase with explicit constants is  $\Delta m^2 c^3 L / (4\hbar E)$ . The formula shows why long baselines and suitable energies are important: oscillations depend on the ratio  $L/E$ .

Equation (24.27) is an approximation. Real three-flavour oscillations involve three mixing angles, a CP-violating phase, two independent mass-squared splittings, and matter effects when neutrinos travel through dense material. Nevertheless, the two-flavour form captures the essential interference mechanism.

#### Example 24.12: Oscillation length scale

Take  $\Delta m^2 = 2.5 \times 10^{-3} \text{eV}^2/c^4$  and neutrino energy  $E = 1.0 \text{ GeV}$ . The first maximum of the second sine in Eq. (24.27) occurs when its argument is  $\pi/2$ :

$$1.267 \frac{\Delta m^2 L}{E} = \frac{\pi}{2}.$$

Therefore

$$L \simeq \frac{\pi E}{2(1.267)\Delta m^2} \simeq \frac{3.1416}{2(1.267)(2.5 \times 10^{-3})} \text{ km} \simeq 496 \text{ km}.$$

The result explains why accelerator-neutrino experiments often use baselines of hundreds of kilometres for GeV-scale neutrinos.

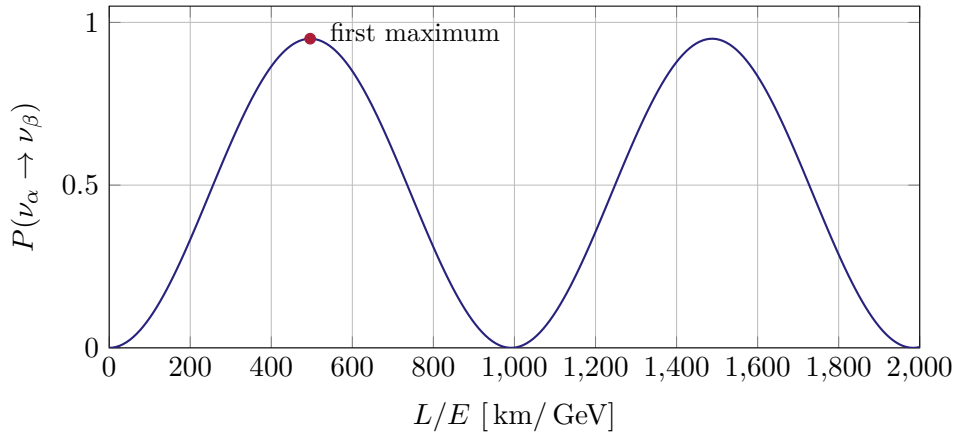


**Figure 24.9:** An ideal two-flavour vacuum probability with  $\sin^2(2\theta) = 0.95$  and  $\Delta m^2 = 2.5 \times 10^{-3} \text{eV}^2/c^4$ . The first maximum is at  $L/E \simeq 496 \text{ km/GeV}$ . This theoretical curve includes no detector smearing, matter effect, or observational data.

## 24.18 From particle physics to the early universe

Cosmology provides a natural laboratory for particle physics because temperature sets a characteristic energy scale. Roughly, a thermal system at temperature  $T$  has characteristic microscopic energies of order  $k_B T$ . At sufficiently high temperature, the thermal bath can produce particle-antiparticle pairs whose rest energies would be inaccessible in a cold environment. As the universe expands and cools, species become nonrelativistic, annihilate, decay, or fall out of thermal equilibrium.

This connection should be used qualitatively in an introductory course. The early universe was not a conventional gas in a box, and a complete treatment requires quantum field theory in an expanding spacetime, kinetic equations, and general relativity. Yet the simple correspondence

$$E_{\text{thermal}} \sim k_B T \quad (24.28)$$

already provides a powerful scale estimate.

For example,  $k_B T = 1 \text{ MeV}$  corresponds to a temperature of order  $10^{10} \text{ K}$ . Such temperatures are far above stellar-surface temperatures but occurred naturally in the first seconds of cosmic history. Nuclear reaction rates and weak interactions at those epochs helped determine the later abundance of light nuclei.

### Key message

High-energy accelerators and the early universe probe the same microscopic laws in very different ways. Accelerators provide controlled, repeatable, localized collisions. Cosmology provides access to enormous volumes, long times, and extreme early conditions, but usually through indirect fossil evidence.

## 24.19 Expansion of the universe and cosmological redshift

On sufficiently large scales, observations are well described by an expanding universe. The most useful conceptual picture is not galaxies flying through a pre-existing static void from one central explosion point. Rather, distances between comoving locations grow as the cosmic scale factor  $a(t)$  increases.

If a light wave is emitted when the scale factor is  $a_{\text{em}}$  and observed when it is  $a_0$ , its cosmological redshift satisfies

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \frac{a_0}{a_{\text{em}}}. \quad (24.29)$$

For very small redshift, one can associate a recession speed  $v \ll c$  with

$$z \simeq \frac{v}{c}, \quad (24.30)$$

and combine this with the low-redshift Hubble relation

$$v \simeq H_0 d. \quad (24.31)$$

At larger redshift, the relation between redshift, distance, and recession rate depends on the cosmological model and cannot be replaced by the simple nonrelativistic Doppler formula.

The numerical value of  $H_0$  is an active observational issue. Measurements based on the early-universe CMB within a standard cosmological model tend to give values near the high-60  $\text{km s}^{-1} \text{ Mpc}^{-1}$  range, while several local distance-ladder determinations give values in the low-to-mid 70  $\text{km s}^{-1} \text{ Mpc}^{-1}$  range. The discrepancy is commonly called the Hubble tension. For worked examples in this chapter, a clearly stated representative value will be used; it should not be interpreted as resolving the tension.

### Example 24.13: Low-redshift Hubble-law estimate

Use a representative value  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$  to estimate the low-redshift recession speed associated with a galaxy at  $d = 100 \text{ Mpc}$ .

$$v \simeq H_0 d = (70)(100) \text{ km/s} = 7000 \text{ km/s}.$$

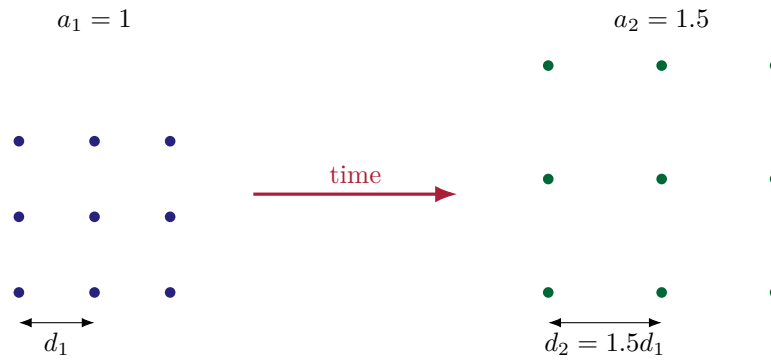


**Figure 24.10:** The same nine comoving positions at two scale factors. All separations grow by the same factor, with no preferred centre. The dot sizes are unchanged: gravitationally bound galaxies and local measuring instruments do not expand with this idealized grid.

The corresponding nonrelativistic redshift estimate is

$$z \simeq \frac{v}{c} \simeq \frac{7000}{3.00 \times 10^5} \simeq 0.023.$$

Because  $z \ll 1$ , the low-redshift approximations are reasonable for this order-of-magnitude calculation.

### Caution

Do not interpret  $v = H_0 d$  as a universal special-relativistic velocity law valid at all distances. In cosmology, recession is tied to the changing spacetime scale factor. At high redshift, a cosmological distance-redshift relation must replace the elementary low- $z$  approximation.

### Guided checks

1. Why does cosmic expansion not require a central point inside the universe from which all galaxies were launched?
2. If  $z = 1$ , is the elementary approximation  $v = zc$  reliable? Why or why not?
3. What observation does a measured redshift provide directly: a wavelength ratio or a model-independent distance?

## 24.20 The cosmic microwave background

The cosmic microwave background is thermal radiation filling the sky with an almost perfect blackbody spectrum at a temperature near 2.7 K. It is a central line of evidence for a hot early universe. When the universe was much hotter, electrons and nuclei formed an ionized plasma. Photons scattered frequently from charged particles, so radiation and matter were tightly coupled. After expansion and cooling allowed stable neutral atoms to form efficiently, the photon mean free path increased dramatically. Those photons have since propagated through the expanding universe and have been redshifted into the microwave region.

The CMB is extremely uniform but not perfectly uniform. Tiny angular temperature anisotropies encode information about early density perturbations, the contents and geometry of the universe, and the acoustic physics of the pre-recombination plasma. Modern cosmological parameter inference uses the statistical pattern of those anisotropies, not simply the mean temperature.

A rough photon energy scale follows from  $E \sim k_B T$ . At  $T = 2.725$  K,

$$k_B T \simeq (8.617 \times 10^{-5} \text{ eV/K})(2.725 \text{ K}) \simeq 2.35 \times 10^{-4} \text{ eV}. \quad (24.32)$$

The blackbody spectrum spans a distribution of photon energies;  $k_B T$  is a characteristic scale, not the energy of every photon.

**Example 24.14: Characteristic wavelength scale of CMB photons**

Estimate a wavelength corresponding to a photon energy  $E = 6.0 \times 10^{-4} \text{ eV}$ , representative of the microwave scale near the peak region of the CMB spectrum. Using  $E = hc/\lambda$  and  $hc \simeq 1.240 \text{ eV } \mu\text{m}$ ,

$$\lambda \simeq \frac{1.240 \text{ eV } \mu\text{m}}{6.0 \times 10^{-4} \text{ eV}} \simeq 2.07 \times 10^3 \mu\text{m} \simeq 2.1 \text{ mm}.$$

This millimetre scale is why the relic background is observed with microwave and millimetre-wave instrumentation.

### A spectrum is different from a sky map

The mean CMB spectrum is close to a blackbody. The spectral radiance per unit frequency is

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/k_B T) - 1}. \quad (24.33)$$

It has units  $\text{W m}^{-2} \text{sr}^{-1} \text{Hz}^{-1}$ . Anisotropy maps instead show the tiny directional departures from a mean temperature. The measured mean temperature is about 2.725 K; the more precise value and its measurement are reviewed in Ref. [9].

Setting  $x = h\nu/(k_B T)$  gives a frequency-spectrum maximum at the positive solution of  $3(1 - e^{-x}) = x$ , namely  $x \simeq 2.82144$ . At  $T = 2.725$  K,  $\nu_{\text{max}} \simeq 160.2 \text{ GHz}$ . This corresponds to  $c/\nu_{\text{max}} \simeq 1.87 \text{ mm}$ . The wavelength spectrum has a different peak:  $\lambda_{\text{max}} T \simeq 2.898 \times 10^{-3} \text{ m K}$ , or  $\lambda_{\text{max}} \simeq 1.06 \text{ mm}$  at this temperature. There is no contradiction:  $B_\nu |d\nu| = B_\lambda |d\lambda|$ , and the Jacobian changes the location of a spectral-density maximum.

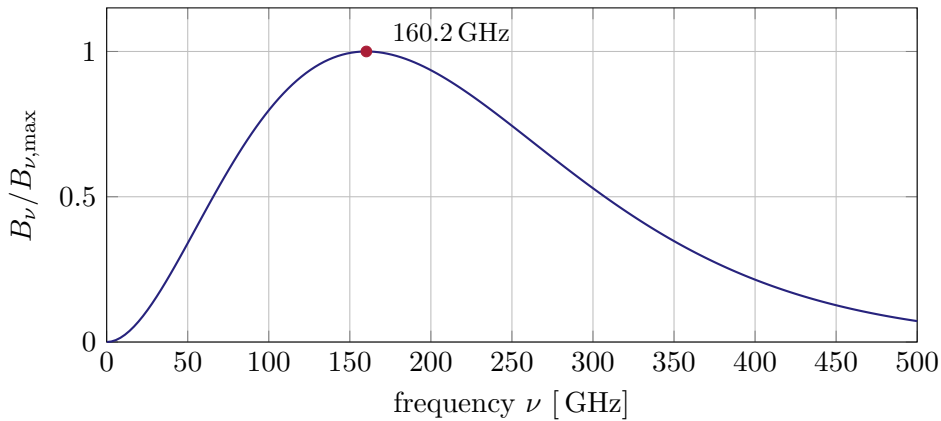


**Figure 24.11:** Normalized Planck frequency spectrum at the illustrative temperature  $T = 2.725$  K. The curve is computed from Eq. (24.33); it is not a measured CMB spectrum. The frequency-density and wavelength-density peaks occur at different locations because the two spectral variables have a nonconstant Jacobian.

## 24.21 A qualitative thermal history of the universe

The phrase “the Big Bang” should be used carefully. In modern cosmology it refers to the early hot, dense, expanding state described by the cosmological model and its extrapolation, not to an

ordinary explosion at a location in space. The theory is strongly supported for a long sequence of epochs after the earliest moments, but it does not by itself provide a complete experimentally established description of an absolute beginning or of physics at arbitrarily high energy.

A simplified timeline is pedagogically useful:

1. **Very early hot phase.** The characteristic energies were far above nuclear scales. A complete treatment of the earliest times may require physics beyond currently tested theories.
2. **Quark-hadron transition region.** As the universe cooled to QCD scales, quarks and gluons became confined predominantly into hadrons.
3. **Weak-interaction freeze-out and neutrino decoupling.** Reaction rates eventually became too slow compared with cosmic expansion to maintain full equilibrium for some species.
4. **Big-bang nucleosynthesis.** During the first minutes, light nuclei such as deuterium, helium, and traces of other light isotopes formed. Their abundances provide a quantitative test of the hot Big Bang model.
5. **Recombination and photon decoupling.** Hundreds of thousands of years later, electrons and nuclei combined efficiently into neutral atoms; photons then propagated freely, producing the CMB observed today.
6. **Structure formation.** Small primordial density differences grew under gravity into stars, galaxies, and larger structures. Dark matter plays a central role in this growth.

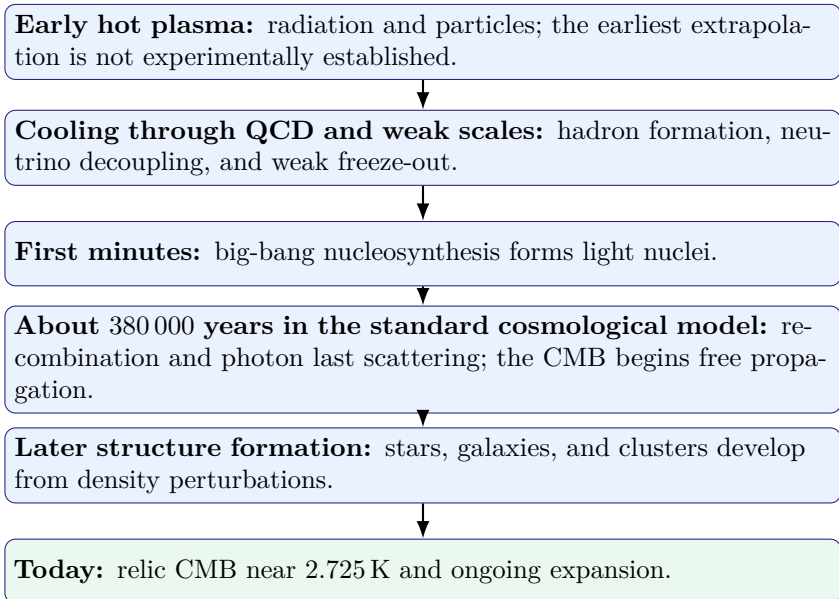


**Figure 24.12:** Chronological order of selected thermal and structural transitions. Spacing is schematic, not proportional to time. Recombination and last scattering have finite duration, and epoch estimates depend on the model.

### Caution

The early-universe timeline spans enormous ranges of time and temperature. A simple evenly spaced diagram cannot represent those scales faithfully. Whenever numerical chronology matters, use a logarithmic time axis and quote the model assumptions associated with the epoch estimate.

## 24.22 Dark matter: evidence and interpretation

Dark matter is inferred primarily from gravitational effects that cannot be explained by the observed distribution of ordinary luminous matter within the standard framework. The evidence is not a single anomalous measurement. It comes from multiple scales and methods, including galaxy rotation, motions in galaxy clusters, gravitational lensing, growth of large-scale structure, and cosmological analyses involving the CMB.

A classic introductory example is the rotation curve of a galaxy. For a test body in an approximately circular orbit of radius  $r$  around a spherically symmetric enclosed mass  $M(r)$ , Newtonian reasoning gives

$$\frac{v_c^2}{r} = \frac{GM(r)}{r^2}, \quad (24.34)$$

so

$$M(r) = \frac{v_c^2 r}{G}. \quad (24.35)$$

If almost all mass were concentrated where most starlight is observed, one would expect  $v_c \propto r^{-1/2}$  sufficiently far outside that mass. Many galaxies instead show approximately flat rotation curves over extended regions, indicating that the enclosed gravitating mass continues to increase with radius.

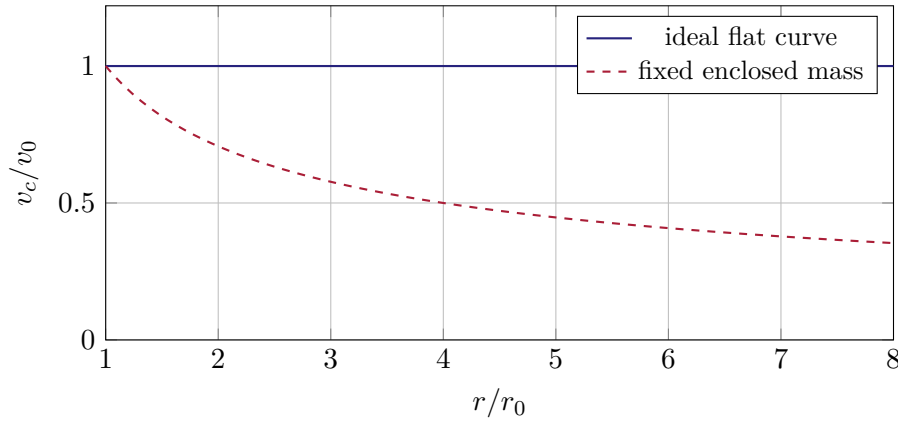


**Figure 24.13:** Two analytical models outside a reference radius  $r_0$ , normalized to agree at  $v_0$ . A constant enclosed mass gives  $v_c/v_0 = (r/r_0)^{-1/2}$  in the spherical approximation; a flat speed requires  $M(r) \propto r$ . These curves illustrate the inference and are not an observational data set or a fit to a disk galaxy.

### Example 24.15: Enclosed mass from a rotation speed

A star or gas cloud moves at  $v_c = 220$  km/s at radius  $r = 20$  kpc from a galaxy centre. Estimate the enclosed mass using Eq. (24.35). Convert

$$v_c = 2.20 \times 10^5 \text{ m/s}, \quad r = 20(3.086 \times 10^{19}) \text{ m} = 6.17 \times 10^{20} \text{ m}.$$

Then

$$M \simeq \frac{(2.20 \times 10^5)^2 (6.17 \times 10^{20})}{6.674 \times 10^{-11}} \text{ kg} \simeq 4.47 \times 10^{41} \text{ kg}.$$

In solar masses, using  $M_\odot \simeq 1.99 \times 10^{30}$  kg,

$$M \simeq 2.25 \times 10^{11} M_\odot.$$

This estimate measures gravitating mass enclosed within the orbit; it does not by itself identify what the unseen component is made of.

Dark matter is therefore a name for a physical component inferred from its gravitational role, not simply a synonym for black holes, cold gas, dust, or any dim object. Ordinary baryonic matter that is difficult to see cannot account for the full cosmological dark-matter phenomenology under the standard interpretation. The microscopic identity of the dominant dark matter remains unknown.

### Key message

Evidence for dark matter is evidence for extra gravitating mass-energy relative to the inventory of ordinary visible matter. Identifying the particle or physical nature of that component is a separate scientific problem.

## 24.23 Dark energy and accelerated expansion

Dark energy is conceptually distinct from dark matter. Dark matter clusters gravitationally and helps form cosmic structure. Dark energy is the name given to the component or effective physics associated with the observed late-time accelerated expansion in the standard cosmological interpretation.

The simplest model is a cosmological constant  $\Lambda$ , corresponding to an energy density that remains constant as the universe expands. More general models allow a dynamical dark-energy component characterized by an equation-of-state parameter

$$w = \frac{p}{\rho c^2}, \quad (24.36)$$

where  $p$  is pressure and  $\rho c^2$  is energy density. A cosmological constant has  $w = -1$ .

Current observational analyses continue to test whether  $w = -1$  is sufficient. Some recent combinations of large-scale-structure, supernova, and CMB data have shown interesting preferences for time-varying dark-energy models, while newer analyses and different data combinations can shift the degree of preference. This is an active research question, not an established discovery of evolving dark energy. The scientifically conservative statement for this course is that accelerated expansion is well established, while the underlying physical explanation remains unknown and the cosmological constant remains the benchmark model.

### Caution

Dark matter and dark energy are not two versions of the same substance. Dark matter is introduced primarily to explain additional gravitational clustering and mass. Dark energy is introduced to describe accelerated cosmic expansion. Their observational roles are different.

## 24.24 What the Standard Model explains – and what it does not

The Standard Model is extraordinarily successful. It organizes the known elementary quarks and leptons, describes electromagnetic, weak, and strong interactions, predicts a huge range of scattering and decay phenomena, and includes the Higgs mechanism. Precision tests have repeatedly confirmed its predictions within experimental uncertainties across many processes.

Yet it is not a complete theory of nature. Important open questions include:

- gravity is not included as a quantum interaction in the Standard Model;
- the observed dark matter is not identified by any established Standard Model particle population;
- neutrino masses require an extension of the minimal massless-neutrino formulation;

- the observed dominance of matter over antimatter in the universe is not explained quantitatively by known Standard Model CP violation alone;
- the pattern of fermion masses, mixing angles, and generations is not derived from a deeper principle within the model;
- the physical origin and magnitude of dark energy are unexplained;
- the theory contains parameters whose measured values must be supplied experimentally.

These limitations do not mean the Standard Model is “wrong” in its tested domain. They mean it is incomplete. This distinction is central to scientific reasoning: a successful effective theory can remain correct in its domain even when embedded in a broader theory.

### Take-home message

A mature view of modern physics holds two ideas simultaneously: the Standard Model and the hot Big Bang cosmological framework are remarkably successful where tested, and major questions remain open. Progress comes from precise experiments that search for where the established frameworks stop being sufficient.

## 24.25 Engineering, computing, and data-science connections

Particle physics and cosmology connect fundamental models to instrument design. An engineering decision should identify the measured quantity, the model used to reconstruct it, and the uncertainty that limits the result.

| System                  | Physics relation                                              | Design consequence                                                                                                      |
|-------------------------|---------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| Magnetic tracker        | $p_T =  q BR$ in a uniform transverse field.                  | Field calibration and fitted curvature jointly determine momentum.                                                      |
| Position measurement    | $s_g \simeq L^2/(8R)$ for a short circular arc.               | A longer lever arm improves high-momentum sensitivity quadratically, but material and alignment affect the benefit.     |
| Calorimeter             | Independent resolution contributions add in quadrature.       | Sampling fluctuations dominate at low energy; calibration uniformity can set the high-energy limit.                     |
| Trigger and acquisition | Stored data rate equals accepted event rate times event size. | Latency, bandwidth, and selection bias must all be controlled.                                                          |
| Redshift spectrograph   | $z = \lambda_{\text{obs}}/\lambda_{\text{em}} - 1$ .          | Wavelength stability is essential; a precise redshift still includes peculiar velocity and a distance-model dependence. |

These are examples of the same measurement chain: a signal is calibrated, converted into a physical quantity, and tested against a prediction. The enrichment modules quantify several links in this chain.

## 24.26 Laboratory and computational connection

Three short computational activities can connect the chapter to practical data reasoning without requiring a specialist high-energy physics software framework.

**Example 24.16: A synthetic Hubble fit with a shared calibration**

The following data are constructed for practice. Distances have been treated as known, each velocity has independent uncertainty  $\sigma_v = 300$  km/s, and a zero intercept is assumed.

|            |      |      |      |      |      |
|------------|------|------|------|------|------|
| $d$ [Mpc]  | 20   | 40   | 60   | 80   | 100  |
| $v$ [km/s] | 1500 | 2600 | 4450 | 5450 | 7100 |

Minimizing  $\chi^2(H) = \sum_i (v_i - Hd_i)^2 / \sigma_v^2$  gives

$$\hat{H} = \frac{\sum_i d_i v_i}{\sum_i d_i^2} = \frac{1547000}{22000} \simeq 70.32 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

For the stated independent errors and fixed distances,

$$\sigma_H = \frac{\sigma_v}{\sqrt{\sum_i d_i^2}} \simeq 2.02 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

If a shared distance calibration makes every distance 2% too large, the fitted slope becomes  $\hat{H}/1.02 \simeq 68.94$  in the same units. This coherent shift does not diminish merely by adding more galaxies. Real analyses also treat distance uncertainty, peculiar-velocity correlations, selection, and the range over which the low-redshift approximation is adequate.

**24.26.1 Reaction checker**

Represent each particle by a small record containing electric charge, baryon number, lepton number, and mass. Given an initial and final particle list, the program should sum additive quantum numbers and test the minimum final rest energy. A minimal pseudocode design is:

```
initial = [particle_1, particle_2, ...]
final   = [particle_a, particle_b, ...]

for quantity in [Q, B, L]:
    compare(sum(initial[quantity]), sum(final[quantity]))

minimum_final_rest_energy = sum(final[mass_energy])
report_result()
```

The exercise teaches both physics and software design: the particle database is separate from the conservation logic, and a failed bookkeeping test can be reported with a specific reason.

**24.26.2 Magnetic-track reconstruction**

Generate points on a circular arc with small random position errors. Fit the circle radius  $R$  and infer  $p_T$  using Eq. (24.25). Repeat for different noise levels and arc lengths. Students will see that high-momentum tracks have large radii and therefore smaller curvature, making momentum estimation more sensitive to finite spatial resolution.

**24.26.3 A low-redshift Hubble diagram**

Create synthetic distances  $d_i$  and velocities

$$v_i = H_0 d_i + \epsilon_i, \quad (24.37)$$

where  $\epsilon_i$  represents measurement scatter or peculiar velocities. Fit a straight line through the origin or with an intercept, compare the inferred  $H_0$ , and study how nearby galaxies can be dominated

by peculiar velocity. This activity teaches a central lesson: even when the physical law is simple, parameter inference requires a noise model and an understanding of systematic effects.

### Method

A useful computational workflow is **model** → **simulated observable** → **noise/systematics** → **reconstruction** → **inferred parameter**. This sequence mirrors real experimental analysis and is more educational than simply evaluating a formula on a calculator.

## 24.27 Common misconceptions and how to correct them

The following checks help connect the microscopic and cosmological models to their domains of validity.

| Tempting interpretation                                                          | Physical correction and a useful check                                                                                                                                   |
|----------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Quarks orbit inside a proton like planets.                                       | Valence content records net quantum numbers. A proton is a quantum QCD state containing quarks, antiquarks, and gluons; constituent dots do not specify orbits.          |
| A virtual carrier borrows energy briefly.                                        | Four-momentum is conserved at each vertex. An internal line can be off shell; it is a term in an amplitude, with no detected classical trajectory.                       |
| Charge conservation proves a reaction can occur.                                 | Check four-momentum, angular momentum, interaction structure, and other relevant quantum numbers. For example, charge balance alone does not permit free proton decay.   |
| The Higgs gives most of a proton's mass.                                         | The Higgs mechanism gives elementary charged fermions their mass terms. Most nucleon mass comes from QCD energy and dynamics.                                            |
| Neutrino flavour change proves total lepton number is violated.                  | Oscillation changes the probability of a flavour measurement. It establishes mass splittings, while the Dirac or Majorana nature of neutrinos remains open.              |
| The Big Bang has a spatial centre, and redshift is always a local Doppler shift. | The homogeneous model expands comoving separations everywhere. For cosmological redshift use $1 + z = a_0/a_{\text{em}}$ ; nearby Doppler reasoning is an approximation. |
| Dark matter and dark energy are identified substances.                           | They describe different inferred roles: gravitating clustered matter and a source of acceleration. Their microscopic identities have not been established.               |

### Guided checks

Explain why the following pairs are different: total laboratory energy and invariant mass; individual lepton flavour and total lepton number; a CMB temperature spectrum and a CMB anisotropy map; matter–dark-energy density equality and the onset of acceleration.

## 24.28 Integrated applications and case studies

The concepts of this chapter become more durable when they are used in complete physical stories. The following case studies deliberately connect particle classification, kinematics, detection, engineering, and inference rather than treating each topic as an isolated formula.

### 24.28.1 Cloud chambers and cosmic-ray tracks

A diffusion cloud chamber contains a supersaturated vapor. A charged particle passing through the chamber ionizes molecules along its path. Droplets condense preferentially around the ions, making the trajectory visible as a thin track. The device therefore converts an invisible microscopic passage into a macroscopic optical record.

Different tracks can look different because ionization loss depends on particle charge and speed. A strongly ionizing slow particle can leave a thicker track than a relativistic minimum-ionizing particle. If the chamber is placed in a magnetic field, curvature additionally contains momentum and charge-sign information. The same physical chain appears in sophisticated collider tracking detectors: ionization or excitation creates a local signal, geometry reconstructs a trajectory, a magnetic field converts curvature into momentum, and multiple tracks are combined into vertices.

Cosmic-ray muons provide an accessible source. Primary cosmic rays interact high in the atmosphere and produce secondary hadrons; pion and kaon decays generate muons and neutrinos. The muon's  $2.2\,\mu\text{s}$  proper lifetime might seem too short for many muons to reach sea level, but relativistic time dilation increases their laboratory lifetime. This turns a tabletop observation into a direct qualitative test of special relativity.

A classroom investigation can proceed in four steps. First, record track lengths and orientations from video frames. Second, classify straight, curved, and highly ionizing tracks qualitatively. Third, estimate whether observed track lengths are compatible with atmospheric muons given relativistic decay. Fourth, discuss selection effects: only particles that enter the sensitive volume and produce enough ionization become visible. The chamber therefore provides a compact lesson in both physics and measurement bias.

### 24.28.2 Positron emission tomography: antimatter in medical imaging

Antimatter is sometimes introduced as an exotic high-energy concept, but positrons are used routinely in medical imaging. In positron emission tomography (PET), a radioactive tracer undergoes beta-plus decay and emits a positron. After losing kinetic energy in tissue, the positron annihilates with an electron, producing predominantly two photons near 511 keV emitted in nearly opposite directions in the pair center-of-momentum frame.

A ring of detectors records pairs of photon interactions in temporal coincidence. The two detections define a line of response along which the annihilation probably occurred. Many such lines are combined computationally to reconstruct a three-dimensional distribution of tracer activity. Modern time-of-flight PET additionally uses the small difference in photon arrival times to localize the event more strongly along the line.

This application connects several parts of Physics 2. Nuclear decay creates the positron. Particle-antiparticle annihilation converts rest energy into photons. Electromagnetic interactions of the photons in scintillators or semiconductor detectors produce measurable signals. Fast electronics measure timing, while tomographic algorithms solve an inverse problem. The image is therefore not a direct photograph of positrons; it is an inference from many detected photon pairs and a detector-response model.

The ideal back-to-back picture also has limitations. The electron-positron pair may have residual momentum, photons scatter in tissue, detector energy resolution is finite, and random coincidences occur. Reconstruction software must model or reject these effects. The case study is a useful reminder that a clean conservation-law diagram is only the first layer of a real instrument.

### 24.28.3 Particle beams in medicine and materials science

Accelerator physics is not restricted to fundamental-particle discovery. Electron, proton, and ion beams are used in cancer therapy, isotope production, imaging, materials modification, and semiconductor processing. The same Lorentz-force principles guide and focus the beam, while energy-loss physics determines how the beam deposits energy in matter.

Proton therapy is a particularly instructive example. Charged particles lose energy through ionization as they traverse tissue, and protons exhibit a pronounced Bragg peak near the end of their range. By adjusting beam energy, a treatment system can place the high-dose region inside a tumor while

reducing dose beyond it compared with some photon-beam configurations. A clinical treatment plan combines accelerator settings, magnetic beam delivery, patient imaging, stopping-power models, and dose optimization.

The connection to particle kinematics is direct but should not be oversimplified. A therapeutic proton at hundreds of MeV is relativistic enough that accurate beam transport and range calculations use relativistic momentum-energy relations. Magnetic rigidity  $p/q$  controls bending. Beam monitors measure position and intensity. Safety systems must terminate the beam rapidly if diagnostics indicate a fault.

Materials science uses related beams for implantation and analysis. Ion implantation introduces dopants into semiconductors with a depth profile determined by ion energy and stopping. Rutherford backscattering and other ion-beam techniques infer composition from scattering kinematics. Thus the conceptual tools used to study fundamental particles also underpin practical technologies at very different energy scales.

#### 24.28.4 Event displays: what is measured and what is drawn

Collider event displays are visually compelling, but they can encourage misconceptions if read as photographs. A display typically overlays reconstructed tracks, calorimeter energy deposits, detector geometry, and sometimes reconstructed jets or missing-momentum vectors. Different colors are graphical choices chosen for readability; they are not the QCD color charge of a quark.

A useful event-display reading strategy is layered. Begin with the beam interaction point and reconstructed vertices. Next inspect charged tracks and their curvature. Then compare tracks with electromagnetic and hadronic calorimeter clusters. Look for muon-system matches. Finally inspect global quantities such as missing transverse momentum. Only after this detector-level description should one propose a physics interpretation.

For example, two isolated high-energy electromagnetic clusters with no matched charged tracks may suggest photons. Their energies and directions can be used to form a diphoton invariant mass. A single event near 125 GeV is not evidence for a Higgs boson by itself. One must study the full mass distribution, background expectation, detector resolution, and rate. Conversely, an event with one high- $p_T$  charged lepton, several jets, one or more displaced heavy-flavour vertices, and substantial missing transverse momentum may be compatible with a top-quark process, but again the interpretation emerges statistically across many events.

The discipline of separating detector description from physics interpretation is broadly useful in data science. It discourages confirmation bias and makes analysis auditable: another researcher can agree on what the detector recorded even while testing a different hypothesis for what produced it.

##### Guided checks

For each case study, identify the full chain *physical process*  $\rightarrow$  *interaction with detector/material*  $\rightarrow$  *electronic or optical signal*  $\rightarrow$  *reconstructed observable*  $\rightarrow$  *physical inference*. Which links are directly measured and which depend on a model?

## 24.29 Main chapter summary

### Summary

- The established elementary matter particles of the Standard Model are quarks and leptons, arranged in three generations.
- Quarks have fractional electric charges and color charge. Hadrons are color-neutral composites; conventional baryons contain three valence quarks and conventional mesons a valence quark-antiquark pair.
- Antiparticles have the same mass as their corresponding particles and opposite additive charges.
- Relativistic particle kinematics is governed by  $E^2 = p^2c^2 + m^2c^4$ . Particle production depends on invariant center-of-mass energy, not merely on the laboratory beam energy.
- Electric charge and energy-momentum are indispensable conservation laws. Baryon and lepton numbers are useful reaction bookkeeping rules, with important theoretical and experimental qualifications.
- The Standard Model describes strong, electromagnetic, and weak interactions through quantum gauge fields. Gravity is not part of the Standard Model.
- The Higgs field participates in electroweak symmetry breaking and elementary-particle mass generation, but most ordinary nucleon mass is associated with QCD dynamics.
- Accelerators create controlled high-energy initial states; detectors reconstruct final states from tracks, calorimeter deposits, vertices, jets, and momentum imbalance.
- Neutrino oscillations arise because flavour and mass eigenstates are not identical.
- Cosmological expansion is described by an evolving scale factor. At low redshift,  $v \simeq H_0 d$  is a useful approximation; precision values of  $H_0$  remain method-dependent and actively studied.
- The CMB is relic thermal radiation from the early universe and carries detailed information in its anisotropies.
- Galaxy dynamics, lensing, structure formation, and cosmological data motivate dark matter. Accelerated expansion motivates dark energy. Their physical origins remain open questions.

The compact formula sheet in Appendix B collects the equations with their conventions and assumptions.

## 24.30 Advanced Topics for Technical and Engineering Students

The five modules below are optional. They develop quantitative tools for students who have mastered the main chapter. Four-vectors require algebra and special relativity; cosmological scalings require elementary calculus. Field-theory amplitudes and the Einstein equations are introduced as models, with their assumptions stated, rather than derived from a graduate-level formalism. Read Modules 1 and 2 for experimental kinematics, Module 3 for quantum structure, Module 4 for cosmology, and Module 5 for computational investigations.

### 24.30.1 Relativistic kinematics and collision geometry

#### Natural units and dimensional discipline

High-energy physics often uses natural units in which

$$\hbar = c = 1. \quad (24.38)$$

Then mass, energy, and momentum are expressed in the same energy unit, typically MeV or GeV. Length and time acquire inverse-energy dimensions. This convention makes relativistic equations compact: Eq. (24.1) becomes

$$E^2 = p^2 + m^2. \quad (24.39)$$

Natural units are not a change in physics. They are a unit convention. Factors of  $c$  and  $\hbar$  can always be restored by dimensional analysis.

Two useful conversion scales are

$$\hbar c \simeq 0.1973 \text{ GeV fm}, \quad \hbar \simeq 6.582 \times 10^{-25} \text{ GeV s}. \quad (24.40)$$

Thus a length scale 1 fm corresponds to a momentum scale of order  $0.2 \text{ GeV}/c$ . Conversely, probing  $10^{-3} \text{ fm}$  requires momentum-transfer scales of order hundreds of GeV.

#### Example 24.17: Resolution scale from momentum transfer

Estimate the characteristic distance scale associated with a momentum transfer  $Q = 100 \text{ GeV}$ . Using the uncertainty-style estimate  $\Delta x \sim \hbar c/Q$ ,

$$\Delta x \sim \frac{0.197 \text{ GeV fm}}{100 \text{ GeV}} \simeq 1.97 \times 10^{-3} \text{ fm} = 1.97 \times 10^{-18} \text{ m}.$$

This is not an exact microscope resolution formula, but it gives the correct inverse relation between momentum transfer and spatial scale.

#### Caution

Natural units can hide dimensions from inexperienced users. When checking a result, temporarily restore  $c$  and  $\hbar$  or use a known conversion such as  $\hbar c = 0.197 \text{ GeV fm}$ . A compact formula is not dimensionally safe merely because it appears in particle-physics notation.

#### Four-momentum and invariant mass

Special relativity becomes especially efficient when energy and momentum are combined into the four-vector

$$p^\mu = \left( \frac{E}{c}, \mathbf{p} \right). \quad (24.41)$$

With metric convention  $(+, -, -, -)$ ,

$$p^\mu p_\mu = \frac{E^2}{c^2} - p^2 = m^2 c^2. \quad (24.42)$$

For a many-particle system, define the total four-momentum

$$P^\mu = \sum_i p_i^\mu. \quad (24.43)$$

Its invariant mass  $M$  satisfies

$$M^2 c^4 = E_{\text{tot}}^2 - P_{\text{tot}}^2 c^2. \quad (24.44)$$

Because  $M$  is Lorentz invariant, it has the same value in every inertial frame. In the center-of-momentum frame  $\mathbf{P}_{\text{tot}} = 0$ , so  $M c^2 = E_{\text{CM}}$ .

Invariant mass is one of the central tools of experimental particle physics. If an unstable parent decays into visible daughters whose four-momenta are reconstructed, then

$$M_{\text{daughters}}^2 c^4 = \left( \sum_i E_i \right)^2 - \left| \sum_i \mathbf{p}_i \right|^2 c^2. \quad (24.45)$$

A resonance can appear as an excess near the parent mass in the distribution of reconstructed invariant mass.

#### Example 24.18: Two-photon invariant mass

Two photons have energies  $E_1$  and  $E_2$  and opening angle  $\theta$ . Since  $p_i = E_i/c$ , the invariant mass of the pair is

$$\begin{aligned} M^2 c^4 &= (E_1 + E_2)^2 - c^2 |\mathbf{p}_1 + \mathbf{p}_2|^2 \\ &= E_1^2 + E_2^2 + 2E_1 E_2 - (E_1^2 + E_2^2 + 2E_1 E_2 \cos \theta) \\ &= 2E_1 E_2 (1 - \cos \theta). \end{aligned}$$

Hence

$$M_{\gamma\gamma} c^2 = \sqrt{2E_1 E_2 (1 - \cos \theta)}. \quad (24.46)$$

If  $E_1 = E_2 = 62.5 \text{ GeV}$  and the photons are back-to-back,  $\theta = \pi$ , giving

$$M_{\gamma\gamma} c^2 = 2(62.5 \text{ GeV}) = 125 \text{ GeV}.$$

The example shows how a massive parent can decay into individually massless particles whose combined invariant mass is nonzero.

#### Guided checks

1. Why can two photons have a nonzero combined invariant mass even though each photon has zero rest mass?
2. What is the invariant mass of two exactly collinear photons travelling in the same direction?
3. Why is invariant mass preferable to total laboratory energy when identifying a parent resonance?

#### Mandelstam $s$ and collider energy

For a two-particle initial state with four-momenta  $p_1$  and  $p_2$ , define

$$s = c^2 (p_1 + p_2)^\mu (p_1 + p_2)_\mu, \quad (24.47)$$

so that  $\sqrt{s}$  is the center-of-mass energy. In natural units this is simply  $s = (p_1 + p_2)^2$ .

For two identical beams colliding head-on with equal total beam energy  $E_b$ ,

$$\sqrt{s} = 2E_b. \quad (24.48)$$

For a projectile of total energy  $E$  hitting an identical target of mass  $m$  at rest,

$$s = 2m^2c^4 + 2mc^2E. \quad (24.49)$$

At high energy,  $\sqrt{s} \propto \sqrt{E}$ , which quantifies the inefficiency of fixed-target kinematics for creating very massive states.

### Example 24.19: Antiproton production threshold in a fixed-target proton collision

Consider the reaction

$$p + p \rightarrow p + p + p + \bar{p}.$$

Baryon number and charge are conserved. At threshold in the center-of-momentum frame, the minimum final invariant mass is  $4m_p$ . Therefore

$$s_{\text{th}} = 16m_p^2c^4.$$

For a beam proton of total energy  $E$  hitting a proton at rest, Eq. (24.49) gives

$$2m_p^2c^4 + 2m_pc^2E = 16m_p^2c^4.$$

Thus

$$E = 7m_pc^2.$$

The kinetic energy is  $K = E - m_pc^2 = 6m_pc^2 \simeq 5.63 \text{ GeV}$ . Although creating a  $p\bar{p}$  pair requires only  $2m_pc^2 \simeq 1.88 \text{ GeV}$  of additional rest energy, momentum conservation in the fixed-target geometry raises the required beam kinetic energy substantially.

### Two-body decay kinematics

Suppose a parent of mass  $M$  decays at rest into two daughters of masses  $m_1$  and  $m_2$ . Momentum conservation requires equal and opposite daughter momenta of magnitude  $p^*$ . Write  $E_2 = Mc^2 - E_1$ , use  $E_i^2 = p^{*2}c^2 + m_i^2c^4$ , and subtract the daughter equations:

$$(Mc^2 - E_1)^2 - E_1^2 = (m_2^2 - m_1^2)c^4.$$

Solving gives

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}c^2, \quad E_2 = \frac{M^2 + m_2^2 - m_1^2}{2M}c^2. \quad (24.50)$$

Then  $p^{*2}c^2 = E_1^2 - m_1^2c^4$ . Factoring its numerator gives

$$(M^2 + m_1^2 - m_2^2)^2 - 4M^2m_1^2 = [M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2].$$

The physical branch requires  $M \geq m_1 + m_2$ . At equality the daughter momenta vanish in the parent rest frame. This derivation and the threshold condition are independent of the interaction causing the decay [6].

The resulting momentum is

$$p^*c = \frac{1}{2Mc^2} \sqrt{[M^2c^4 - (m_1 + m_2)^2c^4][M^2c^4 - (m_1 - m_2)^2c^4]}. \quad (24.51)$$

Equivalently, in natural units,

$$p^* = \frac{\sqrt{\lambda(M^2, m_1^2, m_2^2)}}{2M}, \quad (24.52)$$

where

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \quad (24.53)$$

is the Källén function.

For massless daughters,  $m_1 = m_2 = 0$ , so  $p^*c = Mc^2/2$  and each daughter has energy  $Mc^2/2$ . This is why a scalar particle of mass near 125 GeV decaying at rest to two photons produces two photons of about 62.5 GeV each in the parent rest frame.

#### Example 24.20: Pion decay at rest

For the two-body decay  $\pi^+ \rightarrow \mu^+ + \nu_\mu$ , neglect the neutrino mass. Using  $m_\pi c^2 \simeq 139.6$  MeV and  $m_\mu c^2 \simeq 105.7$  MeV,

$$p^*c = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} c^2.$$

Numerically,

$$p^*c \simeq \frac{(139.6)^2 - (105.7)^2}{2(139.6)} \text{ MeV} \simeq 29.8 \text{ MeV}.$$

Thus the muon and neutrino emerge back-to-back with momentum magnitude about 29.8 MeV/ $c$  in the pion rest frame. The neutrino energy is approximately 29.8 MeV, while the muon energy is larger because it includes its substantial rest energy.

#### Transverse momentum, pseudorapidity, and collider coordinates

At a collider the beam defines a preferred  $z$  axis. Momentum is separated into a transverse component

$$p_T = \sqrt{p_x^2 + p_y^2} \quad (24.54)$$

and longitudinal component  $p_z$ . The polar angle  $\theta$  relative to the beam can be re-expressed using pseudorapidity

$$\eta = -\ln \tan \frac{\theta}{2}. \quad (24.55)$$

Central particles with  $\theta = 90^\circ$  have  $\eta = 0$ . Particles close to the beam direction have large  $|\eta|$ .

For  $p_T \gg mc$  (and exactly for a massless particle), pseudorapidity approximates the true rapidity

$$y = \frac{1}{2} \ln \frac{E + p_z c}{E - p_z c}. \quad (24.56)$$

Rapidity differences are invariant under boosts along the beam direction, which makes rapidity a natural collider variable.

The distance between two reconstructed objects in the detector is often characterized by

$$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}, \quad (24.57)$$

where  $\phi$  is the azimuthal angle and  $\Delta\phi$  is wrapped to  $[-\pi, \pi]$ . This is a dimensionless separation in angular coordinates, not a spatial distance. Jet algorithms and isolation criteria commonly use this geometry.

**Note**

The  $(\eta, \phi)$  coordinate system is an engineering-friendly representation of detector geometry because segmentation is often approximately organized around the beam axis. It also connects naturally to the boost structure of hadron-collider kinematics.

**Cosmic rays as natural high-energy accelerators**

Nature accelerates particles to energies far beyond those achieved in human-made colliders. Cosmic rays striking the atmosphere can have enormous laboratory energy. Yet because atmospheric collisions are effectively fixed-target events, the center-of-mass energy is much smaller than the projectile energy might suggest.

For an ultrarelativistic proton of energy  $E$  hitting a stationary proton,

$$\sqrt{s} \simeq \sqrt{2m_p c^2 E}. \quad (24.58)$$

A cosmic-ray proton with  $E = 10^{17} \text{ eV} = 10^8 \text{ GeV}$  therefore has

$$\sqrt{s} \sim \sqrt{2(0.938)(10^8)} \text{ GeV} \simeq 1.37 \times 10^4 \text{ GeV} = 13.7 \text{ TeV},$$

comparable to the energy scale of a major hadron collider despite a projectile energy of  $10^{17} \text{ eV}$ . The comparison dramatically illustrates the collider advantage.

Cosmic-ray experiments compensate for the uncontrolled initial state by observing huge natural exposures and extensive air showers. They connect particle physics, atmospheric physics, astrophysics, detector engineering, and statistical inference.

**24.30.2 Detectors, event rates, and statistical inference****Cross section, luminosity, and event yield**

A cross section  $\sigma$  quantifies the probability scale for a specified interaction process. It has dimensions of area. The traditional unit is the barn,

$$1 \text{ barn} = 10^{-28} \text{ m}^2, \quad (24.59)$$

with picobarn and femtobarn subunits:

$$1 \text{ pb} = 10^{-12} \text{ barn}, \quad 1 \text{ fb} = 10^{-15} \text{ barn}. \quad (24.60)$$

Collider performance is characterized by luminosity. If the instantaneous luminosity is  $\mathcal{L}(t)$ , the expected number of events for a process of cross section  $\sigma$  is

$$N_{\text{produced}} = \sigma \int \mathcal{L}(t) dt. \quad (24.61)$$

A real analysis observes only a fraction of produced events, so

$$N_{\text{selected}} = \sigma \mathcal{L}_{\text{int}} A \varepsilon, \quad (24.62)$$

where  $A$  represents geometric/kinematic acceptance and  $\varepsilon$  detection and selection efficiency.

**Example 24.21: Expected event yield**

A process has cross section  $\sigma = 2.0\text{pb}$ . A data set has integrated luminosity  $\mathcal{L}_{\text{int}} = 150\text{fb}^{-1}$ . Because  $1\text{pb} = 10^3\text{fb}$ ,

$$\sigma = 2000\text{fb}.$$

The produced yield is

$$N_{\text{produced}} = (2000\text{fb})(150\text{fb}^{-1}) = 3.0 \times 10^5.$$

If  $A\varepsilon = 0.12$ , the selected signal yield is

$$N_{\text{selected}} = 3.6 \times 10^4.$$

This simple relation explains why a tiny cross section can still be studied if the integrated luminosity is sufficiently large.

**Caution**

Cross section is not a literal geometric area occupied by a particle. It is an effective probability measure derived from the scattering amplitude and phase space. The area dimension is historically and mathematically natural, but the concept is probabilistic.

If  $\sigma$  describes production of a parent  $X$ , the selected yield in a particular decay channel is

$$N_{\text{selected}} = \sigma_X \mathcal{L}_{\text{int}} \text{BR}(X \rightarrow f) A\varepsilon.$$

If the quoted cross section already describes the final state  $f$ , its branching fraction is already included and must not be multiplied again. Dimensional check:  $1\text{fb}^{-1} = 10^3\text{pb}^{-1}$ , so a  $1\text{pb}$  process produces 1000 events per inverse femtobarn before selection.

**Luminosity from beam parameters**

For two Gaussian bunches with equal transverse widths in each plane, colliding head-on, a simplified luminosity estimate is

$$\mathcal{L} \simeq \frac{f N_1 N_2}{4\pi\sigma_x\sigma_y}, \quad (24.63)$$

Here  $f$  is the total bunch-pair crossing frequency,  $N_1$  and  $N_2$  are particles per bunch, and  $\sigma_x, \sigma_y$  are the common transverse rms widths of the two bunches. Real machines require correction factors for crossing angles, bunch shape, hourglass effects, and beam dynamics.

Equation (24.63) makes the engineering challenge visible. Increasing bunch population raises collision rate but also strengthens collective effects and radiation backgrounds. Reducing beam size raises luminosity but requires precise focusing and stability. Increasing repetition frequency demands more power, radio-frequency capability, and detector throughput. Accelerator luminosity is therefore not a free parameter; it is the outcome of a coupled design optimization.

**Track sagitta and momentum resolution**

The curvature relation  $p_T \propto BR$  hides an important detector-design question: how accurately can a very large radius be measured? Suppose a charged track is measured across a chord length  $L$  in a uniform transverse magnetic field. For small curvature, the maximum displacement between the circular arc and the chord, called the sagitta  $s_g$ , is approximately

$$s_g \simeq \frac{L^2}{8R}. \quad (24.64)$$

Using  $R[\text{m}] = p_T[\text{GeV}/c]/(0.2998|q/e|B[\text{T}])$ ,

$$s_g \propto \frac{BL^2}{p_T}. \quad (24.65)$$

Thus high-momentum tracks have very small sagitta. Increasing the magnetic field helps linearly, while increasing the lever arm helps quadratically. This is why large detector dimensions can be as valuable as strong magnets for momentum resolution.

If the sagitta uncertainty is approximately  $\sigma_s$ , then at high momentum a rough scaling gives

$$\frac{\sigma_{p_T}}{p_T} \sim \frac{\sigma_s}{s_g} \propto p_T. \quad (24.66)$$

The relative momentum resolution therefore often worsens at very large  $p_T$  when measurement precision dominates. At low momentum, multiple Coulomb scattering in detector material introduces a different contribution that can dominate.

#### Example 24.22: Sagitta scale in a tracker

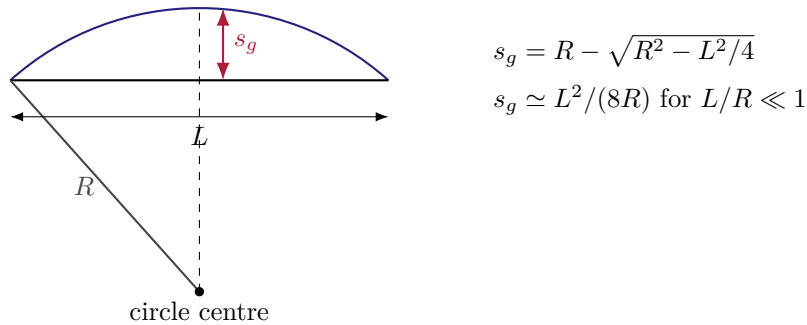
A singly charged particle with  $p_T = 100 \text{ GeV}/c$  crosses a  $B = 4.0 \text{ T}$  field with a measurement lever arm  $L = 1.0 \text{ m}$ . Its radius is

$$R \simeq \frac{p_T}{0.3B} = \frac{100}{1.2} \text{ m} \simeq 83.3 \text{ m}.$$

The sagitta is approximately

$$s_g \simeq \frac{1^2}{8(83.3)} \text{ m} \simeq 1.50 \text{ mm}.$$

A spatial alignment error of only tens of micrometres can therefore matter for precision high-momentum tracking.



$$s_g = R - \sqrt{R^2 - L^2/4}$$

$$s_g \simeq L^2/(8R) \text{ for } L/R \ll 1$$

**Figure 24.14:** Sagitta geometry for a circular track and a measured chord. The approximation follows by expanding  $\sqrt{1 - L^2/(4R^2)}$  to first order in  $L^2/R^2$ . The drawn curvature is enlarged to make the geometry visible.

#### Calorimeter energy resolution and stochastic processes

Calorimeters infer particle energy from showers and deposited signals. A commonly used resolution parameterization is

$$\frac{\sigma_E}{E} \simeq \frac{a_{\text{cal}}}{\sqrt{E/\text{GeV}}} \oplus b_{\text{cal}} \oplus \frac{n_{\text{cal}}}{E/\text{GeV}}, \quad (24.67)$$

Here  $\oplus$  means addition in quadrature, and all three coefficients are dimensionless because energy appears as the ratio  $E/\text{GeV}$ . The coefficient  $a_{\text{cal}}$  is a stochastic term associated with finite shower

sampling or photoelectron statistics,  $b_{\text{cal}}$  is a constant term associated with calibration nonuniformity and other energy-independent fractional effects, and  $n_{\text{cal}}$  represents noise-like contributions that scale roughly as  $1/E$ .

The formula explains why detector optimization is energy dependent. At moderate energy, increasing light yield or sampling quality can reduce the stochastic term. At very high energy, the constant term may become dominant, so precision calibration and mechanical stability become critical. At low energy, electronic noise and pileup contributions may dominate.

Calorimeter response also depends on particle type. Electrons and photons initiate electromagnetic showers through bremsstrahlung and pair production. Hadrons undergo nuclear interactions and generate more complex hadronic showers. A hadronic calorimeter therefore generally has worse energy resolution than a high-quality electromagnetic calorimeter.

This empirical form assumes approximately independent contributions. Parameters must be calibrated for the particular detector and particle type [7].

### Time of flight, Cherenkov radiation, and particle identification

Particles with the same momentum but different masses have different speeds. A time-of-flight system exploits

$$\beta = \frac{v}{c} = \frac{pc}{E} = \frac{pc}{\sqrt{p^2c^2 + m^2c^4}}. \quad (24.68)$$

Over a flight path  $L$ , the arrival time is  $t = L/(\beta c)$ . At sufficiently high momentum all massive particles have  $\beta \rightarrow 1$ , so time-of-flight separation becomes difficult.

Cherenkov detectors use a different speed-dependent effect. In a medium with refractive index  $n$ , Cherenkov radiation is emitted when

$$\beta n > 1. \quad (24.69)$$

The emission angle obeys

$$\cos \theta_C = \frac{1}{\beta n}. \quad (24.70)$$

If the momentum is known from tracking, a measurement of  $\beta$  can infer the mass through

$$m^2c^2 = p^2 \left( \frac{1}{\beta^2} - 1 \right). \quad (24.71)$$

Particle identification therefore often combines independent subsystems rather than relying on one signal.

#### Guided checks

Why does time-of-flight discrimination between a pion and kaon become harder as momentum increases? Why can a Cherenkov threshold be tuned through the refractive index of the radiator? What additional observable is needed to turn a speed measurement into a mass estimate?

### Trigger systems and real-time decision making

At a high-rate collider, it is often impossible or unnecessary to store every raw detector interaction in full detail. A trigger system reduces the event rate while retaining events likely to contain useful physics. This is an engineering and computer-science problem under strict latency, bandwidth, reliability, and radiation constraints.

A trigger may use fast hardware-level summaries followed by more sophisticated software reconstruction. Typical criteria involve high transverse-momentum leptons, photons, jets, missing transverse

momentum, or combinations of objects. The trigger must be efficient for signal processes while keeping output rates within storage and computing limits.

Selection can bias a data sample. If a trigger requires  $p_T > 25$  GeV, the observed distribution below that region is not an unbiased representation of nature. Efficiency must therefore be measured as a function of relevant variables and included in the analysis model.

This provides a general data-science lesson: data acquisition is itself a selection process. A machine-learning model trained only on recorded events inherits the trigger and detector selection unless those effects are corrected.

### Measurement uncertainty and covariance in modern physics

Precision results are almost never described by one uncertainty-free number. Suppose a measured quantity  $f(x_1, \dots, x_n)$  depends on input parameters with covariance matrix  $C$ . For small uncertainties, linear propagation gives

$$\sigma_f^2 \simeq \sum_{i,j} \frac{\partial f}{\partial x_i} C_{ij} \frac{\partial f}{\partial x_j}. \quad (24.72)$$

If inputs are uncorrelated,  $C_{ij} = \delta_{ij}\sigma_i^2$ , recovering the familiar sum in quadrature.

Correlations matter strongly in fits. A detector energy-scale shift can move many reconstructed events coherently. A cosmological calibration parameter can affect many distance measurements in the same direction. Treating correlated systematics as independent random noise typically underestimates uncertainty.

#### Example 24.23: Uncertainty in momentum from field and radius

For a singly charged track  $p_T = 0.3BR$ , suppose  $B$  and  $R$  are independent with uncertainties  $\sigma_B$  and  $\sigma_R$ . Then

$$\left(\frac{\sigma_{p_T}}{p_T}\right)^2 \simeq \left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{\sigma_R}{R}\right)^2.$$

If  $B = 3.8$  T with relative uncertainty 0.1% and  $R = 2.0$  m with relative uncertainty 1.5%, then

$$\frac{\sigma_{p_T}}{p_T} \simeq \sqrt{(0.001)^2 + (0.015)^2} \simeq 0.0150.$$

The radius measurement dominates. Improving the magnetic-field calibration further would have little effect unless track curvature resolution is also improved.

### Advanced case study: from a collision to a measured cross section

A measured cross section is not obtained by dividing a raw event count by luminosity without further thought. Consider a generic process  $pp \rightarrow X \rightarrow \ell^+\ell^-$ . The analysis begins with a trigger and reconstruction selection. A candidate event may require two well-reconstructed, isolated leptons above transverse-momentum thresholds and within a fiducial detector region. The selected sample contains signal, irreducible backgrounds with the same final state, reducible backgrounds in which other objects are misidentified, and detector noise or rare pathologies.

Suppose the observed count is  $N_{\text{obs}}$  and the estimated background is  $N_{\text{bkg}}$ . A naive signal estimate is  $N_{\text{sig}} = N_{\text{obs}} - N_{\text{bkg}}$ . If the efficiency and acceptance are represented by  $A\varepsilon$ , the total cross section for this specified final state may be written schematically as

$$\sigma_{\text{tot}} = \frac{N_{\text{sig}}}{\mathcal{L}_{\text{int}} A\varepsilon}. \quad (24.73)$$

Every factor carries uncertainty. The luminosity has a calibration uncertainty. The lepton efficiency is measured from data control samples. The background estimate may depend on simulation or sideband fits. Momentum or energy calibration can move events across thresholds. A real measurement therefore uses a likelihood that treats several nuisance parameters simultaneously rather than inserting one fixed number into Eq. (24.73).

This case study illustrates why high-energy physics is a measurement science as much as a theory test. The theoretical cross section is defined at an idealized particle or parton level, while the detector records electronic signals. A reliable result constructs a transparent bridge between those levels.

**Fiducial versus total cross sections.** A fiducial cross section uses efficiency corrected within its specified truth-level region:

$$\sigma_{\text{fid}} = \frac{N_{\text{sig}}}{\mathcal{L}_{\text{int}} \varepsilon_{\text{fid}}}. \quad (24.74)$$

There is no acceptance factor extrapolating outside this region; migrations across its boundaries still need correction. In the total expression,  $A$  is the fraction of all specified final-state events inside the region, and  $\varepsilon$  is the selection efficiency conditional on being inside. A fiducial cross section is defined inside a region close to the actual experimental selection, for example charged leptons satisfying specified  $p_T$  and  $\eta$  requirements. It minimizes extrapolation into unmeasured regions. A total cross section extrapolates to the full physical phase space and is often more directly comparable with inclusive theoretical predictions, but the extrapolation is more model dependent. Neither is intrinsically superior; the appropriate choice depends on the scientific question.

**Unfolding.** Detector resolution can migrate events between bins of a measured distribution. If  $\mathbf{t}$  is a truth-level spectrum and  $\mathbf{r}$  the reconstructed spectrum, a linearized detector response can be represented by

$$\mathbf{r} = R\mathbf{t} + \mathbf{b}, \quad (24.75)$$

where  $R$  is a response matrix and  $\mathbf{b}$  background. Recovering  $\mathbf{t}$  from noisy data is an inverse problem. Direct matrix inversion is often unstable; regularization or iterative methods may be used. This is a direct connection between particle physics and numerical analysis.

### Statistical counting, backgrounds, and significance intuition

Event counts fluctuate. For an ideal Poisson process with expected mean  $\mu$ , the probability of observing  $n$  events is

$$P(n | \mu) = \frac{\mu^n e^{-\mu}}{n!}. \quad (24.76)$$

The standard deviation is  $\sqrt{\mu}$ . This leads to the rough intuition that a signal  $S$  on top of a large known background  $B$  becomes statistically easier to resolve as  $S/\sqrt{B}$  increases. However,  $S/\sqrt{B}$  is only an approximation, not a universal discovery-significance formula. Real analyses include nuisance parameters, systematic uncertainties, finite control samples, look-elsewhere effects, and likelihood-based inference.

If both signal and background scale with integrated luminosity, then  $S \propto \mathcal{L}$  and  $B \propto \mathcal{L}$ , so the statistics-dominated ratio behaves as

$$\frac{S}{\sqrt{B}} \propto \sqrt{\mathcal{L}}. \quad (24.77)$$

Doubling sensitivity in this simple regime may therefore require roughly four times as much data. This scaling motivates both higher luminosity and better analysis efficiency/background rejection.

### Method

When reading a particle-physics result, separate four questions: What is the observable? What is the expected background? What uncertainties enter the model? What statistical procedure converts the data-model comparison into a confidence interval, exclusion, or significance?

### Likelihoods, nuisance parameters, and profile inference

A modern analysis often uses a likelihood  $L(\mu, \theta)$ , where  $\mu$  is a parameter of interest and  $\theta$  represents nuisance parameters. A simple binned Poisson likelihood is

$$L(\mu, \theta) = \prod_k \frac{\nu_k(\mu, \theta)^{n_k} e^{-\nu_k(\mu, \theta)}}{n_k!} \times \prod_j C_j(\theta_j), \quad (24.78)$$

where  $n_k$  is the observed count in bin  $k$ ,  $\nu_k$  the expected count, and  $C_j$  constraint terms encoding auxiliary information about systematic effects.

A profile likelihood fixes a trial value of  $\mu$  and maximizes over nuisance parameters. This allows the data to constrain systematic shifts while preserving their prior calibration information. The procedure is more principled than adding every systematic uncertainty in quadrature after the fit when those systematics also change distribution shapes.

The same mathematical architecture appears in cosmological parameter estimation, where a model predicts a data vector and a covariance matrix and a likelihood compares prediction with measurements. The details differ, but the logic of nuisance parameters and correlated uncertainties is universal.

### Bayesian and frequentist viewpoints

Two broad statistical languages are common. In a frequentist approach, parameters are fixed unknowns and probability describes hypothetical data repetitions. Confidence intervals and  $p$ -values are constructed from sampling distributions. In a Bayesian approach, probability also quantifies uncertainty about parameters; Bayes' theorem combines a likelihood with a prior to form a posterior,

$$p(\theta | D) \propto p(D | \theta) \pi(\theta). \quad (24.79)$$

Neither framework eliminates the need for a correct data model. A poorly modeled background remains poorly modeled regardless of statistical philosophy. Likewise, a Bayesian posterior can depend on prior choices when data are weak, while a frequentist interval can depend on construction choices.

For this course, students should learn to read statistical statements carefully rather than adopt slogans. Ask what probability refers to, what model is assumed, how systematics enter, and whether the reported interval or significance answers the scientific question being asked.

### Systematic uncertainty as a design variable

Systematic uncertainty is sometimes treated as an afterthought attached to a final result. In well-designed experiments it influences hardware, calibration, and analysis from the beginning. If jet-energy calibration will dominate a measurement, the detector may include redundant calorimeter systems and in-situ calibration processes. If photometric redshift uncertainty limits a lensing survey, filter selection and spectroscopic calibration samples become central design decisions.

A useful optimization question is not simply “how can we collect more data?” but “which uncertainty component will dominate after we collect more data?” When statistical uncertainty falls below a systematic floor, further exposure yields diminishing returns unless the systematic uncertainty is improved. This principle connects detector design, survey strategy, and algorithm development.

### Advanced case study: combining independent channels

A recurring strategy in particle physics and cosmology is to combine measurements that constrain the same parameter in different ways. Suppose two statistically independent channels measure a parameter  $\mu$  with Gaussian uncertainties  $\sigma_1$  and  $\sigma_2$ . If systematic uncertainties are negligible and the measurements are consistent, the inverse-variance weighted estimator is

$$\hat{\mu} = \frac{\mu_1/\sigma_1^2 + \mu_2/\sigma_2^2}{1/\sigma_1^2 + 1/\sigma_2^2}, \quad \sigma_{\hat{\mu}}^{-2} = \sigma_1^{-2} + \sigma_2^{-2}. \quad (24.80)$$

This simple formula explains why independent measurements improve precision. Real combinations are more complex because systematic uncertainties can be correlated. A luminosity calibration may affect several collider channels coherently; a photometric calibration may affect multiple cosmological observables. The covariance matrix must then replace the independent-error assumption.

Combination is valuable for more than precision. Different channels test consistency. A new particle should have one mass even when reconstructed in different decay modes. A cosmological model should describe CMB, BAO, supernova, and structure-growth measurements simultaneously within a consistent parameter set. A tension localized to one technique suggests a different diagnosis from a discrepancy reproduced by several independent methods.

The engineering lesson is redundancy with diversity. Two identical sensors can reduce random noise, but two physically different measurement principles can reveal hidden systematics. Modern fundamental physics relies heavily on this second kind of redundancy.

### 24.30.3 Fields, symmetries, and neutrinos

#### Symmetry, conservation laws, and the Noether viewpoint

The conservation laws used throughout introductory physics are not an unrelated collection of empirical rules. In modern theoretical physics many arise from continuous symmetries through Noether's theorem. Time-translation symmetry is associated with energy conservation; spatial-translation symmetry with momentum conservation; rotational symmetry with angular momentum conservation. In field theory, internal symmetries are associated with conserved currents and charges.

This viewpoint clarifies an important distinction in particle physics. Electric charge is tied to the exact gauge structure of electromagnetism within the Standard Model and is conserved to extraordinary precision. Baryon and lepton numbers, by contrast, are useful accidental symmetries of the renormalizable Standard Model Lagrangian rather than independent gauge symmetries of the same status. Nonperturbative electroweak effects violate  $B + L$  while preserving  $B - L$  in the minimal framework, and many theories beyond the Standard Model permit additional violation. At Physics 2 level, baryon and lepton number remain excellent reaction-bookkeeping tools, but the advanced student should know why their theoretical status is more subtle than electric charge.

Discrete symmetries provide another layer. Parity  $P$  reverses spatial coordinates, charge conjugation  $C$  exchanges particles and antiparticles, and time reversal  $T$  reverses the direction of time in the formal symmetry operation. The weak charged-current interaction has a chiral structure that violates  $P$  maximally and also violate  $CP$  in certain processes. The combined  $CPT$  symmetry is deeply connected with broad assumptions of local relativistic quantum field theory and implies, among other consequences, equality of particle and antiparticle masses.

The practical lesson is that conservation laws should be learned with their domain and origin. Some are exact consequences of fundamental symmetries within the accepted framework; others are approximate, accidental, or known to be violated in specific sectors.

Noether energy conservation applies to time-translation symmetry. An expanding cosmological spacetime generally has no global time-translation symmetry. Its local covariant conservation law

remains valid, but one cannot assume a conserved global energy of all comoving matter and radiation as though the spacetime were static.

### A deeper look at QCD: confinement and asymptotic freedom

Quantum chromodynamics is the gauge theory of the strong interaction. Quarks carry color charge and gluons mediate color interactions. Unlike photons, gluons themselves carry color charge, so gluons interact directly with other gluons. This non-Abelian structure produces behavior qualitatively different from electromagnetism.

At high momentum-transfer scale  $Q$ , the effective strong coupling decreases. This is *asymptotic freedom*. At low scales the coupling becomes large and perturbative expansions cease to be reliable. A schematic one-loop expression is

$$\alpha_s(Q^2) \simeq \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad (24.81)$$

for  $n_f$  active quark flavours, sufficiently large  $Q$ , and a convention-dependent QCD scale  $\Lambda_{\text{QCD}}$ . The formula is not to be used near  $Q \sim \Lambda_{\text{QCD}}$ , where perturbation theory fails.

Confinement and asymptotic freedom explain the apparently paradoxical pair of observations: quarks are never isolated as low-energy particles, yet at high-energy scattering they can behave approximately like weakly interacting pointlike constituents over short distances. Deep-inelastic scattering made this short-distance partonic structure experimentally visible.

#### Key message

QCD changes character with scale. At high  $Q$ , short-distance calculations can use a relatively small running coupling. At low  $Q$ , strong nonperturbative dynamics produces hadrons. “The strength of the strong force” is therefore not one fixed number independent of scale.

### Partons, PDFs, and factorization intuition

A fast proton in a high-energy collision cannot be modeled as exactly three stationary valence quarks. Its quantum state contains quarks, antiquarks, and gluons carrying different fractions of the proton longitudinal momentum. Parton distribution functions (PDFs) encode the probability-density-like information needed to predict hard scattering rates.

A schematic hadron-collider cross section has the factorized form

$$\sigma_{pp \rightarrow X} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \hat{\sigma}_{ij \rightarrow X}(x_1, x_2; \mu_F, \mu_R). \quad (24.82)$$

Here  $f_i(x, \mu_F)$  is a PDF for parton species  $i$ ,  $x$  is a momentum fraction,  $\hat{\sigma}$  is the short-distance partonic cross section, and  $\mu_F, \mu_R$  are factorization and renormalization scales. This is far beyond what students are expected to calculate in Physics 2, but the structure explains two important experimental facts:

1. the full proton-proton collision energy is not generally available to a single hard parton-parton collision;
2. predictions require both perturbative short-distance physics and empirically constrained information about proton structure.

For a hard system of invariant mass  $M$  produced near zero rapidity in a proton collider of energy  $\sqrt{s}$ , a rough symmetric estimate is

$$x_1 \sim x_2 \sim \frac{M}{\sqrt{s}}. \quad (24.83)$$

Producing very massive systems therefore probes larger momentum fractions, where parton densities are smaller and statistical rates often fall rapidly.

### Pedagogical boundary

Equation (24.82) is presented only to show the architecture of a hadron-collider prediction. DGLAP evolution, factorization proofs, higher-order QCD corrections, resummation, and PDF fitting are beyond the present course.

The probability-density interpretation is most direct at leading order. Beyond leading order, PDFs depend on a factorization convention and are not individually observable probabilities. For the estimate  $x_1 x_2 s \simeq M^2$ , natural units are used, and a leading-order system with negligible transverse recoil is assumed. With explicit  $c$  the right-hand side is  $M^2 c^4$ .

### Electroweak unification and the Higgs mechanism: structural view

At a deeper level, electromagnetic and weak interactions are described together by an electroweak gauge theory. Before symmetry breaking, the fields are organized according to an  $SU(2)_L \times U(1)_Y$  gauge symmetry. The Higgs field develops a nonzero vacuum expectation value  $v \simeq 246$  GeV. Linear combinations of the electroweak gauge fields become the physical photon and  $Z$  boson, while charged combinations become  $W^\pm$ .

At tree level,

$$m_W = \frac{1}{2}gv, \quad m_Z = \frac{1}{2}v\sqrt{g^2 + g'^2}, \quad (24.84)$$

where  $g$  and  $g'$  are electroweak gauge couplings. The photon remains massless. A fermion  $f$  with Yukawa coupling  $y_f$  has mass

$$m_f = \frac{y_f v}{\sqrt{2}}. \quad (24.85)$$

These equations show that the Higgs vacuum scale sets the electroweak mass scale, while the wide hierarchy of fermion masses resides in the Yukawa couplings  $y_f$ .

The measured Higgs-boson mass is close to 125 GeV, and precision measurements continue to test whether its couplings follow the Standard Model pattern. Within present precision, the observed state is consistent with the Standard Model Higgs boson. New physics could still appear as small coupling deviations, additional Higgs states, invisible decays, or entirely different phenomena at higher energy.

### Advanced case study: the Higgs discovery logic

The Higgs boson provides a useful synthesis of collider concepts. It is produced in high-energy collisions through several mechanisms and decays through multiple channels. Some channels have large rates but difficult backgrounds; others have small branching fractions but excellent mass resolution.

Two historically important channels are  $H \rightarrow \gamma\gamma$  and  $H \rightarrow ZZ^* \rightarrow 4\ell$ . The diphoton channel reconstructs a narrow invariant-mass peak from two electromagnetic showers. The four-lepton channel reconstructs charged-particle momenta with very low background and provides excellent kinematic information. The fact that independent detectors and independent final states found compatible excesses near the same mass was essential to the discovery interpretation.

The lesson is not to memorize a list of decay channels. It is to understand complementarity. A measurement program combines channels with different systematic uncertainties and signal-to-background ratios. A particle hypothesis becomes stronger when one mass and one set of couplings consistently explains several final states.

Current Higgs physics continues this logic. The question is no longer merely whether a Higgs-like state exists, but whether its production rates, decay rates, spin-parity properties, self-interactions, and couplings to different particles match the Standard Model. Small deviations could reveal new physics even if no new resonance is directly produced.

### Advanced case study: top quark as a short-lived probe

The top quark is exceptionally heavy and decays through the weak interaction before it can form ordinary long-lived hadrons. Its dominant decay is  $t \rightarrow Wb$ . This makes top events a laboratory for weak interactions, heavy-flavour tagging, jet reconstruction, missing momentum, and spin correlations.

A top-antitop event can have different detector signatures depending on how the two  $W$  bosons decay. If both  $W$  bosons decay hadronically, the event has many jets and no prompt neutrino from the  $W$  decays. If one  $W$  decays leptonically, the event contains a charged lepton, missing transverse momentum, and jets. If both decay leptonically, two charged leptons and two neutrinos appear, making full kinematic reconstruction more difficult.

This is a useful illustration of branching fractions. A single produced process generates multiple experimental channels. The optimal analysis strategy may differ by channel, and combining them improves the overall test.

### Neutrino mixing beyond the two-flavour picture

For three flavours,

$$|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle, \quad \alpha = e, \mu, \tau, \quad (24.86)$$

where  $U$  is the PMNS mixing matrix. The oscillation phase depends on mass-squared differences, not absolute masses. In vacuum, phase differences scale as

$$\Delta\phi_{ij} \sim \frac{\Delta m_{ij}^2 c^3 L}{2\hbar E}. \quad (24.87)$$

Therefore oscillation experiments measure mixing angles and  $\Delta m^2$  values but do not by themselves determine the absolute neutrino mass scale.

When neutrinos pass through matter, coherent forward scattering on electrons changes the effective propagation Hamiltonian. This is the MSW effect. It is important for solar neutrinos and for some long-baseline experiments. The phenomenon is a striking example in which the medium modifies quantum interference without simply absorbing the neutrino beam.

Open neutrino questions include the ordering of the mass eigenstates, the absolute mass scale, the size of leptonic CP violation, whether neutrinos are Dirac or Majorana fermions, and whether additional sterile states exist.

#### Example 24.24: Two-flavour conversion probability

Take  $\sin^2(2\theta) = 0.95$ ,  $\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2/c^4$ ,  $L = 295 \text{ km}$ , and  $E = 0.60 \text{ GeV}$ . Equation (24.27) gives the phase

$$\Phi = 1.267 \frac{(2.5 \times 10^{-3})(295)}{0.60} \simeq 1.56.$$

Thus  $\sin^2 \Phi \simeq 1$  and

$$P \simeq 0.95.$$

The parameters were chosen near an oscillation maximum. A realistic three-flavour experimental

probability is more complicated, especially in matter, but the example shows how  $L/E$  selects sensitivity.

**Two-state interference in vacuum.** With a real mixing angle,

$$|\nu_\alpha\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle, \quad |\nu_\beta\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle.$$

Each component acquires its own propagation phase. For ultrarelativistic coherent propagation over a distance  $L$ , their phase difference is  $\Delta\phi \simeq \Delta m^2 c^3 L / (2\hbar E)$ . Projection onto the second flavour gives, up to a common phase,

$$\mathcal{A}_{\alpha\beta} = \sin\theta \cos\theta (e^{-i\phi_2} - e^{-i\phi_1}),$$

and therefore

$$P_{\alpha\beta} = \sin^2(2\theta) \sin^2(\Delta\phi/2), \quad P_{\alpha\alpha} = 1 - P_{\alpha\beta}.$$

The mass splitting sets the period; mixing sets the amplitude. If either the splitting or the mixing vanishes, flavour conversion disappears. Resolving or separating the mass wave packets can reduce coherence. Matter effects and the third state require the fuller framework [5].

### Advanced case study: neutrino mass measurements are complementary

Oscillation experiments measure mass-squared differences, not the absolute neutrino mass scale. Other approaches address different combinations of masses. Beta-decay endpoint experiments probe an effective electron-neutrino mass through kinematics. Cosmology constrains the sum of neutrino masses through their effect on expansion and structure growth, but the result is model dependent. Neutrinoless double-beta decay, if observed, would test lepton-number violation and the Majorana nature of neutrinos, with an interpretation that also involves nuclear matrix elements.

These methods are complementary precisely because they depend on different physical principles. Agreement could overconstrain the neutrino sector; disagreement might reveal hidden assumptions or new physics. This is a recurring theme in fundamental physics: the strongest conclusion often comes from combining observables that depend differently on the unknown parameter.

### Effective field theory: why incomplete theories can be accurate

Physics often separates scales. If new particles have characteristic mass  $\Lambda$  much larger than the energy  $E$  of an experiment, their detailed dynamics may be inaccessible, yet they can leave suppressed low-energy effects. Effective field theory organizes those effects as an expansion in powers of  $E/\Lambda$ .

Schematically one writes

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{C^{(5)}}{\Lambda} \mathcal{O}^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{C_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \dots, \quad (24.88)$$

Here  $c = \hbar = 1$ ,  $\Lambda$  is an energy scale, and the Lagrangian density has mass dimension four. The dimension-five Weinberg operator uses two lepton doublets and two Higgs fields and can generate a Majorana neutrino mass; it violates lepton number. Omitting it requires an additional assumption, such as imposed lepton-number conservation. The higher-dimensional operators  $\mathcal{O}_i$  respect chosen low-energy symmetries and the coefficients encode unknown high-scale physics. The exact operator basis belongs to advanced particle physics, but the organizing principle is accessible: high-scale effects become progressively smaller at low energy unless a symmetry or coupling enhances them.

This explains how the Standard Model can remain extraordinarily accurate even if it is not fundamental at arbitrarily high scales. New physics does not necessarily produce a dramatic new

particle at the first accessible energy. It may instead shift a precision observable by a tiny amount. Precision measurements and direct high-energy searches are therefore complementary strategies.

Effective-theory thinking also appears outside particle physics. Fluid mechanics ignores molecular details at long wavelength; circuit theory ignores microscopic charge dynamics in many engineering regimes; geometric optics ignores wave structure when wavelength is much smaller than system dimensions. A theory can be intentionally incomplete and still be the correct tool at its scale.

### Key message

“Incomplete” does not mean “incorrect.” An effective theory is judged by whether its assumptions and scale hierarchy are valid for the problem being solved. The Standard Model may be embedded in deeper physics while remaining the quantitatively correct description of presently tested particle processes.

## CP violation and the matter-antimatter problem

If the laws of physics treated matter and antimatter in exactly identical ways under all relevant conditions, producing a large cosmic matter excess would be difficult. The Standard Model does contain CP violation, observed in quark-flavour processes and incorporated through a complex phase in the CKM mixing matrix. The neutrino sector may also contain leptonic CP violation through the PMNS matrix, although its precise size remains an experimental question.

CP violation does not mean that energy or electric charge fails to be conserved. It means that the probability for a process can differ from the probability for its CP-conjugate process. A schematic asymmetry is

$$A_{CP} = \frac{\Gamma(P \rightarrow f) - \Gamma(\bar{P} \rightarrow \bar{f})}{\Gamma(P \rightarrow f) + \Gamma(\bar{P} \rightarrow \bar{f})}. \quad (24.89)$$

A nonzero  $A_{CP}$  signals CP violation in that decay comparison, after experimental effects are controlled.

The amount and pattern of known Standard Model CP violation are not sufficient, under standard assumptions, to explain the observed baryon asymmetry of the universe. This motivates searches for additional CP-violating interactions in flavour experiments, electric-dipole-moment measurements, neutrino oscillations, and possible new-particle sectors.

**Baryogenesis conditions** The observable universe contains far more matter than antimatter. A hot early universe naturally produces particle-antiparticle pairs, so the surviving baryon asymmetry requires an explanation. Sakharov identified three broad ingredients needed for dynamical baryogenesis: baryon-number violation, violation of charge conjugation and CP symmetry, and departure from thermal equilibrium.

The Standard Model contains CP violation and nonperturbative baryon-plus-lepton-number violation at high temperature, but the known effects are not believed to generate the observed asymmetry quantitatively under standard conditions. This makes baryogenesis a major motivation for physics beyond the Standard Model.

The lesson for this chapter is conceptual rather than computational. The presence of antimatter in the theory does not imply a matter-antimatter symmetric universe today. Initial conditions and nonequilibrium dynamics matter, and an observed cosmic asymmetry can become a precision constraint on microscopic physics.

### 24.30.4 Quantitative cosmology and observational tests

### Scale factor, Hubble parameter, and the Friedmann equation

A homogeneous and isotropic expanding universe is described by the scale factor  $a(t)$ . The Hubble parameter is

$$H(t) = \frac{\dot{a}(t)}{a(t)}. \quad (24.90)$$

In general relativity, the first Friedmann equation for a homogeneous universe can be written

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \quad (24.91)$$

Here  $a$  is dimensionless with  $a_0 = 1$ . Accordingly,  $k$  has units  $\text{m}^{-2}$  and the curvature term has units  $\text{s}^{-2}$ ;  $k$  is not restricted to the numerical values  $-1, 0, +1$  in this convention. The density  $\rho$  includes the components other than the explicitly written cosmological constant. Define  $\rho_\Lambda = \Lambda c^2/(8\pi G)$  if it is to be included in a total density budget instead. These are equivalent conventions, but including  $\rho_\Lambda$  inside  $\rho$  and also writing  $\Lambda$  separately would count the same component twice [8].

The critical density is defined by the density that makes the curvature term vanish for a specified  $H$  when  $\Lambda$ -like contributions are included in the total density budget:

$$\rho_c = \frac{3H^2}{8\pi G}. \quad (24.92)$$

Density parameters are then

$$\Omega_i = \frac{\rho_i}{\rho_c}. \quad (24.93)$$

They provide a dimensionless way to discuss the cosmic inventory.

#### Example 24.25: Present critical density

Take  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Convert to SI:

$$H_0 = \frac{70 \times 10^3 \text{ m/s}}{3.086 \times 10^{22} \text{ m}} \simeq 2.27 \times 10^{-18} \text{ s}^{-1}.$$

Then

$$\rho_c = \frac{3H_0^2}{8\pi G} \simeq \frac{3(2.27 \times 10^{-18})^2}{8\pi(6.674 \times 10^{-11})} \simeq 9.2 \times 10^{-27} \text{ kg/m}^3.$$

This density is extraordinarily small by laboratory standards, yet it is cosmologically decisive because it characterizes the mean density on enormous scales.

#### Caution

The critical density is not the density of ordinary matter alone. In cosmological parameterization, the total energy budget can include baryons, dark matter, radiation, neutrinos, and dark energy. Confusing one component with the total is a common source of incorrect conclusions.

The second independent expansion relation is the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3}. \quad (24.94)$$

Here  $\rho$  and  $p$  exclude the separately written cosmological constant, as in Eq. (24.91). Alternatively one may include  $\rho_\Lambda$  and  $p_\Lambda = -\rho_\Lambda c^2$  in the fluid sum and omit the explicit  $\Lambda$  term. Acceleration depends on pressure as well as density; a large matter density alone does not accelerate expansion.

**Example 24.26: Density equality, acceleration, and a model age**

Consider a spatially flat model containing pressureless matter and a cosmological constant, with  $\Omega_{m0} = 0.30$ ,  $\Omega_{\Lambda 0} = 0.70$ , and  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Radiation is neglected in this illustrative age calculation. The expansion law is

$$H(a) = H_0 \sqrt{\Omega_{m0} a^{-3} + \Omega_{\Lambda 0}}.$$

Density equality occurs at  $a_{\text{eq}} = (0.30/0.70)^{1/3} \simeq 0.754$ , or  $z_{\text{eq}} \simeq 0.326$ . Equation (24.94) instead sets the acceleration transition at  $\rho_m = 2\rho_\Lambda$ :  $a_{\text{acc}} \simeq 0.598$  and  $z_{\text{acc}} \simeq 0.671$ .

Because  $\dot{a} = aH$ , the age follows by integrating time:

$$t_0 = \int_0^1 \frac{da}{aH(a)} = \frac{2}{3H_0\sqrt{\Omega_{\Lambda 0}}} \text{arsinh} \sqrt{\frac{\Omega_{\Lambda 0}}{\Omega_{m0}}}.$$

Substitution gives  $t_0 \simeq 13.47 \text{ Gyr}$ , whereas  $H_0^{-1} \simeq 13.97 \text{ Gyr}$ . The Hubble time is therefore a useful scale, while the age requires an expansion history. These results belong to the specified model, not to a model-independent measurement.

**How cosmic energy densities scale with expansion**

For a separately conserved homogeneous fluid in a comoving volume, local energy conservation gives

$$d(\rho c^2 a^3) = -p da^3.$$

Expanding and dividing by  $c^2 a^3$ ,

$$d\rho = -3 \left( \rho + \frac{p}{c^2} \right) \frac{da}{a}.$$

If  $p = w\rho c^2$  with constant  $w$ , integration yields  $\ln \rho = -3(1+w) \ln a + \text{constant}$ . This derivation assumes no energy transfer to another component. Redshift of photon energy does not imply transfer of that energy into dark matter; a conserved total energy of the entire expanding spacetime is generally unavailable.

For a homogeneous component with equation of state  $p = w\rho c^2$  and constant  $w$ , energy conservation in an expanding universe gives

$$\rho(a) \propto a^{-3(1+w)}. \quad (24.95)$$

Important cases are

$$\text{nonrelativistic matter: } w = 0, \quad \rho_m \propto a^{-3}, \quad (24.96)$$

$$\text{radiation: } w = \frac{1}{3}, \quad \rho_r \propto a^{-4}, \quad (24.97)$$

$$\text{cosmological constant: } w = -1, \quad \rho_\Lambda = \text{constant}. \quad (24.98)$$

The extra factor of  $a^{-1}$  for radiation compared with matter arises because photon number density falls as  $a^{-3}$  while each photon also loses energy through cosmological redshift,  $E_\gamma \propto a^{-1}$ .

These different scalings imply cosmic transitions. Radiation dominated the energy density at sufficiently early times; matter later became dominant; a nearly constant dark-energy density can dominate at late times after matter has diluted enough.

**Guided checks**

1. If the scale factor doubles, by what factor does nonrelativistic matter density decrease?
2. If the scale factor doubles, by what factor does radiation energy density decrease?
3. Why can a constant dark-energy density become dynamically important only at late times even if it was present earlier?

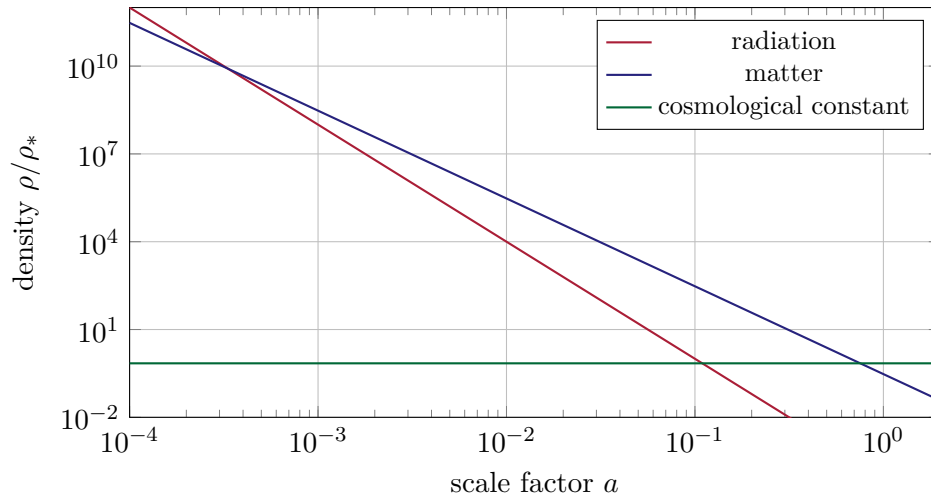


**Figure 24.15:** Illustrative density scalings with arbitrary reference density  $\rho_*$ . The chosen coefficients are  $C_r = 10^{-4}$ ,  $C_m = 0.30$ , and  $C_\Lambda = 0.70$ ; they illustrate a sequence of dominance and are not a precision cosmic inventory. Radiation–matter equality occurs at  $a = C_r/C_m$ , and matter–vacuum equality at  $a = (C_m/C_\Lambda)^{1/3}$ . The acceleration transition is a different condition.

### Temperature scaling and the expanding radiation bath

For freely propagating blackbody radiation in an expanding universe,

$$T \propto \frac{1}{a}, \quad (24.99)$$

so

$$T(z) = T_0(1 + z). \quad (24.100)$$

This follows from the redshifting of photon wavelengths together with preservation of the blackbody form under expansion. If the CMB temperature today is  $T_0 \simeq 2.725$  K, then at redshift  $z = 1000$  the corresponding radiation temperature was of order

$$T \simeq 2.7 \times 10^3 \text{ K}.$$

The detailed recombination history depends on atomic kinetics, baryon density, radiation density, and expansion rate, so the estimate should not be mistaken for a precise recombination calculation.

Entropy conservation introduces a refinement when the number of relativistic particle species changes. Annihilation of a species can transfer entropy to species still in thermal contact. This is why the cosmic neutrino background has a lower temperature than the photon CMB in the standard thermal history. Such details belong to thermal cosmology, but the qualitative message is important:  $T \propto 1/a$  is cleanest when no entropy is being redistributed between components.

### Big-bang nucleosynthesis as a reaction network

Big-bang nucleosynthesis (BBN) links nuclear physics to cosmology. As the early universe cooled through MeV-scale temperatures, weak interactions set the neutron-to-proton ratio, free-neutron

decay modified it, and nuclear reactions eventually built light nuclei. The expansion rate competes with reaction rates: a process remains in equilibrium only while interactions are sufficiently rapid compared with cosmic expansion.

A generic reaction rate per target has the form

$$\Gamma_{\text{int}} \sim n \langle \sigma v \rangle, \quad (24.101)$$

Here  $\Gamma_{\text{int}}$  has units of inverse time, unlike an energy-valued decay width  $\Gamma$ . The symbol  $n$  is the number density of reaction partners and  $\langle \sigma v \rangle$  is a thermally averaged cross-section factor. Freeze-out occurs qualitatively when

$$\Gamma_{\text{int}} \lesssim H. \quad (24.102)$$

This competition between microscopic interaction rate and macroscopic expansion rate is a unifying concept in early-universe physics.

The observed primordial abundances of light elements are therefore not merely historical curiosities. They are quantitative probes of baryon density, nuclear reaction rates, and the expansion history during the first minutes.

### Recombination, optical depth, and the last-scattering surface

Before recombination, free electrons scattered photons efficiently through Thomson scattering. The optical depth accumulated along a photon path from emission to observation is

$$\tau(t_{\text{em}}, t_0) = \int_{t_{\text{em}}}^{t_0} n_e(t) \sigma_T c dt.$$

For a forward time interval its positive increment is

$$d\tau = n_e \sigma_T c dt, \quad (24.103)$$

where  $n_e$  is the free-electron density and  $\sigma_T$  the Thomson cross section. The probability for a photon to propagate without scattering is roughly  $\exp(-\tau)$ .

As the ionization fraction fell, the photon mean free path grew rapidly and the universe became transparent. The CMB therefore originates from a finite-thickness last-scattering region rather than an infinitely sharp geometric surface. Precision calculations track the ionization history and visibility function, but the elementary optical-depth language already connects cosmology to earlier material on scattering, exponential attenuation, and mean free path.

### CMB anisotropies and acoustic physics

Prior to recombination, photons and baryons behaved approximately as a coupled fluid. Gravity pulled overdense regions inward, while radiation pressure resisted compression. The competition produced acoustic oscillations. Their characteristic angular scales are imprinted in the CMB temperature and polarization power spectra.

The positions and heights of acoustic peaks depend on multiple cosmological parameters: baryon density, matter density, expansion history, spatial geometry, primordial fluctuation spectrum, and more. This sensitivity is why CMB observations can infer cosmological parameters with impressive precision – but only within a specified model. A quoted parameter value is therefore always a combination of data and theoretical assumptions.

**Key message**

Precision cosmology is model-based inference. The CMB directly provides sky maps of temperature and polarization. Parameters such as  $H_0$ ,  $\Omega_m$ , and the scalar spectral index are inferred by comparing statistical properties of those maps with cosmological predictions.

**The angular power spectrum** A CMB sky map can be expanded in spherical harmonics,

$$\frac{\Delta T(\theta, \phi)}{T} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi). \quad (24.104)$$

For a statistically isotropic sky, the angular power spectrum is characterized by

$$C_\ell = \langle |a_{\ell m}|^2 \rangle. \quad (24.105)$$

Roughly, angular multipole  $\ell$  corresponds to structures on angular scales of order  $180^\circ/\ell$ . Low  $\ell$  describes large angular scales; high  $\ell$  describes small angular scales.

The acoustic peaks arise because different Fourier modes of the pre-recombination photon-baryon fluid reached different phases of compression or rarefaction by the time photons decoupled. The first peak is closely related to the angular size of the sound horizon. Relative peak heights encode the baryon and matter content. Damping at high  $\ell$  reflects photon diffusion and the finite thickness of the last-scattering surface.

The important computational point is that cosmological inference does not fit individual hot and cold spots. It fits statistical summaries such as temperature and polarization power spectra, together with covariance matrices and instrument beam/noise models.

The brackets in  $C_\ell$  denote an ensemble average. From one ideal, full-sky map one estimates

$$\hat{C}_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2.$$

After subtracting the monopole and dipole, cosmological temperature anisotropies are usually analysed from  $\ell = 2$ . For a statistically isotropic Gaussian sky, even a noiseless full-sky observation has  $\sigma(\hat{C}_\ell)/C_\ell = \sqrt{2/(2\ell + 1)}$ , the cosmic-variance limit. A mask and instrument noise add further complications [9].

**Baryon acoustic oscillations as a standard ruler**

The same acoustic physics that leaves peaks in the CMB also imprints a preferred comoving separation scale in the late-time distribution of matter. This baryon acoustic oscillation (BAO) feature acts as a standard ruler. Galaxy and quasar surveys measure the apparent radial and transverse BAO scale as a function of redshift, constraining the expansion history.

BAO does not provide an absolute ruler from geometry alone. Its calibration is connected to early-universe physics through the sound horizon. Consequently, BAO constraints are especially powerful when combined with CMB information or other calibrations.

This provides another example of model architecture: early-universe plasma physics calibrates a scale, large-scale structure preserves a fossil imprint, and late-time spectroscopy measures how that scale appears after cosmic expansion.

### The distance ladder and calibration chains

Local measurements of  $H_0$  often use a distance ladder. Nearby geometric distance indicators calibrate objects such as Cepheid variables or other standardizable sources. These calibrate Type Ia supernovae at larger distances, which then map the local Hubble flow.

Each rung transfers calibration information. An error in an early rung can propagate coherently through the chain. This is why correlated systematics, metallicity effects, extinction, crowding, detector calibration, and sample selection receive intense scrutiny. Independent geometric anchors and alternative stellar populations can test the robustness of the chain.

The methodological lesson is broadly applicable: a derived quantity can depend on a hierarchy of calibrations. A final uncertainty should account for the covariance of the entire chain, not only the scatter of the last regression.

### The Hubble tension as a measurement lesson

The present Hubble parameter is a particularly instructive example of how different measurement strategies probe the same parameter. A local distance-ladder program calibrates standard candles or related distance indicators and measures the recent expansion directly. An early-universe inference uses CMB data plus a cosmological model to infer what  $H_0$  must be today. These methods depend on different instruments, calibrations, and physical assumptions.

Planck CMB inference in base  $\Lambda$ CDM gives a value near  $67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , whereas the Cepheid-calibrated SH0ES supernova distance ladder gives a value near 73. Other late-universe methods and calibrations give different central values and uncertainties; there is no single uniform result from all local methods. The 2026 review summarizes the continuing tension and the need to test calibration, systematics, and model assumptions [8].

This situation illustrates a general rule: disagreement between high-precision methods is not resolved by averaging them blindly. One must ask whether the methods measure exactly the same model parameter, whether their systematic uncertainties are understood, and whether the assumed theoretical framework is adequate.

#### Note

For classroom numerical exercises, using  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$  is convenient and pedagogically harmless when it is clearly identified as a representative round value rather than a precision world average.

### Dark matter from rotation curves, lensing, and structure

Rotation curves are only one line of evidence. Gravitational lensing measures projected gravitating mass through the bending and distortion of background light. In galaxy clusters, lensing mass maps can differ strongly from the distribution of luminous baryonic matter. Large-scale structure provides another constraint because the growth of density perturbations depends on the amount and clustering properties of matter.

A viable dark-matter model must therefore work across multiple regimes: galactic dynamics, cluster masses, cosmological structure formation, and CMB constraints. This is why replacing particle dark matter by “some unseen stars” does not solve the full problem. The baryonic matter density itself is independently constrained by nucleosynthesis and CMB physics.

At the microscopic level many candidates have been proposed: weakly interacting massive particles, axion-like particles, sterile neutrinos, primordial black holes in allowed mass windows, and other hidden-sector possibilities. No single candidate has been experimentally established as the dominant

dark matter. A scientifically careful introductory text should present these as hypotheses, not discoveries.

**Gravitational lensing as a mass measurement** General relativity predicts that mass-energy bends light. In the weak-lensing regime, images of distant galaxies are distorted by foreground large-scale structure. Individual galaxy shapes are intrinsically irregular, so the signal is extracted statistically from correlated distortions of many galaxies.

For a simple point-mass lens, the characteristic Einstein angle is

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}}, \quad (24.106)$$

where  $D_L$ ,  $D_S$ , and  $D_{LS}$  are angular-diameter distances to the lens, to the source, and between lens and source. The formula shows that lensing depends on both mass and cosmic geometry.

Cluster lensing is particularly important for dark matter because it maps gravitating mass independently of the light emitted by stars or hot gas. Comparing lensing maps with baryonic tracers can reveal a dominant unseen mass component. Weak-lensing cosmology similarly constrains how large-scale structure grows over time.

#### Caution

Gravitational lensing does not identify the microscopic nature of dark matter. It maps gravitational potential or projected mass. Converting that map into a statement about particle species requires additional evidence.

### Particle dark-matter searches: three complementary strategies

If dark matter consists of new particles, experiments can search in several logically distinct ways.

**Direct detection.** Look for scattering of ambient Galactic dark matter from nuclei or electrons in a shielded detector. The challenge is an extremely small expected rate and backgrounds from radioactivity, neutrons, and neutrinos.

**Indirect detection.** Search for photons, antimatter, or neutrinos produced by dark-matter annihilation or decay in regions of high astrophysical density. The challenge is separating a possible signal from conventional astrophysical sources.

**Collider production.** Attempt to produce invisible new particles in high-energy collisions. Since they would escape the detector, search for visible recoil objects plus missing transverse momentum. The challenge is that collider missing momentum is not unique to dark matter and cannot by itself establish cosmological stability or abundance.

Agreement across these strategies would be powerful because the systematic uncertainties are very different. A signal in one channel alone would require careful interpretation.

### Dark energy, $w$ , and the status of evolving models

The cosmological constant corresponds to  $w = -1$  exactly and constant energy density. A phenomenological extension often writes

$$w(a) = w_0 + w_a(1 - a), \quad (24.107)$$

known as the CPL parameterization. Values  $w_0 = -1$ ,  $w_a = 0$  recover a cosmological constant.

Redshift surveys measure baryon acoustic oscillation scales and structure growth, while supernovae constrain relative luminosity distances. Their combinations with CMB data test the expansion history. The 2026 PDG review discusses DESI combinations that can prefer  $w_0$ – $w_a$  extensions; the preference depends on the supernova sample and other data choices. Such a model preference is not an established identification of evolving dark energy [10].

This is a useful case study in scientific communication. A statistical preference for an extended model can depend on data combination, parameterization, systematic treatment, and prior assumptions. Strong claims should wait for robustness across methods.

### Gravitational waves as complementary cosmological probes

Gravitational waves are ripples in spacetime predicted by general relativity and directly detected from compact-object mergers. They lie outside the Standard Model and are not electromagnetic waves, but they provide a useful bridge between particle physics, gravity, and cosmology.

A compact binary waveform encodes masses, spins, distance-related amplitude information, and orbital dynamics. If an independent redshift is identified, a gravitational-wave source can act as a “standard siren” for cosmic expansion. Future observations may therefore contribute independent constraints on  $H_0$  and the expansion history.

The pedagogical value here is methodological: multiple messengers – electromagnetic radiation, neutrinos, cosmic rays, and gravitational waves – probe different physical interactions and systematic uncertainties. Agreement across them strengthens a physical interpretation.

### Relic abundance and the freeze-out idea

A generic particle species in the early universe can remain in thermal equilibrium while reactions creating and destroying it occur much faster than cosmic expansion. As temperature falls, its equilibrium abundance may become exponentially suppressed if it is massive, and eventually the interaction rate can become too slow to track equilibrium.

For a homogeneous, self-conjugate species with pair annihilation into a thermal bath and no particle–antiparticle asymmetry, a number-density equation has the simplified form

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle (n^2 - n_{\text{eq}}^2). \quad (24.108)$$

The  $3Hn$  term describes dilution from expansion. The right-hand side describes annihilation and production relative to equilibrium. Solving this Boltzmann equation quantitatively is beyond the course, but the competition of rates gives strong intuition. Large interaction cross sections keep equilibrium longer and can reduce the surviving relic abundance; small cross sections allow earlier decoupling and potentially larger relic abundance.

This mechanism historically motivated classes of weakly interacting massive dark-matter candidates. The fact that a simple thermal-relic picture can produce an abundance of the observed order for weak-scale interactions is suggestive, not proof that dark matter is a weak-scale WIMP. Null direct-detection and collider searches have pushed many simple models into increasingly constrained regions, while numerous alternative candidates remain viable.

### Horizon scales and causal contact

The finite speed of light limits causal communication. In an expanding universe, the maximum comoving distance from which a signal could have reached an observer since an initial time is related to an integral of  $c/a(t)$ . This motivates the concept of a particle horizon.

The near-uniformity of widely separated CMB regions creates the classical horizon problem when the simplest decelerating hot Big Bang is extrapolated backward: regions now far apart appear

to have had insufficient causal contact to equilibrate before recombination. Cosmic inflation is a proposed early period of accelerated expansion that can place those regions in causal contact before stretching them far apart.

Inflation also provides a mechanism for generating primordial perturbations from quantum fluctuations, later amplified into cosmic structure. However, inflation is a framework with many models, and the precise inflaton field or underlying microphysics is not experimentally established. For this course inflation should be treated as an important extension of the hot Big Bang model, not as a fully identified fundamental particle theory.

### 24.30.5 Simulation, reconstruction, and open investigations

#### Monte Carlo simulation and detector response

Monte Carlo methods are essential because particle reactions are probabilistic and detector response is complex. A simplified simulation chain can be organized as:

1. generate a hard physical process according to a probability distribution;
2. decay unstable particles;
3. model parton showering and hadronization when quarks or gluons are present;
4. transport stable particles through detector material;
5. simulate ionization, showers, electronic response, thresholds, and noise;
6. reconstruct the simulated detector signals with the same algorithms used for data.

The final comparison should therefore be made at the level of reconstructed observables, not between ideal theoretical particles and raw detector channels.

Random sampling does not make a simulation arbitrary. The underlying probability distributions are derived from physical models and calibrated detector descriptions. Reproducibility requires recording generator settings, random seeds when relevant, software versions, calibration conditions, and analysis selections.

#### Method

A strong computational project should validate each layer separately. First check analytic limiting cases. Then check generator-level distributions. Then introduce detector smearing. Only afterward perform a full inference. Debugging is much easier when model, detector, and analysis effects are not mixed prematurely.

#### Advanced case study: why a resonance peak is persuasive

A resonance is a state that can be produced and decay over a finite lifetime. Near an isolated resonance, a simplified amplitude can contain a propagator factor

$$\frac{1}{s - M^2 + iM\Gamma} \quad (24.109)$$

in natural units. Squaring the magnitude produces the familiar Breit–Wigner structure

$$|A(s)|^2 \propto \frac{1}{(s - M^2)^2 + M^2\Gamma^2}. \quad (24.110)$$

The observed detector mass shape is a convolution of intrinsic width, detector resolution, radiation, and event reconstruction. A narrow intrinsic resonance may therefore appear much broader than its natural width if detector resolution dominates.

A convincing resonance claim requires more than a peak-shaped fluctuation. Analysts test whether the peak appears where expected across categories, whether independent decay channels are compatible, whether calibration control samples validate the mass scale, and whether the local excess remains significant after accounting for the possibility of fluctuations over a wider search range. This last issue is related to the look-elsewhere effect.

The logic generalizes. A scientific signal should have a morphology predicted by a model and should survive changes in analysis choices that ought not to destroy a genuine effect. A single tuned histogram is not a robust discovery strategy.

**A reconstruction workflow** Consider a hypothetical neutral resonance  $X$  decaying to two photons. A complete experimental reasoning chain contains several distinct layers:

1. electromagnetic calorimeter clusters are reconstructed and identified as photon candidates;
2. cluster energies and directions are calibrated;
3. the diphoton invariant mass is computed with Eq. (24.46);
4. many events are accumulated into a mass histogram;
5. a smooth background model and a narrow signal model are compared with the data;
6. detector resolution broadens the reconstructed peak beyond any negligible intrinsic width;
7. systematic uncertainties on energy scale, resolution, identification efficiency, and background shape enter the statistical model;
8. a likelihood fit estimates a signal rate and its uncertainty.

No individual event “proves” the resonance. The evidence comes from a distribution and its statistical consistency with competing hypotheses. This logic is general: scientific inference often occurs one level above the raw measurement.

### Open questions as experimentally testable programs

The phrase “physics beyond the Standard Model” can sound vague. In practice it becomes a set of concrete experimental programs:

- searches for new resonances or contact interactions at high energy;
- precision tests of Higgs, electroweak, and flavour observables;
- searches for charged-lepton flavour violation;
- neutrino mass ordering and CP-violation measurements;
- direct, indirect, and collider searches for dark matter;
- proton-decay and baryon-number-violation searches;
- electric-dipole-moment searches for new CP violation;
- cosmological tests of dark energy, neutrino mass, and primordial physics;
- gravitational-wave tests of compact objects and strong-field gravity.

Each program asks a sharply defined question with measurable observables. Progress may come from discovering a new particle, observing a forbidden process, finding a statistically significant

deviation from a Standard Model prediction, or shrinking the allowed parameter space until broad classes of models are excluded.

### Advanced enrichment summary

The advanced material reveals a common mathematical structure across particle physics and cosmology. Lorentz invariants separate physics from coordinate choices. Cross sections and luminosity connect microscopic probabilities to event counts. Running couplings and PDFs encode scale-dependent structure. Oscillation phases connect quantum interference to experimental baselines. The Friedmann equation connects cosmic expansion to energy density. Statistical inference and covariance connect all of these theories to imperfect measurements.

The most important advanced habit is to distinguish three layers:

1. **theory parameters and ideal quantities**, such as a particle mass, cross section, or cosmological density;
2. **experimental observables**, such as tracks, energies, redshifts, angular correlations, and count distributions;
3. **inference machinery**, which maps observables back to parameter constraints with uncertainties and assumptions.

This separation is as valuable in engineering and data science as it is in fundamental physics.

## 24.31 Original practice problems

The following 60 problems are original exercises for this lecture-note series. They are arranged from conceptual classification to quantitative particle kinematics, detector physics, computation, and cosmology. Unless a problem states otherwise, keep factors of  $c$  explicit and show units. Useful approximate constants are collected in Appendix B.

### Conceptual questions

- P24.1.** Explain operationally what physicists mean when they call an electron “elementary.” Why is this statement weaker than a proof that the electron can never have substructure at any smaller scale?
- P24.2.** Classify each of the following as a lepton, quark, hadron, gauge boson, scalar boson, or macroscopic/cosmological object where appropriate:  $e^-$ ,  $u$ , proton,  $\pi^+$ , photon, gluon,  $Z$ , Higgs boson, galaxy. Some categories are nested; state the most informative classification.
- P24.3.** Why does the valence label  $p = (uud)$  not mean that a proton is a rigid system containing exactly three pointlike objects and nothing else?
- P24.4.** State two ways in which a Feynman diagram differs from a classical trajectory diagram. Why can two different diagrams contribute to one measured final state?
- P24.5.** Explain why conservation of electric charge is not sufficient to establish that a proposed particle reaction can occur.
- P24.6.** Distinguish total lepton-number bookkeeping from individual flavour labels. Which phenomenon shows directly that neutrino flavour can change during propagation?
- P24.7.** Explain why the Higgs mechanism is essential for elementary-particle masses but the statement “the Higgs boson makes almost all of the proton mass” is misleading.

- P24.8.** Why can an electron-positron collider provide a cleaner initial state than a proton-proton collider even if the proton collider reaches much larger beam energy?
- P24.9.** A track bends clockwise in a known solenoidal magnetic field. What additional information is required to infer the sign of its electric charge? Why does curvature magnitude alone not identify the particle species?
- P24.10.** Explain why the low-redshift formula  $v \simeq H_0 d$  should not be interpreted as a universal special-relativistic speed formula at arbitrary cosmological distance.  
Explain why a redshift measurement alone does not specify a distance without a cosmological model or an independent distance indicator.
- P24.11.** Give two independent qualitative reasons why the CMB supports a hot early-universe picture.
- P24.12.** Compare dark matter and dark energy in terms of their dominant observational role. Why is it misleading to treat them as two varieties of the same phenomenon?
- P24.13.** Why does a flat galactic rotation curve suggest that enclosed gravitating mass continues to increase with radius beyond the bright stellar disk?
- P24.14.** What does it mean to say that a parameter tension may indicate either underestimated systematics or incomplete theory? Why should one not choose between these explanations solely from the existence of disagreement?
- P24.15.** Explain the meaning of freeze-out in the condition  $\Gamma_{\text{int}} \lesssim H$  for an early-universe reaction.

### Analytical and quantitative problems

- P24.16.** Use quark charges to calculate the electric charges of the states  $uud$ ,  $udd$ ,  $uuu$ , and  $ddd$ . Identify the proton and neutron among them. Which two have charges  $+2e$  and  $-e$  respectively?
- P24.17.** The mesons  $\pi^+ = (u\bar{d})$  and  $\pi^- = (d\bar{u})$ . Verify their charges. Then determine the charge of  $K^0 = (d\bar{s})$  and  $K^- = (s\bar{u})$ .  
Also calculate the strangeness of  $K^0 = (d\bar{s})$  and  $\Lambda^0 = (uds)$ . Check charge, baryon number, and strangeness in  $\pi^- p \rightarrow K^0 \Lambda^0$ .
- P24.18.** Check electric charge, baryon number, and total lepton number for neutron beta decay  $n \rightarrow p + e^- + \bar{\nu}_e$ . Then explain qualitatively which fundamental interaction causes the flavour change.
- P24.19.** Test the proposed decay  $\mu^- \rightarrow e^- + \gamma$  using electric charge and simple lepton-flavour bookkeeping. Is charge conservation enough to rule it out? Discuss why experiments can still search for this reaction as evidence of physics beyond the minimal Standard Model.  
State the distinction between traditional lepton-family bookkeeping and the tiny flavour-changing amplitudes possible when neutrinos are massive.
- P24.20.** A particle has rest energy  $mc^2 = 0.500 \text{ GeV}$  and momentum  $p = 2.00 \text{ GeV}/c$ . Find its total energy and kinetic energy.
- P24.21.** A proton has kinetic energy  $10.0 \text{ GeV}$ . Using  $m_p c^2 = 0.938 \text{ GeV}$ , find its total energy, momentum, Lorentz factor, and speed as a fraction of  $c$ .
- P24.22.** A photon has energy  $2.50 \text{ GeV}$ . Find its momentum in  $\text{GeV}/c$  and its wavelength in metres using  $hc = 1.240 \text{ eV } \mu\text{m}$ .

- P24.23.** An electron and positron collide head-on with equal beam energies 3.00 GeV. Neglecting the electron mass, find  $\sqrt{s}$ . Is production of a  $\tau^+\tau^-$  pair kinematically possible if  $m_\tau c^2 \simeq 1.777$  GeV?
- P24.24.** Derive Eq. (24.49) for a projectile of mass  $m$  and total energy  $E$  striking an identical target at rest. Start from total energy and total momentum and use  $E^2 - p^2c^2 = m^2c^4$ .
- P24.25.** Use the fixed-target formula to find the beam total energy required for  $p + p \rightarrow p + p + X$  at threshold if  $X$  has mass energy  $m_X c^2 = 3.0$  GeV and all final particles can be treated as one composite system at threshold. Use  $m_p c^2 = 0.938$  GeV.
- P24.26.** A particle of mass  $M = 5.0$  GeV/ $c^2$  decays at rest to two identical daughters of mass 1.0 GeV/ $c^2$ . Use Eq. (24.51) to find the daughter momentum.
- P24.27.** Two photons have energies 40 GeV and 60 GeV with opening angle  $120^\circ$ . Find their invariant mass.
- P24.28.** A muon with proper lifetime  $2.20 \mu\text{s}$  has energy  $E = 20m_\mu c^2$ . Find  $\gamma$ ,  $\beta$ , and the mean flight distance in the laboratory.
- P24.29.** An unstable hadron has decay width  $\Gamma = 1.0$  MeV. Estimate its mean lifetime using  $\hbar = 6.582 \times 10^{-22}$  MeV s.
- P24.30.** A singly charged track has radius  $R = 1.50$  m in a  $B = 2.0$  T magnetic field. Find  $p_T$ . If  $R$  is measured with 2% relative uncertainty and  $B$  with 0.2%, estimate the relative uncertainty on  $p_T$  assuming independence.
- P24.31.** A track is measured over a lever arm  $L = 1.2$  m with curvature radius  $R = 120$  m. Estimate the sagitta. If the sagitta resolution is  $20 \mu\text{m}$ , estimate the corresponding fractional curvature uncertainty  $\sigma_s/s_g$ .
- P24.32.** A process has cross section 50 fb and is studied with  $300\text{fb}^{-1}$  of integrated luminosity. If the combined acceptance and efficiency is 0.35, estimate the expected selected signal yield.
- P24.33.** A calorimeter has resolution  $\sigma_E/E = 10\%/\sqrt{E/\text{GeV}} \oplus 1\%$ . Estimate the fractional and absolute resolution at  $E = 25$  GeV and at  $E = 400$  GeV. Which term dominates at each energy?
- P24.34.** A particle travels 2.0 m. Compare the time of flight for a pion and a proton, each with momentum 1.0 GeV/ $c$ . Use  $m_\pi c^2 = 0.140$  GeV and  $m_p c^2 = 0.938$  GeV.
- P24.35.** In a Cherenkov radiator with  $n = 1.030$ , find the minimum  $\beta$  required for emission and the limiting Cherenkov angle as  $\beta \rightarrow 1$ .
- P24.36.** A galaxy at low redshift is at  $d = 250$  Mpc. Using  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , estimate its recession speed and the corresponding small- $z$  redshift. Comment on whether the approximation is still comfortably nonrelativistic.
- P24.37.** A spectral line emitted at 500 nm is observed at 750 nm. Find the redshift  $z$ . Use  $1+z = a_0/a_{\text{em}}$  to determine the scale-factor ratio between observation and emission.
- P24.38.** The CMB temperature today is 2.725 K. Estimate the radiation temperature at redshifts  $z = 100$ , 1000, and  $10^6$  using  $T = T_0(1+z)$ . Convert the last temperature to a characteristic  $k_B T$  energy in eV.
- P24.39.** Use  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$  to estimate the Hubble time  $1/H_0$  in seconds and years. Explain why this is only an order-of-magnitude age estimate, not an exact age of the universe.

- P24.40.** A galaxy has a flat rotation speed  $v_c = 180 \text{ km/s}$  out to  $r = 30 \text{ kpc}$ . Estimate the enclosed mass in solar masses. Use  $M_\odot = 1.99 \times 10^{30} \text{ kg}$ .

### Computational and data-analysis problems

- P24.41.** Build a reaction checker for electric charge, baryon number, and total lepton number. Use it on neutron beta decay, muon decay, electron-positron annihilation to two photons, and the hypothetical proton decay  $p \rightarrow e^+ \pi^0$ . Your program should report which rule fails, but include a printed warning that passing these checks does not prove a reaction is dynamically allowed.
- P24.42.** Generate 1000 synthetic charged tracks with momenta uniformly distributed from 1 to 100 GeV/c in a 4 T field. Convert momenta to radii, compute exact sagitta for a 1 m chord using Fig. 24.14, and add Gaussian sagitta noise of width  $30 \mu\text{m}$ . Reconstruct the momenta and compare the small-curvature approximation at low and high momentum. Plot fractional momentum error versus true momentum and explain the trend.
- P24.43.** Simulate a two-photon resonance of mass  $125 \text{ GeV}/c^2$ . At rest, generate back-to-back photon directions isotropically and give each photon 62.5 GeV. Smear each energy by 1.5% and each direction by a small angular resolution. Reconstruct  $M_{\gamma\gamma}$  for  $10^4$  events and estimate the mass resolution from the histogram width.
- P24.44.** Generate 50 synthetic low-redshift galaxies with distances from 10 to 200 Mpc and velocities  $v = 70d + \epsilon$ , where  $\epsilon$  is Gaussian with width 300 km/s. Fit  $H_0$  with and without the nearest 20 Mpc galaxies. Repeat the experiment many times and compare the spread of fitted slopes.
- P24.45.** Download or construct a small public-style galaxy rotation data set with radius and circular speed. Compute  $M(r) = v_c^2 r / G$  under the spherical approximation and plot  $M(r)$ . Compare with the cumulative luminous-mass profile if one is available, clearly labeling any model dependence.
- P24.46.** Implement the two-flavour neutrino probability in Eq. (24.27). Plot  $P$  as a function of  $L/E$  for several mixing angles. Verify that changing  $\Delta m^2$  rescales the oscillation period in  $L/E$ .
- P24.47.** Plot the density scalings  $\rho_r \propto a^{-4}$ ,  $\rho_m \propto a^{-3}$ , and  $\rho_\Lambda = \text{const}$  on logarithmic axes for  $10^{-6} \leq a \leq 2$ . Choose arbitrary normalizations such that matter and radiation cross at one visible point and matter and dark energy at another. Interpret each crossover physically.
- P24.48.** Use synthetic blackbody spectra at  $T = 3000 \text{ K}$  and  $T = 2.725 \text{ K}$  to illustrate how peak wavelength scales with  $1/T$ . Normalize the curves separately if necessary for visibility. Explain how cosmic expansion can transform an early thermal spectrum while preserving its blackbody form.

### Engineering and applied problems

- P24.49.** A tracker designer can either increase the magnetic field by 20% or increase the measurement lever arm by 10%. Using  $s_g \propto BL^2/p_T$ , compare the fractional sagitta improvement for the two choices at fixed momentum. Discuss at least two engineering constraints that could make the larger improvement impractical.
- P24.50.** A trigger system accepts events at 1 kHz with an average stored event size of 2.5 MB. Estimate the data rate in GB/s and the storage required for one day of continuous operation. Then explain why real experiments must account for duty cycle, compression, metadata, replication, and processing overhead.

- P24.51.** A calorimeter upgrade reduces the stochastic term from  $12\%/\sqrt{E}$  to  $8\%/\sqrt{E}$  but leaves a 1.5% constant term unchanged. Here  $E$  in the stochastic term denotes its numerical value in GeV. Compare the resolution at  $E = 10$  GeV and  $E = 1$  TeV. At which energy is the upgrade more valuable?
- P24.52.** A neutrino beam facility can choose between a 300 km baseline at 0.6 GeV and a 1300 km baseline at 2.5 GeV. Compare  $L/E$  for the two designs. Why can similar  $L/E$  still lead to different physics sensitivity in a full experiment?
- P24.53.** A redshift survey instrument must measure a spectral feature near 500 nm to determine  $z$  with uncertainty  $10^{-4}$ . Estimate the required wavelength precision near  $z = 0$ . Discuss why calibration stability, line profile, signal-to-noise, and peculiar velocities can matter even if the spectrograph meets the nominal wavelength precision.

### Challenge problems

- P24.54.** Derive the two-body momentum formula Eq. (24.51) from energy conservation  $Mc^2 = E_1 + E_2$  and equal/opposite momentum in the parent rest frame. Show explicitly how the Källén function factorization arises.
- P24.55.** A parent particle of mass  $M$  moves with Lorentz factor  $\gamma$  and decays to two photons. In the parent rest frame the photons are back-to-back with equal energy. Derive the laboratory photon energies as a function of the rest-frame emission angle relative to the boost. Verify that the reconstructed two-photon invariant mass remains  $M$ .
- P24.56.** Starting from conservation of energy in a comoving volume,  $d(\rho c^2 a^3) = -p d(a^3)$ , derive  $\rho \propto a^{-3(1+w)}$  for constant  $w$ . Interpret the cases  $w = 0$ ,  $1/3$ , and  $-1$ .
- P24.57.** Assume a spatially flat universe containing only pressureless matter and a cosmological constant. Write

$$H(a) = H_0 \sqrt{\Omega_m a^{-3} + \Omega_\Lambda}.$$

Find the scale factor at which the two density terms are equal for  $\Omega_m = 0.30$  and  $\Omega_\Lambda = 0.70$ , then convert to redshift.

Using the acceleration equation, determine the redshift at which accelerated expansion begins. Explain why it differs from the density-equality redshift.

- P24.58.** In a toy resonance search, expected signal and background in a mass window are  $S = 80$  and  $B = 400$ . A background normalization uncertainty of 10% is present. Compare the naive statistical fluctuation  $\sqrt{B}$  with the systematic uncertainty  $0.1B$ . Explain why collecting much more luminosity eventually stops improving sensitivity as  $\sqrt{\mathcal{L}}$  unless systematics improve.
- P24.59.** Use natural units  $c = 1$  in this problem;  $M$  denotes the numerical rest energy. In the parton model, a central system of mass  $M$  produced in a collider of energy  $\sqrt{s}$  requires  $x_1 x_2 s = M^2$  and at zero rapidity  $x_1 = x_2$ . Derive  $x = M/\sqrt{s}$ . Evaluate  $x$  for  $M = 125$  GeV and 2 TeV at  $\sqrt{s} = 14$  TeV. Discuss qualitatively why the heavier state is generally rarer even before considering the hard-interaction matrix element.
- P24.60.** Show that for a massless particle pseudorapidity equals rapidity. Begin with  $p_z = p \cos \theta$  and  $E = pc$  and simplify Eq. (24.56).

### 24.32 Problem-solving workshop

Use the following sequence to organize a solution. It works for a threshold, a detector estimate, a decay, or a cosmological inference.

**Method**

1. State the frame and the objects. Distinguish total energy from kinetic energy, and a proper lifetime from a laboratory time.
2. List exact conservation conditions and the model-dependent rules. Choose an invariant when several frames are involved.
3. Write the relation before substituting numbers. Keep the charge in units of  $e$ , luminosity in units inverse to the cross section, and cosmological densities in one convention.
4. Separate production, branching fraction, acceptance, and efficiency. For inference, identify background and calibration uncertainties.
5. Check dimensions and a limiting case. State what is measured, what is reconstructed, and what remains dependent on the model.

For a particle threshold, the final rest-mass sum sets the minimum invariant energy; it usually does not equal the required fixed-target kinetic energy. For a Hubble fit, the slope and its scatter are statistical outputs, while the distance calibration is a separate shared uncertainty.

### 24.33 Selected solutions to representative problems

These 20 solutions demonstrate the method as well as the numerical result. Problem numbers are linked to the statements. Approximate inputs justify only the precision reported; several final digits are retained in intermediate steps.

#### Solution to P24.16.: baryon charges

For  $uud$ ,

$$Q = \frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +e,$$

so this is the proton. For  $udd$ ,

$$Q = \frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0,$$

so this is the neutron. The hypothetical  $uuu$  state has charge  $+2e$  and  $ddd$  has charge  $-e$ . The important check is that three fractional charges combine into an integer total charge.

#### Solution to P24.19.: charged-lepton flavour violation

For  $\mu^- \rightarrow e^- + \gamma$ , charge is  $-1$  on each side, and total lepton number is  $+1$  on each side. There is adequate rest energy, and two-body momentum conservation is possible. Traditional family bookkeeping changes  $(L_e, L_\mu) = (0, 1)$  into  $(1, 0)$ , so it rejects the process in an approximation that treats these family numbers as exact.

With massive, mixed neutrinos, individual family numbers are not exact. The Standard Model extended only to include neutrino masses and mixing allows an extraordinarily small loop-induced rate. An experimentally accessible rate would therefore be evidence for additional physics. The useful conclusion is a severe dynamical suppression, rather than an absolute prohibition by electric charge or total lepton number.

#### Solution to P24.21.: 10 GeV kinetic-energy proton

The total energy is

$$E = K + m_p c^2 = 10.0 + 0.938 = 10.938 \text{ GeV}.$$

Then

$$pc = \sqrt{E^2 - m_p^2 c^4} = \sqrt{10.938^2 - 0.938^2} \text{ GeV} \simeq 10.898 \text{ GeV},$$

so  $p \simeq 10.90 \text{ GeV}/c$ . The Lorentz factor is

$$\gamma = \frac{E}{m_p c^2} \simeq 11.66.$$

Therefore

$$\beta = \sqrt{1 - \gamma^{-2}} \simeq 0.9963.$$

The proton is highly relativistic, so  $pc$  is close to  $E$  but not exactly equal.

### Solution to P24.23.: $e^+e^-$ collider threshold

Each beam has  $3.00 \text{ GeV}$  and the momenta are opposite, so the total laboratory momentum is approximately zero and

$$\sqrt{s} \simeq 6.00 \text{ GeV}.$$

The  $\tau$ -pair threshold is

$$2m_\tau c^2 \simeq 3.554 \text{ GeV}.$$

Therefore the reaction is kinematically allowed, leaving center-of-mass energy for the final-state kinetic motion.

### Solution to P24.24.: fixed-target invariant energy

Let the beam four-momentum be  $(E/c, \mathbf{p})$  and the target four-momentum  $(mc, \mathbf{0})$ . Then

$$\begin{aligned} s &= c^2(p_1 + p_2)^2 \\ &= (E + mc^2)^2 - p^2 c^2 \\ &= E^2 + 2Emc^2 + m^2 c^4 - p^2 c^2. \end{aligned}$$

Using  $E^2 - p^2 c^2 = m^2 c^4$  gives

$$s = 2m^2 c^4 + 2mc^2 E.$$

This derivation shows explicitly that the result is an invariant statement, not a memorized special formula.

### Solution to P24.25.: generic fixed-target threshold

Let the final threshold invariant mass be

$$M_f c^2 = (2m_p + m_X) c^2.$$

The initial invariant is

$$s = 2m_p^2 c^4 + 2m_p c^2 E.$$

Set  $s = M_f^2 c^4$  and solve:

$$E = \frac{M_f^2 c^4 - 2m_p^2 c^4}{2m_p c^2}.$$

With  $m_p c^2 = 0.938 \text{ GeV}$  and  $m_X c^2 = 3.0 \text{ GeV}$ ,

$$M_f c^2 = 4.876 \text{ GeV}.$$

Therefore

$$E \simeq \frac{(4.876)^2 - 2(0.938)^2}{2(0.938)} \text{ GeV} \simeq 11.74 \text{ GeV}.$$

The beam kinetic energy is  $E - m_p c^2 \simeq 10.80 \text{ GeV}$ .

**Solution to P24.27.: two-photon invariant mass**

Equation (24.46) gives

$$M^2 c^4 = 2(40)(60)(1 - \cos 120^\circ) \text{ GeV}^2.$$

Since  $\cos 120^\circ = -1/2$ ,

$$M^2 c^4 = 4800 \times 1.5 \text{ GeV}^2 = 7200 \text{ GeV}^2.$$

Thus

$$Mc^2 \simeq 84.9 \text{ GeV}.$$

The mass is less than the total energy 100 GeV because the pair has nonzero net momentum in the laboratory.

**Solution to P24.28.: boosted muon decay length**

If  $E = 20m_\mu c^2$ , then  $\gamma = 20$  and

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \simeq 0.99875.$$

Therefore

$$\ell = \beta\gamma c\tau \simeq (0.99875)(20)(3.00 \times 10^8)(2.20 \times 10^{-6}) \text{ m} \simeq 1.32 \times 10^4 \text{ m}.$$

The boost increases the mean laboratory flight distance by almost a factor of 20 relative to the nonrelativistic  $c\tau$  scale.

**Solution to P24.30.: track momentum and uncertainty**

The transverse momentum is

$$p_T \simeq 0.3BR = (0.3)(2.0)(1.50) = 0.900 \text{ GeV}/c.$$

For independent  $B$  and  $R$  uncertainties,

$$\left(\frac{\sigma_{p_T}}{p_T}\right)^2 = \left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{\sigma_R}{R}\right)^2.$$

Thus

$$\frac{\sigma_{p_T}}{p_T} = \sqrt{(0.002)^2 + (0.020)^2} \simeq 0.0201.$$

The result is about 2.0%, dominated by the radius measurement.

**Solution to P24.31.: sagitta**

With  $L = 1.2 \text{ m}$  and  $R = 120 \text{ m}$ ,

$$s_g \simeq \frac{1.2^2}{8(120)} \text{ m} = 1.50 \times 10^{-3} \text{ m} = 1.50 \text{ mm}.$$

For  $\sigma_s = 20 \mu\text{m} = 0.020 \text{ mm}$ ,

$$\frac{\sigma_s}{s_g} \simeq \frac{0.020}{1.50} = 1.33\%.$$

At high momentum, this geometric uncertainty translates approximately into a comparable fractional curvature and momentum uncertainty, before including multiple scattering or alignment systematics.

**Solution to P24.32.: event yield**

Convert  $\sigma = 50\text{fb}$  and use  $\mathcal{L}_{\text{int}} = 300\text{fb}^{-1}$ :

$$N_{\text{produced}} = 50 \times 300 = 15000.$$

Then

$$N_{\text{selected}} = 15000(0.35) = 5250.$$

The units cancel because femtobarn times inverse femtobarn is dimensionless. The number is an expectation value; an actual observed count fluctuates.

**Solution to P24.33.: calorimeter resolution**

At 25 GeV,

$$\frac{10\%}{\sqrt{25}} = 2.0\%,$$

so

$$\frac{\sigma_E}{E} = \sqrt{(2.0\%)^2 + (1.0\%)^2} = 2.24\%,$$

and  $\sigma_E \simeq 0.56\text{ GeV}$ . The stochastic term is larger.

At 400 GeV,

$$\frac{10\%}{\sqrt{400}} = 0.50\%,$$

so

$$\frac{\sigma_E}{E} = \sqrt{(0.50\%)^2 + (1.0\%)^2} = 1.12\%,$$

and  $\sigma_E \simeq 4.47\text{ GeV}$ . The constant term now dominates.

**Solution to P24.34.: pion and proton time of flight**

For momentum  $p = 1.0\text{ GeV}/c$ ,

$$\beta = \frac{pc}{\sqrt{p^2c^2 + m^2c^4}}.$$

For the pion,

$$\beta_\pi = \frac{1}{\sqrt{1 + 0.140^2}} \simeq 0.9904.$$

For the proton,

$$\beta_p = \frac{1}{\sqrt{1 + 0.938^2}} \simeq 0.729.$$

Over  $L = 2.0\text{ m}$ ,

$$t_\pi = \frac{L}{\beta_\pi c} \simeq 6.74\text{ ns}, \quad t_p \simeq 9.15\text{ ns}.$$

The difference is about 2.4 ns, easily measurable with a sufficiently precise timing system.

**Solution to P24.37.: redshift and scale factor**

The redshift is

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} = \frac{750 - 500}{500} = 0.5.$$

Therefore

$$1 + z = 1.5 = \frac{a_0}{a_{\text{em}}},$$

so

$$a_{\text{em}} = \frac{a_0}{1.5}.$$

If the present scale factor is normalized to  $a_0 = 1$ , then  $a_{\text{em}} = 2/3$ .

**Solution to P24.38.: CMB temperature scaling**

Using  $T = T_0(1 + z)$  with  $T_0 = 2.725 \text{ K}$ ,

$$\begin{aligned} T(z = 100) &\simeq 275 \text{ K}, \\ T(z = 1000) &\simeq 2728 \text{ K}, \\ T(z = 10^6) &\simeq 2.725 \times 10^6 \text{ K}. \end{aligned}$$

For the last case,

$$k_B T \simeq (8.617 \times 10^{-5} \text{ eV/K})(2.725 \times 10^6 \text{ K}) \simeq 235 \text{ eV}.$$

This remains far below nuclear MeV scales, reminding us that nucleosynthesis occurred at much larger redshift than  $10^6$ .

**Solution to P24.39.: Hubble time**

Convert

$$H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \simeq 2.27 \times 10^{-18} \text{ s}^{-1}.$$

Then

$$H_0^{-1} \simeq 4.41 \times 10^{17} \text{ s} \simeq 1.40 \times 10^{10} \text{ yr}.$$

The number is close to the actual cosmic age scale, but not exactly equal to the age because  $H(t)$  has changed over cosmic history.

**Solution to P24.40.: galaxy mass**

Convert  $v_c = 180 \text{ km/s} = 1.80 \times 10^5 \text{ m/s}$  and

$$r = 30 \text{ kpc} = 30(3.086 \times 10^{19}) \text{ m} = 9.26 \times 10^{20} \text{ m}.$$

Then

$$M = \frac{v_c^2 r}{G} \simeq \frac{(1.80 \times 10^5)^2 (9.26 \times 10^{20})}{6.674 \times 10^{-11}} \simeq 4.49 \times 10^{41} \text{ kg}.$$

In solar masses,

$$M \simeq \frac{4.49 \times 10^{41}}{1.99 \times 10^{30}} M_\odot \simeq 2.26 \times 10^{11} M_\odot.$$

This is an enclosed gravitating mass estimate under a spherical circular-orbit approximation.

**Solution to P24.50.: trigger data rate**

At 1000 events/s and 2.5 MB per event,

$$R = 2500 \text{ MB/s} = 2.5 \text{ GB/s}$$

using decimal storage units. In one day,

$$V = (2.5 \text{ GB/s})(86400 \text{ s}) = 216000 \text{ GB} = 216 \text{ TB}.$$

This simple number excludes replication, derived data formats, calibration streams, metadata, failed runs, compression, and processing outputs.

**Solution to P24.56.: density law from energy conservation**

Write

$$d(\rho c^2 a^3) = -p da^3.$$

Expand the left side:

$$c^2 a^3 d\rho + 3\rho c^2 a^2 da = -3p a^2 da.$$

Divide by  $c^2 a^3$  and insert  $p = w\rho c^2$ :

$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}.$$

Integrating gives

$$\ln \rho = -3(1+w) \ln a + \text{const},$$

so

$$\rho \propto a^{-3(1+w)}.$$

For matter  $w = 0$ ; for radiation  $w = 1/3$ ; for a cosmological constant  $w = -1$ .

**Solution to P24.57.: matter-dark-energy equality**

Set

$$\Omega_m a^{-3} = \Omega_\Lambda.$$

Then

$$a = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} = \left(\frac{0.30}{0.70}\right)^{1/3} \simeq 0.754.$$

The corresponding redshift is

$$z = \frac{1}{a} - 1 \simeq 0.326.$$

This is equality of the density terms, not exactly the moment when cosmic acceleration begins; the acceleration condition depends on pressure as well as density.

For pressureless matter plus a cosmological constant, the acceleration equation can be written

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + 2\rho_\Lambda).$$

Acceleration begins when  $\rho_m = 2\rho_\Lambda$ . With  $a_0 = 1$ ,

$$a_{\text{acc}} = \left(\frac{0.30}{2(0.70)}\right)^{1/3} \simeq 0.598, \quad z_{\text{acc}} = a_{\text{acc}}^{-1} - 1 \simeq 0.671.$$

The transition occurs earlier than density equality at  $z \simeq 0.326$ , because vacuum pressure contributes to the acceleration as well as its density.

**24.34 Bridge beyond Physics 2**

This chapter closes the Physics 2 sequence but opens several possible next courses. A course in quantum mechanics develops states, operators, angular momentum, spin, and scattering more systematically. Quantum field theory replaces particle-number-fixed quantum mechanics with fields whose excitations can be created and destroyed. Particle physics develops gauge symmetries, the Standard Model Lagrangian, collider phenomenology, flavour physics, QCD, and neutrino physics. General relativity provides the geometric theory underlying modern cosmology. Astrophysics and cosmology develop stellar evolution, compact objects, structure formation, CMB physics, gravitational lensing, and observational inference.

The recurring intellectual pattern is the same one encountered throughout this volume: define measurable quantities carefully, identify symmetries and conservation laws, choose the simplest valid model, quantify uncertainties, and test predictions against observation. Fundamental physics is not a collection of isolated facts. It is a connected modelling discipline.

# Appendix A

## Notation dictionary and terminology

This appendix collects the notation used in Chapter 24 and clarifies terms that are often confused in introductory particle physics and cosmology. The goal is not to reproduce a specialist data booklet, but to provide a compact vocabulary for revision and problem solving.

### Particle-physics notation

| Symbol or term             | Meaning in this chapter                                                                                              |
|----------------------------|----------------------------------------------------------------------------------------------------------------------|
| $e$                        | magnitude of the elementary electric charge; electron charge is $-e$                                                 |
| $u, d, s, c, b, t$         | up, down, strange, charm, bottom, and top quark flavours                                                             |
| $\bar{q}$                  | antiquark corresponding to quark $q$                                                                                 |
| $e, \mu, \tau$             | charged-lepton flavours                                                                                              |
| $\nu_e, \nu_\mu, \nu_\tau$ | neutrino flavour states associated with weak interactions                                                            |
| $\gamma$                   | photon, gauge boson of electromagnetism                                                                              |
| $g$                        | gluon; context distinguishes it from gravitational acceleration                                                      |
| $W^\pm, Z$                 | massive weak-interaction gauge bosons                                                                                |
| $H$                        | Higgs boson                                                                                                          |
| $p, n$                     | proton and neutron                                                                                                   |
| $\pi, K$                   | pion and kaon families of mesons                                                                                     |
| $Q$                        | electric charge; also sometimes a momentum-transfer scale in advanced contexts, so context matters                   |
| $B$                        | baryon number in reaction bookkeeping; magnetic field is written explicitly as $B$ only where context is unambiguous |
| $L$                        | lepton number in reaction bookkeeping or detector lever arm in geometric contexts                                    |
| $E$                        | relativistic total energy                                                                                            |
| $K$                        | kinetic energy                                                                                                       |
| $\mathbf{p}$               | three-momentum                                                                                                       |
| $p_T$                      | momentum transverse to the collider beam axis                                                                        |
| $\eta$                     | pseudorapidity                                                                                                       |
| $y$                        | rapidity                                                                                                             |
| $\phi$                     | azimuthal angle around the beam axis                                                                                 |
| $\Delta R$                 | angular separation in the $(\eta, \phi)$ plane                                                                       |
| $\sqrt{s}$                 | center-of-mass energy of a two-particle initial state                                                                |
| $M$                        | invariant mass or parent-particle mass, depending on context                                                         |
| $\Gamma$                   | decay width (energy); $\Gamma_{\text{int}}$ denotes an interaction rate (inverse time)                               |
| $\tau$                     | mean proper lifetime                                                                                                 |
| $\sigma$                   | cross section; in detector contexts may also denote a standard deviation                                             |
| $\mathcal{L}$              | instantaneous luminosity                                                                                             |
| $\mathcal{L}_{\text{int}}$ | integrated luminosity                                                                                                |
| $A\varepsilon$             | combined acceptance and efficiency factor                                                                            |

|                 |                                              |
|-----------------|----------------------------------------------|
| $\alpha_s$      | strong coupling                              |
| $f_i(x, \mu_F)$ | parton distribution function for species $i$ |

## Cosmology notation

| Symbol or term     | Meaning in this chapter                                                                         |
|--------------------|-------------------------------------------------------------------------------------------------|
| $a(t)$             | cosmic scale factor                                                                             |
| $H(t) = \dot{a}/a$ | Hubble parameter at cosmic time $t$                                                             |
| $H_0$              | present Hubble parameter                                                                        |
| $z$                | cosmological redshift                                                                           |
| $\rho$             | mass-equivalent density; $\rho c^2$ is energy density in the convention used here               |
| $p$                | pressure in the cosmological equation of state; context distinguishes it from particle momentum |
| $w = p/(\rho c^2)$ | equation-of-state parameter                                                                     |
| $\rho_c$           | critical density                                                                                |
| $\Omega_i$         | density parameter $\rho_i/\rho_c$                                                               |
| $\Lambda$          | cosmological constant                                                                           |
| $k$                | curvature parameter, with units $\text{m}^{-2}$ for dimensionless $a$                           |
| $T_0$              | present CMB temperature                                                                         |
| $C_\ell$           | CMB angular power spectrum coefficient                                                          |
| $D_L, D_S, D_{LS}$ | angular-diameter distances used in lensing geometry                                             |

## Terminology pairs that should not be confused

**Particle versus antiparticle.** An antiparticle is not a negative-energy version of a particle. It is a physical state with the same mass and spin and opposite additive charges where applicable.

**Hadron versus baryon.** A baryon is a type of hadron. Mesons are also hadrons. Therefore “hadron” is the broader category.

**Valence quark versus parton.** Valence content summarizes net flavour quantum numbers. A high-energy proton contains quarks, antiquarks, and gluons in a scale-dependent partonic description.

**Real versus virtual particle.** A real asymptotic particle can in principle be detected as an external state. A virtual internal line is part of a perturbative amplitude representation and does not satisfy the same on-shell relation as a directly observed free particle.

**Rest mass versus invariant mass.** For one stable particle the rest mass is its invariant mass. For a system of several particles the combined invariant mass can be nonzero even if every constituent is massless.

**Cross section versus detector area.** A cross section has dimensions of area but represents interaction probability, not the physical geometric size of a detector element or necessarily the geometric size of a particle.

**Luminosity versus optical brightness.** Collider luminosity is a beam-overlap rate density used to convert cross section into event rate. It is unrelated to astronomical luminosity except for the shared everyday word.

**Redshift versus distance.** Redshift is an observable wavelength ratio. Distance is inferred through a cosmological model or an independent distance indicator.

**Dark matter versus dark energy.** Dark matter clusters and contributes to gravitational mass. Dark energy is associated with accelerated expansion. The terms describe different phenomenological roles.

**Big Bang versus explosion.** The hot Big Bang model describes an early hot, dense, expanding universe. It does not require matter to explode from one spatial point into surrounding empty space.

# Appendix B

## Compact formula and reference sheet

### Formula summary

This sheet uses explicit  $c$  unless natural units are stated.  $E$  is total energy,  $\tau$  is the proper lifetime, and  $\sqrt{s}$  has energy units. Equations describe specified idealizations; a full solution must check their assumptions and units.

### Relativistic kinematics

$$E^2 = p^2 c^2 + m^2 c^4, \quad (\text{B.1})$$

$$K = E - mc^2, \quad (\text{B.2})$$

$$\gamma = \frac{E}{mc^2} = \frac{1}{\sqrt{1 - \beta^2}}, \quad (\text{B.3})$$

$$\beta = \frac{v}{c}, \quad (\text{B.4})$$

$$p = \gamma mv. \quad (\text{B.5})$$

For photons,

$$E = pc = hf = \frac{hc}{\lambda}. \quad (\text{B.6})$$

For a multiparticle system,

$$M^2 c^4 = E_{\text{tot}}^2 - P_{\text{tot}}^2 c^2. \quad (\text{B.7})$$

For two photons,

$$M_{\gamma\gamma}^2 c^4 = 2E_1 E_2 (1 - \cos \theta). \quad (\text{B.8})$$

### Thresholds and center-of-mass energy

At threshold in the center-of-momentum frame,

$$\sqrt{s}_{\text{th}} = \sum_i m_i c^2. \quad (\text{B.9})$$

For equal-mass head-on beams with equal total energies,

$$\sqrt{s} = 2E_b. \quad (\text{B.10})$$

For projectile  $a$  of total energy  $E_a$  on target  $b$  at rest,

$$s = m_a^2 c^4 + m_b^2 c^4 + 2m_b c^2 E_a. \quad (\text{B.11})$$

For identical projectile and target masses  $m$ ,

$$s = 2m^2 c^4 + 2mc^2 E. \quad (\text{B.12})$$

## Two-body decay

For  $M \rightarrow 1 + 2$  at rest,

$$p^*_c = \frac{\sqrt{[M^2c^4 - (m_1 + m_2)^2c^4][M^2c^4 - (m_1 - m_2)^2c^4]}}{2Mc^2}. \quad (\text{B.13})$$

For two massless daughters,

$$E_1 = E_2 = \frac{Mc^2}{2}. \quad (\text{B.14})$$

## Lifetime and decay length

$$N(t_*) = N_0 e^{-t_*/\tau} \quad (t_* : \text{proper time}), \quad (\text{B.15})$$

$$t_{1/2} = \tau \ln 2, \quad (\text{B.16})$$

$$\tau = \frac{\hbar}{\Gamma}, \quad (\text{B.17})$$

$$\ell = \beta\gamma c\tau. \quad (\text{B.18})$$

## Magnetic tracking and detector geometry

For a singly charged relativistic track transverse to  $B$ ,

$$p_T [\text{GeV}/c] \simeq 0.2998 B [\text{T}] R [\text{m}]. \quad (\text{B.19})$$

Sagitta for lever arm  $L \ll R$ :

$$s_g \simeq \frac{L^2}{8R}. \quad (\text{B.20})$$

Pseudorapidity and angular distance:

$$\eta = -\ln \tan \frac{\theta}{2}, \quad (\text{B.21})$$

$$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}. \quad (\text{B.22})$$

## Cross sections and event counts

$$N_{\text{produced}} = \sigma \mathcal{L}_{\text{int}}, \quad (\text{B.23})$$

$$N_{\text{selected}} = \sigma \mathcal{L}_{\text{int}} A \varepsilon. \quad (\text{B.24})$$

Unit conversions:

$$1 \text{ barn} = 10^{-28} \text{ m}^2, \quad (\text{B.25})$$

$$1 \text{ pb} = 10^3 \text{ fb}, \quad (\text{B.26})$$

$$1 \text{ fb}^{-1} = 10^3 \text{ pb}^{-1}. \quad (\text{B.27})$$

For a Poisson process,

$$P(n|\mu) = \frac{\mu^n e^{-\mu}}{n!}, \quad \sigma_n = \sqrt{\mu}. \quad (\text{B.28})$$

The quantity  $S/\sqrt{B}$  is only a rough statistics-dominated intuition, not a general significance definition.

## Quark charges and simple quantum numbers

$$Q_u = Q_c = Q_t = +\frac{2}{3}e, \quad (\text{B.29})$$

$$Q_d = Q_s = Q_b = -\frac{1}{3}e. \quad (\text{B.30})$$

For introductory baryon-number bookkeeping,

$$B_q = +\frac{1}{3}, \quad B_{\bar{q}} = -\frac{1}{3}. \quad (\text{B.31})$$

A conventional baryon has  $B = 1$ ; antibaryon  $B = -1$ ; meson  $B = 0$ . Strangeness is  $S = N_{\bar{s}} - N_s$  and is conserved in strong and electromagnetic reactions.

## Neutrino oscillation approximation

For two flavours,

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta) \sin^2 \left( 1.267 \frac{\Delta m^2 [\text{eV}^2/c^4] L [\text{km}]}{E [\text{GeV}]} \right). \quad (\text{B.32})$$

This form assumes coherent, ultrarelativistic vacuum propagation. The real three-flavour problem includes several mixing parameters and matter effects.

## Cosmological expansion

$$H(t) = \frac{\dot{a}}{a}, \quad (\text{B.33})$$

$$1 + z = \frac{a_0}{a_{\text{em}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}}, \quad (\text{B.34})$$

$$v \simeq H_0 d \quad (z \ll 1), \quad (\text{B.35})$$

$$T(z) = T_0(1 + z) \quad \text{for freely redshifting radiation.} \quad (\text{B.36})$$

First Friedmann equation ( $a$  dimensionless;  $k$  in  $\text{m}^{-2}$ ;  $\rho$  excludes the explicit  $\Lambda$  component):

$$H^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}. \quad (\text{B.37})$$

Critical density:

$$\rho_c = \frac{3H^2}{8\pi G}. \quad (\text{B.38})$$

For constant  $w = p/(\rho c^2)$ ,

$$\rho \propto a^{-3(1+w)}. \quad (\text{B.39})$$

Therefore matter  $\propto a^{-3}$ , radiation  $\propto a^{-4}$ , and a cosmological constant remains constant.

## Galaxy dynamics and lensing

Circular-speed mass estimate in spherical approximation:

$$M(r) = \frac{v_c^2 r}{G}. \quad (\text{B.40})$$

Point-lens Einstein angle:

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}}. \quad (\text{B.41})$$

## Useful numerical scales

| Quantity         | Approximate value used in exercises                                |
|------------------|--------------------------------------------------------------------|
| $c$              | $2.998 \times 10^8 \text{ m/s}$                                    |
| $G$              | $6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ |
| $e$              | $1.602 \times 10^{-19} \text{ C}$                                  |
| $\hbar c$        | $0.1973 \text{ GeV fm}$                                            |
| $hc$             | $1.240 \text{ eV } \mu\text{m}$                                    |
| $k_B$            | $8.617 \times 10^{-5} \text{ eV/K}$                                |
| 1 pc             | $3.086 \times 10^{16} \text{ m}$                                   |
| 1 kpc            | $3.086 \times 10^{19} \text{ m}$                                   |
| 1 Mpc            | $3.086 \times 10^{22} \text{ m}$                                   |
| $M_\odot$        | $1.99 \times 10^{30} \text{ kg}$                                   |
| $m_p c^2$        | $0.938 \text{ GeV}$                                                |
| $m_\mu c^2$      | $105.7 \text{ MeV}$                                                |
| $\tau_\mu$       | $2.20 \mu\text{s}$                                                 |
| $T_{\text{CMB}}$ | about $2.725 \text{ K}$                                            |

# Appendix C

## Computational recipes and reproducible analysis

The following recipes are written in language-independent pseudocode. They can be implemented in Python, Julia, C++, MATLAB, or another scientific language. Students should include units in comments or use a units package to prevent silent conversion errors.

### Recipe 1: relativistic four-vector class

A minimal four-vector object stores  $(E, p_x, p_y, p_z)$  and supports addition.

```
class FourVector:
    E, px, py, pz

    add(other):
        return FourVector(E+other.E,
                           px+other.px,
                           py+other.py,
                           pz+other.pz)

    mass2():
        return E**2 - px**2 - py**2 - pz**2    # c = 1

    pt():
        return sqrt(px**2 + py**2)
```

Validation tests should include a photon with  $E = |\mathbf{p}|$ , which must have  $m^2 \simeq 0$ , and two back-to-back photons whose combined invariant mass equals their total center-of-mass energy.

### Recipe 2: reaction checker

Store each particle as a record:

```
particle = {
    "name": "proton",
    "charge": +1,
    "baryon": +1,
    "lepton": 0,
    "mass_GeV": 0.938
}
```

For quarks, use exact integer charge units  $e/3$  and baryon units  $1/3$  (or rational arithmetic). Add strangeness for strong or electromagnetic reactions, and compare the available invariant energy with the final mass sum. Then sum each additive quantity over initial and final lists. A robust

implementation should distinguish three outputs: `passes bookkeeping`, `fails bookkeeping`, and `not enough information for dynamical allowance`. This prevents the program from claiming that conservation checks alone establish a physical reaction.

### Recipe 3: circle fit for a magnetic track

Generate noisy  $(x, y)$  points satisfying approximately

$$(x - x_c)^2 + (y - y_c)^2 = R^2. \quad (\text{C.1})$$

Fit  $(x_c, y_c, R)$  by nonlinear least squares or an algebraic circle fit. Convert the radius to  $p_T$ . Study bias by changing the number of hits, lever arm, noise, and multiple-scattering model. Plot residuals as well as fitted momentum.

### Recipe 4: invariant-mass histogram

Construct each photon from its smeared positive energy and a unit direction, so  $|\mathbf{p}| = E$  in natural units. Then, for each event:

```
p = photon1 + photon2
m2 = p.mass2()
if m2 < -tolerance:
    raise Error("check units and four-vector construction")
m = sqrt(max(m2, 0)) # clamp only floating-point roundoff
histogram.fill(m)
```

Never take the square root of a negative reconstructed  $m^2$  without understanding whether the value comes from numerical precision, inconsistent units, or unphysical smeared inputs.

### Recipe 5: synthetic Hubble regression

Generate distances  $d_i$ , peculiar velocities  $u_i$ , and measurement errors  $\epsilon_i$ :

$$v_i = H_0 d_i + u_i + \epsilon_i. \quad (\text{C.2})$$

Fit a slope by weighted least squares. Repeat many pseudo-experiments to estimate bias and variance. Compare fits that include and exclude very nearby galaxies. Report the assumed scatter model together with the fitted value.

### Recipe 6: logarithmic cosmology plot

To visualize component scaling, define

$$\begin{aligned} \rho_r(a) &= C_r a^{-4}, \\ \rho_m(a) &= C_m a^{-3}, \\ \rho_\Lambda(a) &= C_\Lambda. \end{aligned}$$

Sample  $a$  logarithmically, for example  $10^{-6} \leq a \leq 2$ . Plot both axes logarithmically. Mark equality points. A log plot converts power laws into straight lines, making exponents visually interpretable.

## Recipe 7: two-flavour oscillation scan

Define

```
phase = 1.267 * dm2_eV2_over_c4 * L_km / E_GeV  
P = sin(2*theta)**2 * sin(phase)**2
```

Scan in  $L/E$  and verify that the period is inversely proportional to  $\Delta m^2$ . Test edge cases:  $\theta = 0$  gives no oscillation;  $\Delta m^2 = 0$  gives no relative phase; maximal mixing has  $\sin^2(2\theta) = 1$ .

## Recipe 8: reproducibility checklist

A computational result intended for assessment should record:

- code version or archived script;
- input data source and exact file version;
- numerical constants and units;
- random seed for stochastic simulations when reproducibility is required;
- fitting method and convergence criteria;
- plots with labeled axes and units;
- at least one analytic or limiting-case validation;
- a short statement of known model limitations.

A correct-looking plot is not sufficient evidence that the calculation is correct.

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