

Electric Charge and Coulomb's Law

Physics II - Chapter 1

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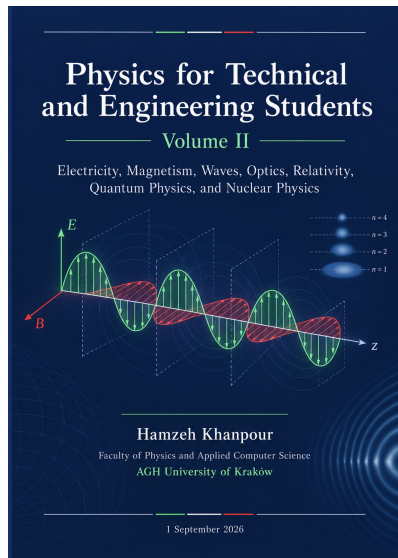
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Our Physics II course book



Physics for Technical and Engineering Students - Volume II

Hamzeh Khanpour

Electricity, Magnetism, Waves, Optics, Relativity, Quantum Physics, and Nuclear Physics

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Role in this course

- Written for technical and engineering students and aligned with the Physics II progression.
- Connects physical motivation to equations, modelling assumptions, worked reasoning and applications.
- Supports lectures, auditorium-class problem solving, and independent study between classes.

From mechanics to electrostatics



Mechanics tells us how forces affect motion. Electric charge introduces a new interaction—from paper attracted by a rubbed rod to electrostatic discharge in electronics.

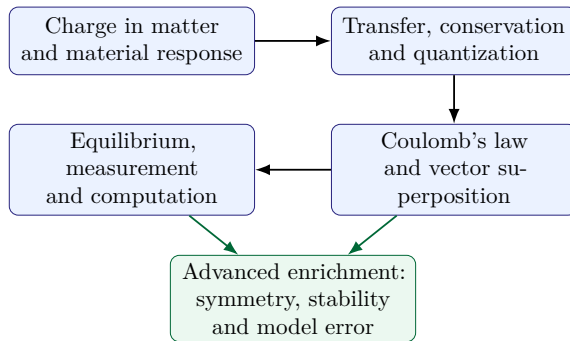


Figure 1.1: The route from observations of charging to quantitative models. The advanced branch refines the same force-based ideas. Electric fields, flux and potential are developed in later chapters.

Chapter support: §1.1; Figure 1.1.

What you should be able to do



- Distinguish net charge, mobile carriers, and polarization; track charge through contact and grounding.
- Select a point-charge model and calculate forces using signed charges and vector superposition.
- Combine electrostatics with mechanical equilibrium; interpret what a measurement actually determines.
- Validate a numerical sum and recognize limits of a model.

Starting point and destination

Use mechanics, vectors, trigonometry, and elementary calculus. The final ten slides add stability, uncertainty, and approximation error. Electric fields and potential belong to later chapters.

Charge is a property, not a substance

Net charge is an algebraic sum

An object is neutral when its positive and negative charges balance: $Q = \sum_i q_i = 0$. Neutral does *not* mean charge-free.

Constituent	Charge	Ordinary solid metal
Electron	$-e$	Some electrons are mobile.
Proton	$+e$	Bound in nuclei.
Neutron	0 net	Bound in nuclei.

A positive metal object usually has an **electron deficit**, not extra mobile protons. An ion has an electron imbalance relative to the neutral atom or molecule.

Follow the electrons



Two initially neutral bodies A and B form an isolated pair. Transfer N electrons from A to B. Each electron carries charge $-e$.

Keep the sign of the transferred charge

The charge carried from A to B is $\Delta q = -Ne$.

$$Q'_A = 0 - \Delta q = 0 - (-Ne) = +Ne,$$

$$Q'_B = 0 + \Delta q = 0 + (-Ne) = -Ne.$$

$$Q'_A + Q'_B = +Ne - Ne = 0.$$

Check: A loses negative charge and becomes positive; B gains negative charge and becomes negative. Both still contain charged particles. The total of the isolated pair is unchanged.

Materials: mobility and response

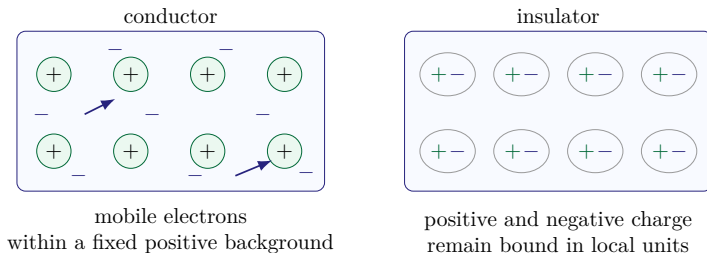


Figure 1.3: Schematic charge mobility in initially neutral materials. Equal numbers of signs are drawn only as a bookkeeping aid. In a conductor electrons can redistribute through the sample; in an insulator bound charge can shift locally. Neither drawing is a literal atomic model.

Conductor: carriers can redistribute macroscopically.

Insulator: charge is locally bound under ordinary conditions.

Semiconductor: doping, temperature, and illumination strongly control the mobile-carrier population and conductivity.

Chapter support: §1.3; Definition 1.2; Figure 1.3.

Why neutral matter can be attracted

Charge separation is not net charging

Near an external charge, mobile electrons in a conductor redistribute. In an insulator, bound positive and negative charges can shift slightly relative to one another: **polarization**.

The nearer, oppositely charged region can be attracted more strongly than the farther, like-charged region is repelled. Position matters, not just total charge.

What does attraction establish?

Attraction alone does not establish the sign-or even the presence-of net charge. A neutral polarizable object can be attracted. The interaction must vary across the object for this net-force explanation.

Chapter support: §§1.3.1–1.3.2.

Contact is not charge creation



- Contact and separation can transfer charge. In the electron-transfer model, what one object gains the other loses.
- Rubbing supplies mechanical work; it does not create net charge. The microscopic carrier mechanism depends on the materials and surfaces.
- Conductivity lets charge move *within* an object. A conducting path is needed for charge to leave the chosen system.

Predict the difference

A metal on an insulating support can retain net charge. Holding it in a conducting hand can open a path to Earth and change the experiment.

Chapter support: §§1.4.1–1.4.2.

Charging by induction: keep the order

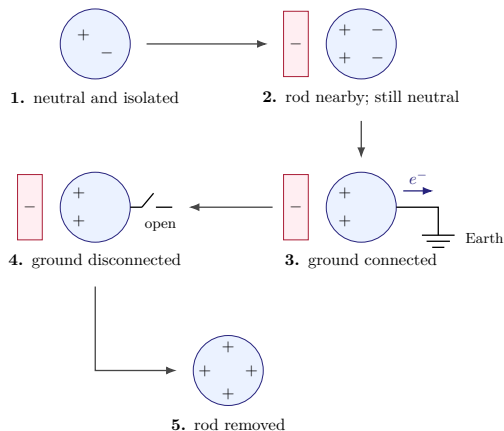


Figure 1.4: Charging by induction with a negative rod. In step 3 electrons leave the sphere through the ground connection. In step 4 the sphere is already positively charged but remains polarized. Only after the connection is broken is the rod removed. The signs indicate qualitative imbalance, not equal counted packets between panels.

Chapter support: §§1.4.3–1.4.4; Figure 1.4.

Induction: which system is isolated?



With grounding. A negative rod pushes electrons from the sphere to Earth. Disconnect ground first: the sphere retains positive charge. Remove the rod first while still grounded: charge can return and neutralize it.

Without grounding: two touching neutral spheres

Bring a negative rod near the left sphere. Electrons move to the right. Separate the spheres *before* removing the rod: left becomes positive, right negative.

$$Q_{\text{left}} + Q_{\text{right}} = 0.$$

The pair is isolated; the rod supplied a force, not these electrons.

Check: Does conservation give each charge magnitude?

Answer: No; geometry and material response are also needed.

Chapter support: §§1.4.3–1.4.5; guided checks.

Conservation is signed bookkeeping



For an isolated system, $Q_{\text{total}} = \sum_i q_i$ is constant. Individual charges can change by transfer.

Example 1.1: initially neutral beads

A plastic bead transfers $N = 2.5 \times 10^{10}$ electrons to a metal bead. Treat the pair as isolated; only electrons are transferred. Find the two final charges.

$$Ne = (2.5 \times 10^{10})(1.602 \times 10^{-19} \text{ C}) \simeq 4.0 \text{ nC}.$$

$$q_{\text{plastic}} = +4.0 \text{ nC}, \quad q_{\text{metal}} = -4.0 \text{ nC}.$$

Check: The signs follow electron loss/gain; their sum remains zero. A few nanocoulombs already represent billions of electrons.

Chapter support: §1.5; Eq. (1.1); Example 1.1.

Quantization: microscopic steps, large counts



Elementary charge

Ordinary free charge states occur in integer multiples of e :

$$q = ne, \quad n \in \mathbb{Z}, \quad e = 1.602\,176\,634 \times 10^{-19} \text{ C}.$$

This SI value is exact; calculations here use $1.602 \times 10^{-19} \text{ C}$.

Example 1.2. An object carries $q = -0.80 \mu\text{C}$. How many excess electrons does this represent?

$$N = \frac{|q|}{e} = \frac{0.80 \times 10^{-6}}{1.602 \times 10^{-19}} = 4.99 \times 10^{12} \simeq 5.0 \times 10^{12}.$$

The count is positive; the negative *net charge* identifies an electron excess. Such large counts hide discrete steps macroscopically.

Chapter support: §1.5.1; Eqs. (1.5)–(1.8); Example 1.2.

Your turn: a charge-transfer ledger



Problem 10

Sphere A initially carries $+8.0 \text{ nC}$ and sphere B carries -3.0 nC . Electrons with total charge -2.0 nC move from B to A. No charge enters or leaves the pair.

Find the final charge of each sphere and the number of transferred electrons. Check charge conservation.

Before using numbers:

- Which sphere gains negative charge?
- Is the other sphere becoming more negative or less negative?
- Is the number of transferred particles a signed quantity?

Chapter support: §1.17, Problem 10; solution on the next slide.

Solution: subtracting a negative charge



The transferred carrier charge is $\Delta q = -2.0 \text{ nC}$. Add it to A and remove it from B:

$$Q'_A = +8.0 + (-2.0) = +6.0 \text{ nC},$$

$$Q'_B = -3.0 - (-2.0) = -1.0 \text{ nC}.$$

Conservation: $Q'_A + Q'_B = +5.0 \text{ nC}$, the initial total.

$$N = \frac{|\Delta q|}{e} = \frac{2.0 \times 10^{-9}}{1.602 \times 10^{-19}} \simeq 1.2 \times 10^{10} \text{ electrons}.$$

Interpretation

A becomes less positive; B becomes less negative. A negative carrier charge does not imply a negative number of carriers.

Chapter support: §1.18, detailed solution of Problem 10.

Current is a rate of charge transfer



Choose a positive direction through a cross-section. Let q_{passed} be the cumulative *signed* charge crossing in that direction:

$$I = \frac{dq_{\text{passed}}}{dt}, \quad 1 \text{ A} = 1 \text{ C s}^{-1}.$$

If 3.0×10^9 electrons per second move toward $+x$,

$$I = -(3.0 \times 10^9 \text{ s}^{-1})(1.602 \times 10^{-19} \text{ C}) \simeq -4.8 \times 10^{-10} \text{ A}.$$

Conventional positive current points opposite to electron motion.

Do not confuse two quantities

Charge stored in a body and charge passed through a section are different. Relating their rates requires a stated boundary and sign convention. Circuit theory comes later.

Chapter support: §1.5.2; Eq. (1.9) and its numerical illustration.

Coulomb's law: magnitude and direction

Two point charges at rest in vacuum

$$F = k_e \frac{|q_1 q_2|}{r^2}, \quad k_e = \frac{1}{4\pi\epsilon_0}.$$

The force lies on the line joining them. Like charges repel; opposite charges attract.

$$k_e \simeq 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}, \quad \epsilon_0 \simeq 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}.$$

These rounded constants are not exact SI definitions.

Magnitude first, direction second: $F \geq 0$ does not determine an axis component. Doubling r at fixed charges divides the magnitude by four.

The inverse-square dependence is an empirical law.

Chapter support: §1.6; Eqs. (1.10)–(1.12).

First choose a defensible model



- Identify the target, source charges and positions, medium, and supports holding the sources fixed.
- Small objects: size must be small compared with separation. For spheres of radius R and surface gap d , use $r = d + 2R$.
- Check internal charge structure. A neutral polarized body is not generally equivalent to a zero-charge point.
- A spherical *charge distribution* has special symmetry; a conducting sphere near an external charge may not.

Range of the electrostatic approximation

Sources are stationary here. A slow-motion extension requires a specified laboratory frame and negligible magnetic, retardation, and radiation effects.

Chapter support: §1.6.1; point-charge model and its limitations.

Worked example: two fixed beads



Example 1.3

Two small beads are fixed on an insulating rail, with $q_1 = +3.0 \mu\text{C}$, $q_2 = -2.0 \mu\text{C}$ and centre separation $r = 0.15 \text{ m}$. Find the force on bead 1 due to bead 2.

Assume point charges at rest and a negligible effect of the medium.

$$\begin{aligned} F_{12} &= k_e \frac{|q_1 q_2|}{r^2} \\ &= (8.99 \times 10^9) \frac{(3.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.15)^2} \text{ N} \simeq 2.40 \text{ N}. \end{aligned}$$

Direction: toward bead 2, because the signs are opposite.

Model check: this newton-scale ideal result is not proof that microcoulomb charges can be maintained on small exposed beads in air.

Chapter support: §1.6.1; Example 1.3; Eqs. (1.13)–(1.15).

Units and scaling catch different errors



$$[k_e] = \frac{[F][r]^2}{[q]^2} = \text{N m}^2 \text{C}^{-2}.$$

Unit check. For $2.0 \mu\text{C}$ and $3.0 \mu\text{C}$ at 4.0 cm ,

$$F = (8.99 \times 10^9) \frac{(2.0 \times 10^{-6})(3.0 \times 10^{-6})}{(0.040)^2} = 33.7 \text{ N}.$$

Both charge conversions multiply; missing them causes a factor 10^{12} error before any distance error.

Independent scaling checks

Both charges multiplied by b : $F \mapsto b^2 F$.

All distances multiplied by $s > 0$: $F \mapsto F/s^2$.

Reverse every charge sign: every pair product is unchanged.

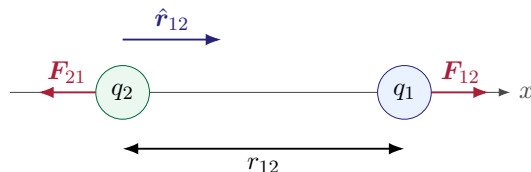
Chapter support: §1.6.2; unit/scaling illustration.

The target comes first

$\mathbf{F}_{12}$ means force **on 1 due to 2**. Define

$$\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2,$$

$$\mathbf{F}_{12} = k_e q_1 q_2 \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|^3}.$$



shown for like charges: repulsion

Figure 1.5: Vector convention for Coulomb's law. The vector $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ points from charge 2 to charge 1. For like charges, $\mathbf{F}_{12}$ points in the same direction as $\hat{\mathbf{r}}_{12}$, while $\mathbf{F}_{21} = -\mathbf{F}_{12}$.

Use **signed charges** in this vector law. Do not add a second attraction/repulsion sign afterward.

Chapter support: Conventions; §1.7; Eq. (1.18); Figure 1.5.

Components and free-body diagrams



For source-to-target differences $\Delta x = x_1 - x_2$, and similarly for y, z ,

$$r = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2},$$

$$(F_{12,x}, F_{12,y}, F_{12,z}) = \frac{k_e q_1 q_2}{r^3} (\Delta x, \Delta y, \Delta z).$$

The displacement has length r , so the magnitude still varies as $1/r^2$. Use the full three-dimensional distance, not a projection.

One diagram, one target

Anchor each force-vector tail on the object being analysed. $\mathbf{F}_{12} = -\mathbf{F}_{21}$, but the two forces act on different objects. They do not cancel in the free-body diagram of charge 1.

A planar direction needs the signs of both components, or $\text{atan2}(F_y, F_x)$. A zero vector has no direction.

Chapter support: §§1.7.1–1.7.3; Eqs. (1.19), (1.23).

Worked example: an off-axis source



Example 1.4. Target $q_0 = +3.0 \mu\text{C}$ is at the origin; source $q_s = -2.0 \mu\text{C}$ is at $(0.30, 0.40) \text{ m}$. Find the force on the target, using the point-charge model.

$$\Delta \mathbf{r} = (-0.30\hat{\mathbf{i}} - 0.40\hat{\mathbf{j}}) \text{ m}, \quad r = 0.50 \text{ m}.$$

The target is attracted toward the source: both force components must be positive.

$$\begin{aligned} \mathbf{F} &= (8.99 \times 10^9)(-6.0 \times 10^{-12}) \frac{-0.30\hat{\mathbf{i}} - 0.40\hat{\mathbf{j}}}{(0.50)^3} \text{ N} \\ &\simeq (0.129\hat{\mathbf{i}} + 0.173\hat{\mathbf{j}}) \text{ N}. \end{aligned}$$

Check: $|\mathbf{F}| = 0.216 \text{ N} = k_e |q_0 q_s| / (0.50)^2$; the direction agrees with attraction, not with $\Delta \mathbf{r}$ alone.

Chapter support: Example 1.4; Eqs. (1.20)–(1.22).

Superposition adds vectors



Force on one target from many prescribed sources

$$\mathbf{F}_0 = \sum_i \mathbf{F}_{0i} = k_e \sum_i q_0 q_i \frac{\mathbf{r}_0 - \mathbf{r}_i}{|\mathbf{r}_0 - \mathbf{r}_i|^3}.$$

Calculate every contribution *on the same target*, then add its components. The target is not one of its own sources.

- Choose axes and predict force directions before inserting numbers.
- Add magnitudes only in a collinear case with directions already accounted for.
- For conducting bodies, first determine the redistributed source charge. Superposition does not determine that distribution for you.

Chapter support: §1.8; Eqs. (1.24)–(1.25).

Worked example: three charges on a line



Example 1.6. A target $q_0 = +2.0 \mu\text{C}$ is at $x = 0$. Fixed sources are $q_1 = +4.0 \mu\text{C}$ at -0.30 m and $q_2 = -3.0 \mu\text{C}$ at $+0.20 \text{ m}$. Find $\mathbf{F}_0$.

Directions: source 1 repels the target to the right; source 2 attracts it to the right. Both contributions point along $+x$.

$$F_{01} = (8.99 \times 10^9) \frac{(2.0 \times 10^{-6})(4.0 \times 10^{-6})}{0.30^2} = 0.799 \text{ N},$$

$$F_{02} = (8.99 \times 10^9) \frac{(2.0 \times 10^{-6})(3.0 \times 10^{-6})}{0.20^2} = 1.35 \text{ N},$$

$$\mathbf{F}_0 = (F_{01} + F_{02})\hat{\mathbf{i}} \simeq 2.15\hat{\mathbf{i}} \text{ N}.$$

Check: opposite *source signs* need not mean opposite force directions on the target.

Chapter support: §1.8; Example 1.6; Eqs. (1.26)–(1.28).

Two dimensions: geometry before arithmetic

Example 1.7. Find the net force on q_0 from the two fixed point charges shown. Predict the signs of F_x and F_y first.

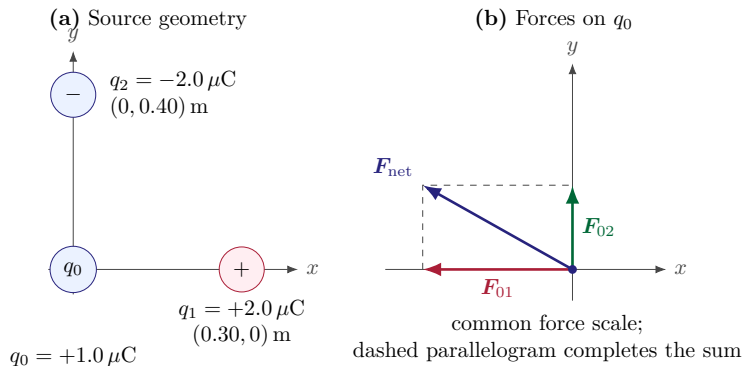


Figure 1.6: The geometry used in Example 1.7 and its separate force diagram. Force lengths are proportional to 0.200 N and 0.112 N. The resultant is their actual vector sum, not a third independent interaction. Source-position and force scales differ.

Chapter support: §1.8; Example 1.7; Figure 1.6.

Solution: components, magnitude, quadrant



Source 1 repels along $-x$; source 2 attracts along $+y$:

$$\mathbf{F}_{01} = -\frac{(8.99 \times 10^9)(1.0 \times 10^{-6})(2.0 \times 10^{-6})}{0.30^2} \hat{\mathbf{i}} \simeq -0.200 \hat{\mathbf{i}} \text{ N},$$

$$\mathbf{F}_{02} = +\frac{(8.99 \times 10^9)(1.0 \times 10^{-6})(2.0 \times 10^{-6})}{0.40^2} \hat{\mathbf{j}} \simeq +0.112 \hat{\mathbf{j}} \text{ N}.$$

$$\mathbf{F}_0 \simeq (-0.200 \hat{\mathbf{i}} + 0.112 \hat{\mathbf{j}}) \text{ N}, \quad |\mathbf{F}_0| = 0.229 \text{ N}.$$

The force lies in quadrant II. Using unrounded components,

$$\theta = \tan^{-1} \left(\frac{0.112375}{0.199778} \right) = 29.4^\circ \quad \text{above } -x.$$

Check: not $0.200 + 0.112 = 0.312 \text{ N}$; the forces are perpendicular.

Chapter support: Example 1.7; Eqs. (1.29)–(1.35).

Your turn: a negative off-axis target



Problem 17

A target charge $q_0 = -2.0 \text{ nC}$ is at $(0.10, -0.10) \text{ m}$. A fixed source $q_s = +3.0 \text{ nC}$ is at $(0.40, 0.30) \text{ m}$. Treat both as point charges and find the force vector, its magnitude, and its direction.

Explain before calculating: Why should both force components be positive even though the target charge is negative?

Then translate both positions by $(-0.10, +0.10) \text{ m}$. Does the force change?

A useful order

Target and source $\rightarrow$ displacement $\rightarrow$ distance $\rightarrow$ signed vector law $\rightarrow$ physical check.

Chapter support: §1.17, Problem 17; solution on the next slide.

Solution: signs and translation invariance



$$\Delta \mathbf{r} = (-0.30\hat{\mathbf{i}} - 0.40\hat{\mathbf{j}}) \text{ m}, \quad r = 0.50 \text{ m}, \quad q_0 q_s = -6.0 \times 10^{-18} \text{ C}^2.$$

$$\begin{aligned} \mathbf{F} &= (8.99 \times 10^9)(-6.0 \times 10^{-18}) \frac{-0.30\hat{\mathbf{i}} - 0.40\hat{\mathbf{j}}}{0.50^3} \text{ N} \\ &= (1.29456\hat{\mathbf{i}} + 1.72608\hat{\mathbf{j}}) \times 10^{-7} \text{ N} \\ &\simeq (1.3\hat{\mathbf{i}} + 1.7\hat{\mathbf{j}}) \times 10^{-7} \text{ N}. \end{aligned}$$

From unrounded components, $F = 2.1576 \times 10^{-7} \text{ N}$ and $\theta = \tan^{-1}(4/3) = 53.1^\circ$ above $+x$.

Physical check: attraction points up and right toward the source. A common translation cancels in $\mathbf{r}_0 - \mathbf{r}_s$, so it leaves the force unchanged.

Chapter support: §1.18, detailed solution of Problem 17.

Oblique forces at a triangle vertex

Example 1.8.

Three identical point charges $+Q$ are held in a horizontal plane at the vertices of an equilateral triangle of side a .

Find the force on the top charge, then evaluate it for

$$Q = 20 \text{ nC}, \quad a = 0.10 \text{ m}.$$

The diagram's upward direction lies in that horizontal plane.

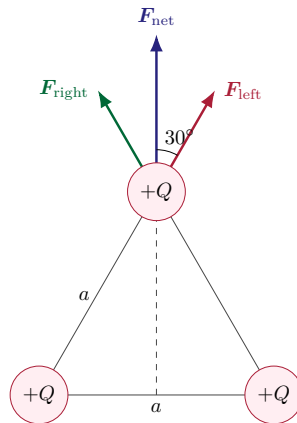


Figure 1.7: Equal source charges repel the top charge along the sides extended upward. Horizontal components cancel; vertical components add. The diagram identifies the 30° projection angle. The resultant arrow is schematic in length.

Chapter support: §1.8.1; Example 1.8; Figure 1.7.

Solution: project before adding



Each source repels the target with magnitude $F_* = k_e Q^2/a^2$, making 30° with $+y$ in the plane:

$$F_x = F_* \sin 30^\circ - F_* \sin 30^\circ = 0,$$

$$F_y = 2F_* \cos 30^\circ = \sqrt{3} \frac{k_e Q^2}{a^2}.$$

$$F_* = (8.99 \times 10^9) \frac{(20 \times 10^{-9})^2}{0.10^2} = 3.596 \times 10^{-4} \text{ N},$$

$$\boxed{\mathbf{F}_{\text{net}} \simeq 6.2 \times 10^{-4} \hat{\mathbf{j}} \text{ N.}}$$

Checks: $F_* < F_{\text{net}} < 2F_*$; doubling a divides it by four. Reversing all three signs changes no pair product. A support must oppose the in-plane resultant to hold the top charge fixed.

Chapter support: Example 1.8; Eqs. (1.36)–(1.37).

Worked example: a symmetric source pair



Example 1.9. Fixed sources $+Q$ are at $(0, +a)$ and $(0, -a)$; target $+q$ is at $(x, 0)$ with $x > 0$. Find the net force.

Both sources repel. Their transverse components cancel; their axial components add. For either source,

$$r = \sqrt{x^2 + a^2}, \quad F_* = \frac{k_e q Q}{x^2 + a^2}, \quad \cos \theta = \frac{x}{\sqrt{x^2 + a^2}}.$$

Therefore $F_x = 2F_* \cos \theta$ and $F_y = 0$, so

$$\mathbf{F}_{\text{net}} = \frac{2k_e q Q x}{(x^2 + a^2)^{3/2}} \hat{\mathbf{i}}.$$

Checks: zero at the centre; for $x \gg a$, $F_x \simeq 2k_e q Q/x^2$, as for total charge $2Q$ at the origin. The projection factor is essential.

Chapter support: §1.8.2; Example 1.9; Eqs. (1.38)–(1.42).

A resultant need not decrease monotonically

With $u = x/a$, write $F_x/(2k_e qQ/a^2) = f(u) = u/(1 + u^2)^{3/2}$.

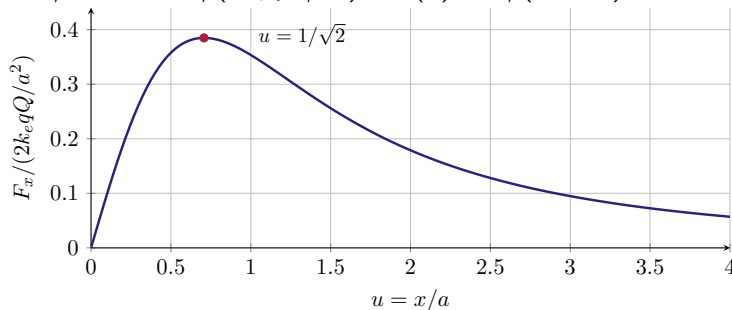


Figure 1.8: Calculated resultant of the equal source pair in Eq. (1.43), for $qQ > 0$. The cancellation at the centre and the large-distance decrease produce a maximum at a finite distance.

Since $f'(u) = (1 - 2u^2)/(1 + u^2)^{5/2}$, the peak is at $u = 1/\sqrt{2}$. Near the centre cancellation weakens; far away the force falls as $1/u^2$.

Chapter support: §1.8.2; Eq. (1.43); Figure 1.8.

Where can the electric forces cancel?



Example 1.10. Fixed charges $+9.0 \mu\text{C}$ at $x = 0$ and $+1.0 \mu\text{C}$ at $x = 0.40 \text{ m}$ act on a positive test charge. Find its zero-electric-force point between them.

For $0 < x < 0.40 \text{ m}$, the repulsions oppose one another:

$$k_e \frac{q_0 q_1}{x^2} = k_e \frac{q_0 q_2}{(0.40 - x)^2} \Rightarrow \frac{9.0}{x^2} = \frac{1.0}{(0.40 - x)^2}.$$

$$\frac{3}{x} = \frac{1}{0.40 - x} \Rightarrow 3(0.40 - x) = x \Rightarrow \boxed{x = 0.30 \text{ m}.}$$

Check: the weaker charge is closer, compensating its smaller magnitude. The sources require supports. Mechanical equilibrium means *all* forces sum to zero, not only the electric forces.

Chapter support: §1.9; Example 1.10; Eqs. (1.44)–(1.50).

Opposite sources: test the regions



Example 1.11. Sources $+4Q$ and $-Q$ ($Q > 0$) are fixed at 0 and L . Where is the electric force on a positive target zero?

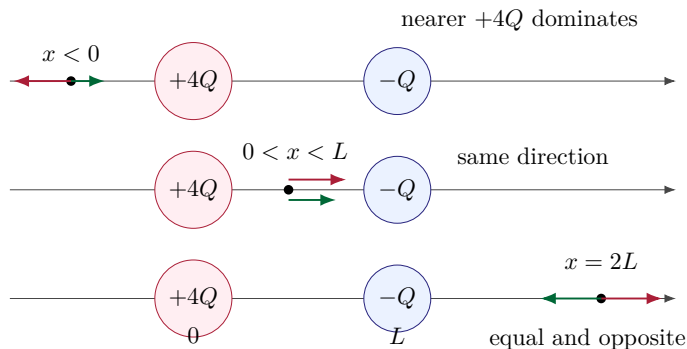


Figure 1.9: Region test for a positive target near sources $+4Q$ and $-Q$. Arrows show forces on the black target point; unequal lengths in the first two panels are qualitative. Only to the right of the smaller-magnitude source can cancellation occur.

Chapter support: §1.9.1; Example 1.11; Figure 1.9.

Solution: a root must belong to the model

The only possible region is $x > L$: forces oppose, and the smaller source is nearer. The distances are x and $x - L$.

$$k_e \frac{q_0(4Q)}{x^2} = k_e \frac{q_0 Q}{(x - L)^2} \Rightarrow \frac{4}{x^2} = \frac{1}{(x - L)^2}.$$

Since both distances are positive,

$$\frac{2}{x} = \frac{1}{x - L} \Rightarrow 2(x - L) = x \Rightarrow \boxed{x = 2L}.$$

Why not the other squared-equation root?

$x = 2L/3$ equates magnitudes inside the interval, where the forces point in the *same* direction. It is not a zero-force point.

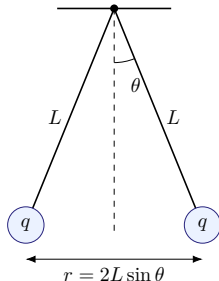
Reversing a nonzero test charge reverses both forces but not their zero. It can change stability, which is a separate question.

Chapter support: Example 1.11; Eqs. (1.51)–(1.55); §1.16.2.

Suspended balls: what is actually measured?

Example 1.12. Identical balls of mass $m = 0.20 \text{ g}$ hang from threads of length $L = 0.50 \text{ m}$ at one point. Equal charges give $\theta = 6.0^\circ$ from vertical. Estimate $|q|$ using point charges and $g = 9.81 \text{ m s}^{-2}$.

(a) Apparatus geometry



(b) Right ball

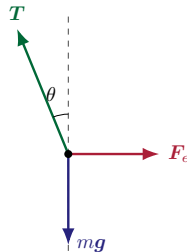


Figure 1.10: Suspended equal charges and the forces on one ball. Angles are exaggerated for clarity. The force arrows are qualitative; the equilibrium equations determine their magnitudes. The distance in Coulomb's law is the centre-to-centre separation r .

Chapter support: §1.9.2; Example 1.12; Figure 1.10.

From force balance to inferred charge



On the right ball: weight downward, tension along the thread, and electric repulsion horizontally outward. At rest,

$$T \cos \theta = mg, \quad T \sin \theta = F_e \quad \Rightarrow \quad F_e = mg \tan \theta.$$

Geometry gives $r = 2L \sin \theta$. For equal charges,

$$F_e = k_e \frac{q^2}{r^2} \quad \Rightarrow \quad |q| = r \sqrt{\frac{F_e}{k_e}}.$$

The angle determines both the force and the distance

$$|q| = 2L \sin \theta \sqrt{\frac{mg \tan \theta}{k_e}}.$$

No small-angle approximation was needed to obtain this result.

Chapter support: Example 1.12; Eqs. (1.56)–(1.60).

Result and limits of the ball experiment



Convert $m = 0.20 \text{ g} = 2.0 \times 10^{-4} \text{ kg}$:

$$F_e = (2.0 \times 10^{-4})(9.81) \tan 6.0^\circ = 2.06 \times 10^{-4} \text{ N},$$

$$r = 2(0.50) \sin 6.0^\circ \simeq 0.105 \text{ m},$$

$$|q| = r\sqrt{F_e/k_e} \simeq 1.6 \times 10^{-8} \text{ C} = 16 \text{ nC}.$$

Unrounded values give about 15.8 nC. Units check: $\sqrt{F_e/k_e}$ has units C/m.

What the experiment does not establish

Repulsion does not distinguish two positive charges from two negative ones. Equal deflection does not prove equal charges: without justified equal-charge preparation, the geometry determines $|q_1 q_2|$, not each charge separately.

Chapter support: Example 1.12; Eqs. (1.61)–(1.65) and following discussion.

Charge sharing needs symmetry

For two identical isolated spheres in a symmetric environment,

$$q'_A = q'_B = \frac{q_A + q_B}{2}.$$

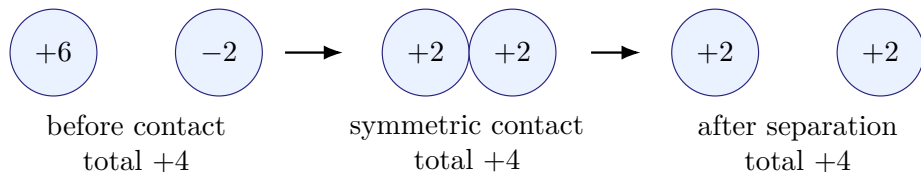


Figure 1.11: Charge sharing between identical isolated conducting spheres in a symmetric environment. Values are in a common charge unit. Equal final charges follow from symmetry in addition to conservation; they are not a consequence of conservation alone.

Conservation fixes the sum; **symmetry** fixes equal division. Unequal spheres or nearby objects that distort the distribution need additional analysis. Insulating supports must prevent leakage.

Chapter support: §1.10; Eq. (1.66); Figure 1.11.

Share charge, then calculate force



Example 1.13. Identical small conducting spheres carry $+12\text{ nC}$ and -4 nC . They touch under symmetric isolated conditions, separate, then are placed 0.25 m apart. Find their charges and force.

$$Q_{\text{total}} = +12 - 4 = +8\text{ nC}, \quad q'_A = q'_B = +4\text{ nC}.$$

At large separation compared with their radii, use point charges:

$$\begin{aligned} F &= k_e \frac{|q'_A q'_B|}{r^2} \\ &= (8.99 \times 10^9) \frac{(4.0 \times 10^{-9})^2}{(0.25)^2} = 2.30 \times 10^{-6}\text{ N}. \end{aligned}$$

Direction and check: repulsive, since both final charges are positive; their sum remains $+8\text{ nC}$. Do not use their initial charges in the final force calculation.

Chapter support: §1.10; Example 1.13; Eqs. (1.67)–(1.71).

Sequential contact: update the state



Example 1.14. Three identical spheres initially carry $q_A = +12 \text{ nC}$, $q_B = 0$, $q_C = -6 \text{ nC}$. A touches B and separates; then B touches C and separates. Find the final charges.

State (charges in nC)	A	B	C	Total
Initially	+12	0	-6	+6
After A touches B	+6	+6	-6	+6
After B touches C	+6	0	0	+6

First share $12 + 0$ between A and B. Then share the *updated* $6 - 6$ between B and C; A no longer participates.

Assumptions and check

At each contact: an isolated, symmetric pair, with the third sphere far away. The total remains $+6 \text{ nC}$.

Chapter support: §1.10.1; Example 1.14; Eqs. (1.72)–(1.75).

What do before-and-after forces identify?



Two identical spheres are measured at the same separation r before and after symmetric, isolated contact. On the right sphere, take rightward repulsion as positive:

$$P = q_1 q_2 = \frac{F_{\text{before}} r^2}{k_e}, \quad |S| = |q_1 + q_2| = 2r \sqrt{\frac{F_{\text{after}}}{k_e}}.$$

Here F_{after} is the repulsive force magnitude. An independent polarity test must determine the sign of S .

$$t^2 - St + P = 0 \quad \Rightarrow \quad q_{1,2} = \frac{S \pm \sqrt{S^2 - 4P}}{2}.$$

Inference limits: this gives an *unordered* pair. Without a sign reference, reversing both charges reproduces the same forces. A negative inferred discriminant flags inconsistent data or model assumptions, not imaginary physical charges.

Chapter support: §1.10.2; Eq. (1.76). Further reading: Example 1.15.

Electric force versus gravitation



Example 1.16. Compare the electric and gravitational attractions between an electron and proton at the same separation:

$$F_e = \frac{k_e e^2}{r^2}, \quad F_g = \frac{G m_e m_p}{r^2} \Rightarrow \frac{F_e}{F_g} = \frac{k_e e^2}{G m_e m_p}.$$

Use $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$, and $m_p = 1.67 \times 10^{-27} \text{ kg}$:

$$\frac{F_e}{F_g} = \frac{(8.99 \times 10^9)(1.602 \times 10^{-19})^2}{(6.67 \times 10^{-11})(9.11 \times 10^{-31})(1.67 \times 10^{-27})} \simeq 2.3 \times 10^{39}.$$

Check and meaning: the ratio is dimensionless and r cancels. It is specific to this particle pair, not a universal force ratio. Bulk matter is nearly neutral, whereas ordinary mass contributions add with the same sign.

Chapter support: §1.11; Example 1.16.

Engineering screen: force versus weight



Example 1.17. Two small grains each have $m = 2.0 \times 10^{-7} \text{ kg}$ and $q = 50 \text{ pC}$; centre separation is $r = 5.0 \text{ mm}$. Compare the ideal mutual force with each grain's weight.

$$F_e = (8.99 \times 10^9) \frac{(50 \times 10^{-12})^2}{(5.0 \times 10^{-3})^2} = 8.99 \times 10^{-7} \text{ N},$$
$$mg = (2.0 \times 10^{-7})(9.81) = 1.962 \times 10^{-6} \text{ N},$$
$$\eta = \frac{F_e}{mg} \simeq 0.46.$$

Equal magnitudes would require $|q| = r\sqrt{mg/k_e} \simeq 74 \text{ pC}$.

Interpretation: electrostatics is relevant on this scale. Equal magnitudes are not equilibrium unless directions also balance. Adhesion, drag, geometry, and charge leakage remain to be assessed.

Chapter support: §1.12; Example 1.17; Eq. (1.80). Ideal point-charge screening model.

An electroscope observes a response

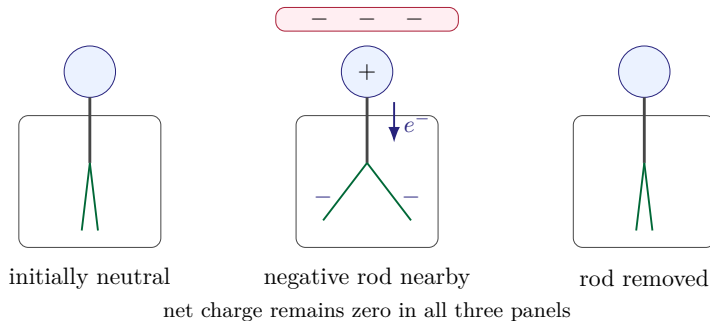


Figure 1.12: Induction in a neutral electroscope, with no contact or ground path. Charge signs indicate induced imbalance; leaf angles are schematic. This reversible response cannot by itself determine the sign of an unknown rod.

A negative rod moves electrons into the leaves, so they repel while **total charge stays zero**. Remove the rod: the response reverses unless contact, grounding, or leakage has changed the charge.

Chapter support: §§1.13.1–1.13.3; Figure 1.12.

A torsion balance: angle to force to charge



Example 1.19. An ideal balance has torsion constant $\kappa = 2.0 \times 10^{-6} \text{ N m rad}^{-1}$ and arm length $\ell = 0.10 \text{ m}$. A $+2.0 \text{ nC}$ source at $r = 0.10 \text{ m}$ produces a repulsive deflection $\phi = 0.100 \text{ rad}$. Infer the bead charge.

With force perpendicular to the deflected arm, equilibrium gives

$$\kappa\phi = \ell F \quad \Rightarrow \quad F = \frac{(2.0 \times 10^{-6})(0.100)}{0.10} = 2.0 \times 10^{-6} \text{ N}.$$

$$q = + \frac{Fr^2}{k_e Q} = \frac{(2.0 \times 10^{-6})(0.10)^2}{(8.99 \times 10^9)(2.0 \times 10^{-9})} \simeq +1.1 \text{ nC}.$$

Repulsion fixes the sign within the point-charge model. Calibrate κ , measure the actual deflected geometry, and test $Fr^2 = \text{constant}$ only while the charge product is fixed.

Chapter support: §1.13.4; Example 1.19; Eq. (1.84). Ideal model, not measured data.

From the vector law to a reliable algorithm



For target $(q_0, \mathbf{r}_0)$ and each distinct source $(q_i, \mathbf{r}_i)$,

$$\Delta \mathbf{r}_i = \mathbf{r}_0 - \mathbf{r}_i, \quad r_i = |\Delta \mathbf{r}_i|, \quad \Delta \mathbf{F}_i = k_e q_0 q_i \frac{\Delta \mathbf{r}_i}{r_i^3}.$$

Direct summation

Start with three zero component sums. For each source, compute its displacement and distance, reject a zero distance, and add the signed contribution. Do not include the target as its own source.

Validate the implementation: pair attraction/repulsion; a known analytical sum; symmetry; common translation; and scaling every distance by two, which divides force by four.

An arbitrary small denominator changes the model. It is not a harmless fix for coincident point charges.

A numerical sum with an analytical check



Example 1.20. Find the force on $q_0 = +1.0 \mu\text{C}$ at the origin from these fixed sources. Positions are in metres; forces in newtons.

q_i	(x_i, y_i)	$(F_{0i,x}, F_{0i,y})$
$+1.0 \mu\text{C}$	$(+0.20, 0)$	$(-0.22475, 0)$
$-1.0 \mu\text{C}$	$(0, +0.20)$	$(0, +0.22475)$
$+2.0 \mu\text{C}$	$(-0.20, 0)$	$(+0.44950, 0)$

The common unit force is $k_e(10^{-6})^2/(0.20)^2 = 0.22475 \text{ N}$.

$$\mathbf{F} = (0.22475\hat{\mathbf{i}} + 0.22475\hat{\mathbf{j}}) \text{ N}, \quad F \simeq 0.318 \text{ N}, \quad \theta = 45^\circ.$$

Equal resultant components check the direction. Near exact cancellation, judge residuals against a nonzero characteristic force, not against zero.

Chapter support: §1.14; Example 1.20. Guard digits retained for code validation.

Core synthesis: equations need a model



Charge: an isolated total is conserved; ordinary free charge is quantized.

$$Q_{\text{isolated}} = \sum_i q_i = \text{constant}, \quad q = ne.$$

Prescribed point sources: use signed charges and displacements.

$$\mathbf{F}_0 = k_e \sum_i q_0 q_i \frac{\mathbf{r}_0 - \mathbf{r}_i}{|\mathbf{r}_0 - \mathbf{r}_i|^3}.$$

Equilibrium: sum *all* forces on the target: $\sum \mathbf{F} = \mathbf{0}$.

Symmetric identical spheres: $q'_A = q'_B = (q_A + q_B)/2$.

Before accepting an answer

Check target, sources, signs, SI units, centre distances, symmetry or limits, and whether the model describes the object or experiment.

Chapter support: §1.15. Suspended-ball inference is developed on slides 36–38.

Reading and the bridge to Chapter 2



Primary text: Hamzeh Khanpour, *Electric Charge and Coulomb's Law*, Chapter 1, first edition (1 September 2026).

Comparative reading

- Halliday, Resnick and Walker, *Fundamentals of Physics*, Vol. 2, 12th ed., Wiley (2022), Chapter 21.
- Serway and Jewett, *Physics for Scientists and Engineers with Modern Physics*, 9th ed., Brooks/Cole (2014), §§23.1–23.3.
- Ling, Sanny and Moebs, *University Physics*, Vol. 2, OpenStax (2016; supplied 2018 revision), §§5.1–5.3.

Next chapter: separate the source from the test charge.

For prescribed sources, dividing force by a small nonzero test charge leads to the electric field. The same signs, geometry, and superposition remain essential.

Advanced Topics for Technical and Engineering Students

Advanced: one missing source



Example 1.21 - predict, then solve. Six $+Q$ charges would form a regular hexagon of radius R around a central $+q$, but the positive- x vertex is empty. Find the force on $+q$.

Rotation symmetry makes the completed hexagon's central resultant zero. The missing positive source would have repelled the target toward $-x$:

$$\mathbf{F}_{\text{removed}} = -\frac{k_e q Q}{R^2} \hat{\mathbf{i}}.$$

Subtract that contribution from the complete sum:

$$\mathbf{F}_{\text{remaining}} = \mathbf{0} - \mathbf{F}_{\text{removed}} = +\frac{k_e q Q}{R^2} \hat{\mathbf{i}}.$$

Interpretation and check: the target is pushed toward the empty vertex. No force acts *from* empty space; that point is a direction reference. A negative target reverses the resultant.

Chapter support: §1.16.1; Example 1.21. Fixed point charges; $Q, q > 0$.

Advanced: equilibrium is only a starting point



Two fixed $+Q$ sources are at $(\pm a, 0, 0)$. A positive target q at their midpoint has zero resultant. Displace it along x , with $|x| < a$:

$$F_x(x) = k_e q Q \left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2} \right].$$

For $|u| \ll 1$, the first-order Taylor expansion is $(1+u)^{-2} = 1 - 2u + O(u^2)$. Applying it to both terms gives

$$F_x(x) \simeq -\frac{4k_e q Q}{a^3} x = -K_x x, \quad K_x = \frac{4k_e q Q}{a^3} > 0.$$

The force opposes the displacement: restoring *along this axis*. If a guide allows only axial motion, $m\ddot{x} = -K_x x$ and $\omega_x = \sqrt{K_x/m}$.

Check: K_x has units N/m, not the units of k_e . Stability requires information near equilibrium, not just at one point.

Chapter support: §1.16.2; Eqs. (1.90)–(1.91); $|x| \ll a$.

Advanced: the transverse test



Keep sources $+Q$ at $(\pm a, 0, 0)$, but displace the same positive target to $(0, y, 0)$. The horizontal components cancel; each transverse component is $k_e q Q y / (a^2 + y^2)^{3/2}$.

$$F_y(y) = \frac{2k_e q Q y}{(a^2 + y^2)^{3/2}} \simeq + \frac{2k_e q Q}{a^3} y \quad (|y| \ll a).$$

A restoring axis is not a three-dimensional trap

For $q > 0$, the axial force opposes a small x displacement, but the transverse force reinforces a small y displacement. The midpoint is therefore unstable to unrestricted motion.

Reversing q reverses both gradients: the transverse force becomes restoring, but the axial force becomes destabilizing. An unstable direction remains.

A guide, time-dependent forces, or material responses change these fixed-source assumptions and require a new analysis.

Chapter support: §1.16.2; Eqs. (1.91)–(1.92).

Advanced: a charged bead on a spring

Charges are fixed: bead $-q$ and source $+Q$, with $q, Q > 0$. A frictionless guide balances vertical forces; the spring stiffness is K .

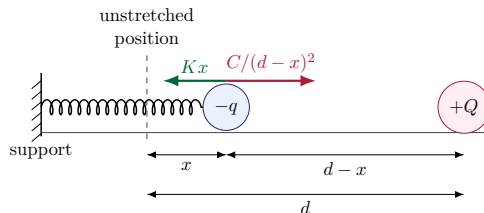


Figure 1.16: A spring opposes attraction toward a fixed positive source. The displacement x is measured from the unstretched position, so the electrical separation is $d - x$. Arrow lengths are schematic; equality occurs only at equilibrium.

$$F(x) = -Kx + \frac{C}{(d-x)^2}, \quad 0 < x < d.$$

Here $C = k_e qQ > 0$ has units N m^2 .

Chapter support: §1.16.3; Eq. (1.94); Figure 1.16.

Advanced: the equilibrium branches

Set $u = x/d$, $\mu = C/(Kd^3)$. Force balance gives $\mu = h(u) = u(1 - u)^2$.

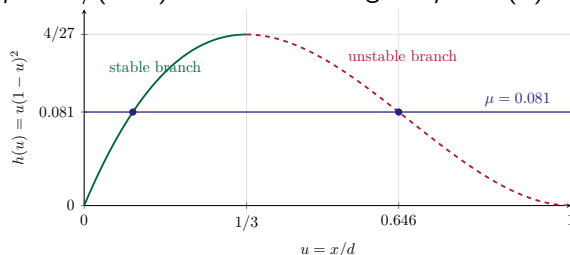


Figure 1.17: The normalized spring–charge equilibrium curve. A horizontal line at the prescribed coupling μ intersects the curve at the possible equilibria. The smaller-displacement branch is restoring; the other branch is unstable. The peak marks loss of the stable operating point.

$$h'(u) = (1 - u)(1 - 3u) \Rightarrow u_{\max} = \frac{1}{3}, \quad h_{\max} = \frac{4}{27}.$$

At the peak the linear stiffness vanishes; this does not establish stability.

Chapter support: §1.16.3; Eqs. (1.95)–(1.97); Figure 1.17.

Advanced: only one stable operating point



Example 1.23. Find the spring equilibria for $K = 0.010 \text{ N/m}$, $d = 0.020 \text{ m}$, $C = 6.48 \times 10^{-9} \text{ N m}^2$.

$$\mu = \frac{6.48 \times 10^{-9}}{(0.010)(0.020)^3} = 0.081, \quad (u - 0.1)(u^2 - 1.9u + 0.81) = 0.$$

Root u	Position $x = du$	Interpretation
0.100	2.0 mm	Stable along guide
0.645862...	12.9 mm	Unstable
1.254138...	Outside $0 < x < d$	Reject for this model

$$K_{\text{eff}} = -F'(x_*) = K - \frac{2C}{(d - x_*)^3} = K \frac{1 - 3u_*}{1 - u_*}.$$

At $u_* = 0.1$, $K_{\text{eff}} = 0.010(0.7/0.9) \simeq 7.8 \times 10^{-3} \text{ N/m}$ and $Kx = C/(d - x)^2 = 2.0 \times 10^{-5} \text{ N}$. The undamped bead oscillates. Fixed-charge and voltage-driven models differ.

Chapter support: §1.16.3; Example 1.23.

Advanced: uncertainty is not model bias



For the suspended balls, $|q| = 2L \sin \theta \sqrt{mg \tan \theta / k_e}$. With g, k_e fixed, differentiate its logarithm:

$$\frac{d|q|}{|q|} = \frac{dL}{L} + \frac{1}{2} \frac{dm}{m} + A(\theta)d\theta, \quad A(\theta) = \cot \theta + \frac{1}{2 \sin \theta \cos \theta}.$$

For small, independent *standard uncertainties*, first-order propagation gives

$$\left(\frac{\sigma_q}{|q|} \right)^2 \simeq \left(\frac{\sigma_L}{L} \right)^2 + \frac{1}{4} \left(\frac{\sigma_m}{m} \right)^2 + A(\theta)^2 \sigma_\theta^2.$$

Angle uncertainty is in radians. The same angle determines distance and force; count it once through the combined sensitivity.

Two different limitations

Measurement uncertainty concerns inputs within a model. Polarization, unequal charges, or lack of equilibrium can bias the model itself. Repetition does not remove a wrong model.

Chapter support: §1.16.4; Eqs. (1.98)–(1.99).

Advanced: which measurement matters most?



Example 1.24. Use independent standard uncertainties:

$$m = (0.200 \pm 0.005) \text{ g}, \quad L = (0.500 \pm 0.002) \text{ m}, \quad \theta = (6.0 \pm 0.2)^\circ.$$

At 6° , $A \simeq 14.32 \text{ rad}^{-1}$ and $\sigma_\theta = 0.2\pi/180 = 0.003491 \text{ rad}$.

Input	Fractional contribution to charge uncertainty
Length	$\sigma_L/L = 0.002/0.500 = 0.0040$
Mass	$\sigma_m/(2m) = 0.005/(2 \cdot 0.200) = 0.0125$
Angle	$A\sigma_\theta \simeq 0.0500$

$$\frac{\sigma_q}{|q|} \simeq \sqrt{0.0040^2 + 0.0125^2 + 0.0500^2} = 0.0517, \quad |q| = (15.8 \pm 0.8) \text{ nC}.$$

The angle dominates. Improving only the mass precision does little; improved precision still requires a valid physical model.

Chapter support: §1.16.4; Example 1.24.

Advanced: quantify the point-charge error



Return to sources $+Q$ at $(0, \pm a)$ and target $+q$ at $(x, 0)$, $x > 0$. Replacing the pair by $2Q$ at the origin gives

$$F_{\text{exact}} = \frac{2k_e q Q x}{(x^2 + a^2)^{3/2}}, \quad F_{\text{point}} = \frac{2k_e q Q}{x^2}.$$

Their ratio is $F_{\text{exact}}/F_{\text{point}} = (1 + a^2/x^2)^{-3/2}$.

Relative error, with the exact force in the denominator

$$\varepsilon = \frac{F_{\text{point}} - F_{\text{exact}}}{F_{\text{exact}}} = \left(1 + \frac{a^2}{x^2}\right)^{3/2} - 1.$$

For $a/x \ll 1$, $(1 + v)^{3/2} = 1 + \frac{3}{2}v + O(v^2)$ gives $\varepsilon \simeq \frac{3}{2}(a/x)^2$.

The point replacement overestimates this force. State the geometry, observable, and error denominator before quoting a tolerance.

Chapter support: §1.16.5; Eq. (1.101); same geometry as Example 1.9.

Advanced: how far is far enough?



Example 1.25. For the symmetric pair, require a point-replacement error below 1% relative to the exact force. How large must x/a be?

$$\left(1 + \frac{a^2}{x^2}\right)^{3/2} < 1.01 \Rightarrow \frac{a^2}{x^2} < (1.01)^{2/3} - 1.$$

$$\frac{x}{a} > \frac{1}{\sqrt{(1.01)^{2/3} - 1}} \simeq 12.3.$$

At $x = 10a$, the error is $(1.01)^{3/2} - 1 \simeq 1.50\%$: “ten times farther” is not a 1% criterion here.

The engineering conclusion

A credible answer needs force prediction, stability, measurement uncertainty, and an adequate model. A long decimal answer guarantees none of these.

Chapter support: §1.16.5; Example 1.25; synthesis of §§1.16.1–1.16.5.

