

Electric Fields and Field Lines

Physics II - Chapter 2

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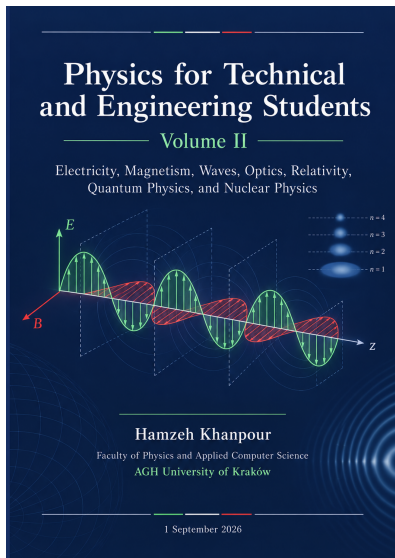
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Our Physics II course book



Physics for Technical and Engineering Students - Volume II

Hamzeh Khanpour

Electricity, Magnetism, Waves, Optics, Relativity, Quantum Physics, and Nuclear Physics

Faculty of Physics and Applied Computer Science, AGH University of Kraków

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Role in this course

- Written for technical and engineering students and aligned with the Physics II progression.
- Connects physical motivation to equations, modelling assumptions, worked reasoning and applications.
- Supports lectures, auditorium-class problem solving, and independent study between classes.

Why introduce an electric field?



Compute the source's influence once. Then predict the force on any small charge placed at the observation point:

$$\text{source charges} \longrightarrow \mathbf{E}(\mathbf{r}) \longrightarrow \mathbf{F} = q\mathbf{E}.$$

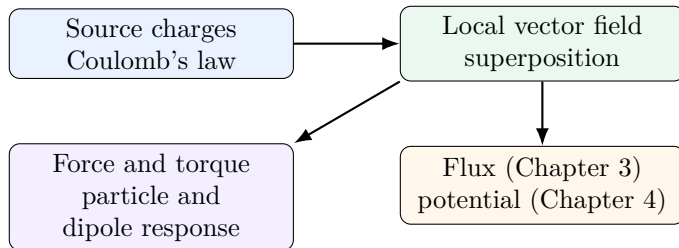


Figure 2.1: The field separates the source calculation from the response of a particle or dipole. The same local vectors also prepare the flux and potential descriptions developed in subsequent chapters.

Chapter support: Chapter 2, §2.1; Figure 2.1.

Learning route and prerequisites



- **Describe a field:** distinguish source, observation point and probe; connect field vectors with signed forces.
- **Calculate a field:** use Coulomb's law, vector superposition and symmetry; integrate a specified continuous charge distribution.
- **Predict response:** calculate particle deflection and dipole torque, work and rotational stability.
- **Test a model:** check units, limits and numerical accuracy; distinguish measurement uncertainty from source disturbance.

Prerequisites: signed charge, Cartesian vectors, trigonometry, Newton's law and elementary integration. Slides 51–60 develop advanced response, accuracy and source inference.

Chapter support: Chapter 2, §§2.1–2.15.

Source, observation point and probe



Source: charges at positions $\mathbf{r}_i$ generate the field. **Observation point:** $\mathbf{r}$ specifies where that field is evaluated. **Probe:** a small charge q_0 samples it.

Operational definition

$$\mathbf{E}(\mathbf{r}) = \lim_{q_0 \rightarrow 0^+} \frac{\mathbf{F}_{\text{ext}}(\mathbf{r})}{q_0}.$$

The positive probe is sufficiently weak that it does not measurably rearrange the source. Its force defines the field direction.

The field exists at an empty point. The limit expresses negligible disturbance; it does not require continuously divisible elementary charge.

Chapter support: Chapter 2, §2.2, Definition 2.1.

The responding charge feels the external field



For a point particle, exclude its own singular field:

$$\mathbf{F}_i = q_i \mathbf{E}_{\text{ext}}(\mathbf{r}_i) = k_e q_i \sum_{j \neq i} q_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}.$$

At a fixed point in an unchanged source field, a positive charge feels force along $\mathbf{E}$; a negative charge feels force opposite to it. Acceleration follows from $m\mathbf{a} = q\mathbf{E}$.

$$\mathbf{F}_{\text{body}} = \int \rho_{\text{body}}(\mathbf{r}) \mathbf{E}_{\text{ext}}(\mathbf{r}) d^3r.$$

For an extended body, $Q\mathbf{E}$ follows only when the field is uniform across it. Neutrality alone guarantees neither zero force in a varying field nor zero torque.

Chapter support: Chapter 2, §2.2.

Example 2.2 — field, force and acceleration



Example 2.2 — complete statement

An electron enters a region where the electric field is uniform and has magnitude $E = 2.0 \times 10^4 \text{ N C}^{-1}$ directed upward. Find the force and acceleration of the electron, neglecting gravity.

Take upward as $+y$. With $q = -e$, $e = 1.602 \times 10^{-19} \text{ C}$ and $m_e = 9.11 \times 10^{-31} \text{ kg}$:

$$F_y = -eE = -3.204 \times 10^{-15} \text{ N},$$
$$a_y = F_y/m_e = -3.52 \times 10^{15} \text{ m s}^{-2}.$$

Both vectors point downward. The magnitude ratio $|a_y|/g = 3.59 \times 10^{14}$, using $g = 9.81 \text{ m s}^{-2}$, justifies neglecting gravity. **Units:** $[E] = \text{N/C} = \text{V/m}$; field strength is force per charge, not acceleration.

Chapter support: Chapter 2, §2.3; Example 2.2.

Derive the point-charge field



For a source q at $\mathbf{r}'$ and a positive nonperturbing probe q_0 at $\mathbf{r}$, define the **source-to-observation** displacement:

$$\mathbf{R} = \mathbf{r} - \mathbf{r}', \quad R = |\mathbf{R}|, \quad \hat{\mathbf{R}} = \mathbf{R}/R.$$

Coulomb's force divided by the probe charge gives

$$\mathbf{F}_{\text{ext}} = k_e q q_0 \frac{\mathbf{R}}{R^3} \implies \boxed{\mathbf{E}(\mathbf{r}) = k_e q \frac{\mathbf{R}}{R^3} = \frac{k_e q}{R^2} \hat{\mathbf{R}}}, \quad R > 0.$$

Use signed q once. A negative source automatically reverses the direction. The magnitude is $k_e |q|/R^2$; the inverse-square denominator multiplies a length vector.

Chapter support: Chapter 2, §2.4; coordinate form of Coulomb's field, $k_e = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$.

Point-source direction and radial symmetry

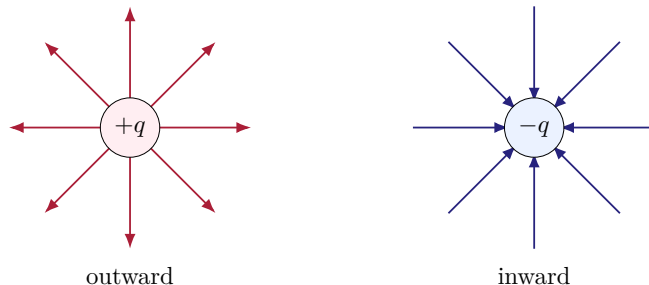


Figure 2.2: Radial field lines in a plane through an isolated point charge. Arrows indicate direction along the lines, not displacement from the charge to a field point or a calibrated field magnitude. The three-dimensional field is spherically symmetric. At equal distances, $+q$ and $-q$ give equal magnitudes and opposite directions. Doubling charge doubles the field; tripling distance divides its magnitude by nine.

Chapter support: Chapter 2, §2.4; Figure 2.2.

Example 2.4 — an off-axis field



Example 2.4 — complete statement

A charge $q = -2.0 \text{ nC}$ is at $(0.10, 0.00) \text{ m}$. Find the field at $P = (0.40, 0.40) \text{ m}$.

Subtract source coordinates from observation coordinates:

$$\mathbf{R} = (0.30\hat{\mathbf{i}} + 0.40\hat{\mathbf{j}}) \text{ m}, \quad R = 0.50 \text{ m}.$$

$$\begin{aligned}\mathbf{E} &= \frac{(8.99 \times 10^9)(-2.0 \times 10^{-9})}{(0.50)^3} (0.30\hat{\mathbf{i}} + 0.40\hat{\mathbf{j}}) \\ &= (-43.152\hat{\mathbf{i}} - 57.536\hat{\mathbf{j}}) \text{ N/C}.\end{aligned}$$

Thus $\mathbf{E} \simeq -(43\hat{\mathbf{i}} + 58\hat{\mathbf{j}}) \text{ N/C}$, toward the source. Before rounding, $|\mathbf{E}| = 71.92 \text{ N/C}$ and $E_y/E_x = 4/3$, matching the displacement geometry.

Chapter support: Chapter 2, §2.4; Example 2.4.

Superposition adds vectors at one point

At one observation point, $\mathbf{E}(\mathbf{r}) = \sum_i \mathbf{E}_i(\mathbf{r})$.

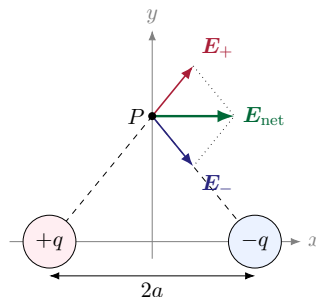


Figure 2.3: Vector addition on the perpendicular bisector of opposite equal charges. Both component arrows are parallel to their exact source-to-point directions, with the negative-source contribution reversed. The horizontal resultant equals their vector sum.

Chapter support: Chapter 2, §2.5; Figure 2.3.

Example 2.5 — opposite charges at a midpoint



Example 2.5 — complete statement

Two point charges lie on the x axis. Charge $q_1 = +3.0 \text{ nC}$ is at $x = 0$ and charge $q_2 = -1.0 \text{ nC}$ is at $x = 0.40 \text{ m}$. Find the electric field at the midpoint.

The midpoint is $x = 0.20 \text{ m}$. Away from $q_1 > 0$ and toward $q_2 < 0$ both mean **rightward** here.

$$E_{1x} = \frac{(8.99 \times 10^9)(3.0 \times 10^{-9})}{0.20^2} = 674.25 \text{ N/C}.$$

Similarly, $E_{2x} = 224.75 \text{ N/C}$.

$$\mathbf{E} = (899 \text{ N/C})\hat{\mathbf{i}} \simeq (9.0 \times 10^2 \text{ N/C})\hat{\mathbf{i}}.$$

Opposite source signs give no cancellation at this midpoint: the field directions agree. Each contribution has units $k_e q/R^2 = \text{N/C}$.

Chapter support: Chapter 2, §2.5; Example 2.5.

Example 2.6 — symmetry with signed components



Example 2.6 — complete statement

Two identical charges $+q$ are fixed at $x = \pm a$ on the x axis. Find the electric field at the point $P = (0, y)$.

For $q, a > 0$, let $R = \sqrt{a^2 + y^2}$. The two source-to-point vectors are (a, y) and $(-a, y)$:

$$\mathbf{E}_L = \frac{k_e q}{R^3} (a\hat{\mathbf{i}} + y\hat{\mathbf{j}}), \quad \mathbf{E}_R = \frac{k_e q}{R^3} (-a\hat{\mathbf{i}} + y\hat{\mathbf{j}}).$$

$$\mathbf{E}(0, y) = \frac{2k_e q y}{(a^2 + y^2)^{3/2}} \hat{\mathbf{j}}.$$

The field points upward for $y > 0$, downward for $y < 0$, and vanishes at $y = 0$. For $|y| \gg a$, it tends to $2k_e q y / |y|^3 \hat{\mathbf{j}}$: the far field of total charge $2q$.

Chapter support: Chapter 2, §2.5; Example 2.6.

Triangle geometry before the component sum

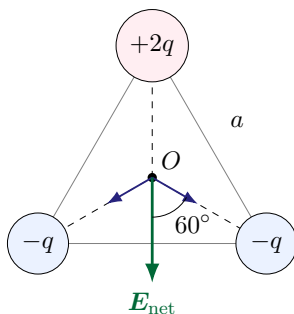


Figure 2.4: Charges on an equilateral triangle. At the centroid, each field from a lower negative charge is at 60° to the downward vertical. Their vertical components and the field from the upper positive charge all point downward. Arrow lengths indicate directions schematically.

Take the centroid as the origin and upward as $+y$. The complete problem and solution follow on slide 14.

Chapter support: Chapter 2, §2.5; Figure 2.4.

Example 2.8 — direction first, magnitude check



Example 2.8 — complete statement

Three charges are placed at the vertices of an equilateral triangle of side a . The top vertex carries charge $+2q$, and the two bottom vertices each carry charge $-q$. Find the direction of the electric field at the centre of the triangle.

The bottom horizontal components cancel. Both bottom negative sources pull the positive probe downward; the top positive source pushes it downward.

$$E_{\text{top}} = \frac{2k_e q}{(a/\sqrt{3})^2} = \frac{6k_e q}{a^2}, \quad E_b = \frac{3k_e q}{a^2}.$$

$$\mathbf{E}_O = -(E_{\text{top}} + 2E_b \cos 60^\circ)\hat{\mathbf{j}} = -\frac{9k_e q}{a^2}\hat{\mathbf{j}}.$$

The downward projection is $1/2$: a bottom vertex has $y = -a/(2\sqrt{3})$ and distance $a/\sqrt{3}$. Using $\cos 30^\circ$ would use the wrong reference axis.

Chapter support: Chapter 2, §2.5; Example 2.8.

Example 2.7 — locate a like-charge field null



Example 2.7 — complete statement

A charge $+4q$ is fixed at $x = 0$ and a charge $+q$ is fixed at $x = L$. Find the point on the x axis between the charges where the electric field is zero.

With $q, L > 0$ and $0 < x < L$, the $+4q$ field points right and the $+q$ field points left. Only here can their magnitudes cancel:

$$\frac{4k_e q}{x^2} = \frac{k_e q}{(L - x)^2} \implies \frac{2}{x} = \frac{1}{L - x} \implies \boxed{x = \frac{2L}{3}}.$$

The distances are $2L/3$ and $L/3$; the larger charge's squared distance is four times larger. Outside the pair, both fields point the same way, so no other axial root exists. **Qualification:** a field null is a cancellation at one point; it does not establish a field-free neighbourhood or stable three-dimensional equilibrium.

Chapter support: Chapter 2, §2.5; Example 2.7.

Practice A4 — analyse directions before algebra



Problem A4 — complete statement

A charge $+q$ is at $x = 0$ and $-4q$ at $x = L$. Find all finite zero-field points on the axis, considering the three intervals separately.

Take $q > 0$ and $L > 0$. The source points $x = 0, L$ are excluded. Right is positive; determine directions separately in each open interval.

- $x < 0$: $(E_{+q}, E_{-4q}) = (\leftarrow, \rightarrow)$. Cancellation is possible: the smaller charge can be nearer.
- $0 < x < L$: $(\rightarrow, \rightarrow)$. The two fields add; no root exists.
- $x > L$: $(\rightarrow, \leftarrow)$, but $4/(x - L)^2 > 1/x^2$. The larger, nearer negative source dominates.

The complete remaining algebra follows on slide 17.

Chapter support: Chapter 2, §2.16, A4; §2.17.

Practice A4 — complete root and signed check



For $+q$ at 0 and $-4q$ at L , the interval analysis leaves only $x < 0$. Write $x = -s$ with $s > 0$:

$$E_x = -\frac{k_e q}{s^2} + \frac{4k_e q}{(L+s)^2} = 0.$$

$$\frac{1}{s^2} = \frac{4}{(L+s)^2} \implies L+s = 2s \implies \boxed{x = -L}.$$

At $x = -L$, the distances are L and $2L$:

$$E_x = -\frac{k_e q}{L^2} + \frac{4k_e q}{(2L)^2} = 0.$$

This is the only finite axial root. The magnitude equation alone also produces $x = L/3$ after squaring; that point fails the direction test because both fields point right.

Chapter support: Chapter 2, A4; Example 2.39.

What a field-line diagram means

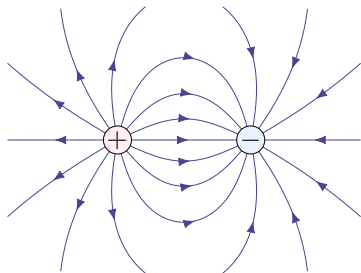


- **Tangent:** its arrow gives the local field direction, defined by force on a positive probe.
- **Endpoints:** lines begin on positive charge and end on negative charge, or extend to or from infinity.
- **Noncrossing:** at a regular point with $\mathbf{E} \neq 0$, the field has one direction. Distinct field-line tangents cannot cross there. Direction is undefined at a null.
- **Density:** magnitude comparisons require an appropriate common seeding convention. Illustrative lines do not supply a calibrated magnitude scale.

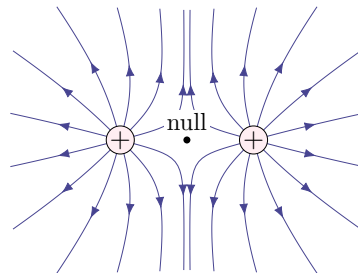
A field line is not generally a particle trajectory: particle velocity retains its history, while force sets its acceleration. A moving positive particle need not travel along the local field tangent.

Chapter support: Chapter 2, §2.6.

Two-charge field-line patterns



(a) $+q$ and $-q$



(b) $+q$ and $+q$

Figure 2.5: Field lines obtained by integrating the normalized Coulomb field of two point charges in the plane of the page. Curves stop near the sources or at the panel boundary. In (a), arrows run from positive to negative charge; in (b), lines leave both positive charges and the midpoint is a null. Seed choices are illustrative and do not calibrate line density as field strength.

Chapter support: Chapter 2, §2.6; Figure 2.5.

Problem C6 — audit the diagram and correct it

Problem C6 — complete statement

A student draws field lines for $+2q$ and $-q$ with six lines leaving the positive charge and six entering the negative charge, and two regular curves crossing at a point where $\mathbf{E} \neq 0$. Diagnose both errors if line count is intended to be proportional to charge magnitude.

Crossing error: a regular nonzero field has one direction. Two distinct field-line tangents cannot cross at the stated point. **Count error:** proportional counting requires a 2:1 ratio. If six lines leave $+2q$, only three enter $-q$; the other three continue to infinity. Alternatively, six lines entering $-q$ require twelve leaving $+2q$, with six continuing outward. Absolute line count is conventional; the ratio represents relative charge magnitudes. The diagram is a two-dimensional representation of a three-dimensional field, not a literal measurement of its magnitude.

Chapter support: Chapter 2, C6; Example 2.38.

Dipole coordinates and the exact axial field

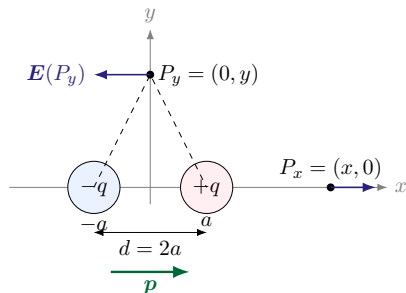


Figure 2.6: Coordinates for the exact axial and perpendicular-bisector fields of a dipole. The field on the bisector is opposite to $\mathbf{p}$; on the exterior positive axis it is parallel to $\mathbf{p}$.

The moment points from $-q$ to $+q$: $\mathbf{p} = 2aq\hat{\mathbf{i}}$, with $d = 2a$.

$$E_x = k_e q \left[\frac{1}{(x - a)^2} - \frac{1}{(x + a)^2} \right] = \frac{4k_e q a x}{(x^2 - a^2)^2}, \quad x > a.$$

Chapter support: §2.7; Figure 2.6.

Dipole bisector: neutral does not mean zero field



At $P = (0, y)$, the source-to-point vectors are $-a\hat{i} + y\hat{j}$ from $+q$ and $a\hat{i} + y\hat{j}$ from $-q$. Their distances are equal.

$$\begin{aligned} \mathbf{E}(0, y) &= \frac{k_e q}{(a^2 + y^2)^{3/2}} [(-a\hat{i} + y\hat{j}) - (a\hat{i} + y\hat{j})] \\ &= -\frac{2k_e q a}{(a^2 + y^2)^{3/2}} \hat{i} = -\frac{k_e \mathbf{p}}{(a^2 + y^2)^{3/2}}. \end{aligned}$$

The y components cancel; the x components add opposite to $\mathbf{p}$.

Exact centre and reflection checks

$\mathbf{E}(0, 0) = -2k_e q \hat{i} / a^2$ is finite and nonzero. The result is even in y : above and below the midpoint the field has the same direction.

Chapter support: §2.7.

Example 2.10: dipole far-field estimate



Example 2.10 — complete statement

An electric dipole consists of charges $\pm 5.0 \text{ nC}$ separated by 4.0 mm . Estimate the electric field on the dipole axis at a point 0.20 m from the dipole centre, on the positive side of the dipole moment.

$$a = d/2 = 2.0 \text{ mm} \text{ and } p = qd = 2.0 \times 10^{-11} \text{ C m.}$$

$$E_{\text{ax}} \simeq \frac{2k_e p}{x^3} = \frac{2(8.99 \times 10^9)(2.0 \times 10^{-11})}{(0.20)^3} = 44.95 \text{ N/C} \simeq 45 \text{ N/C.}$$

The field is parallel to $\mathbf{p}$. Here $a/x = 0.010$, so the axial underestimate is only 0.020% .

$$|\mathbf{E}_{\perp}| \simeq \frac{k_e p}{r^3}, \quad \mathbf{E}_{\perp} \parallel -\mathbf{p} : \quad |\mathbf{E}_{\text{ax}}| = 2|\mathbf{E}_{\perp}| \quad (r \gg a).$$

On either exterior axial side the far field is parallel to $\mathbf{p}$.

Chapter support: §2.7; Example 2.10.

Student practice A8: define the accuracy requirement

Problem A8 — complete statement

For the same dipole, determine the minimum exterior axial distance x/a for the far-field approximation to have error below 1% relative to the exact field. Give the condition also in units of the separation $d = 2a$.

The referenced dipole has fixed sources $-q$ at $-a$ and $+q$ at $+a$ in vacuum, $q, a > 0$, $d = 2a$, $\mathbf{p} = 2aq\hat{\mathbf{i}}$. Work at $x > a$. The exact and approximate positive components are

$$E_{\text{exact}} = \frac{2k_e p}{x^3} \left(1 - \frac{a^2}{x^2}\right)^{-2}, \quad E_{\text{dip}} = \frac{2k_e p}{x^3}.$$

Use the *exact* field in the error denominator:

$$\delta = \frac{E_{\text{exact}} - E_{\text{dip}}}{E_{\text{exact}}}.$$

Chapter support: Problem A8; §2.7; Solution A8 (Example 2.41).

A8 solution: half-separation and separation differ



Put $u = a^2/x^2$. The exterior domain gives $0 < u < 1$ and $1 - u > 0$.

$$\delta = 1 - (1 - u)^2 = 2u - u^2,$$

$$\delta < 0.01 \iff (1 - u)^2 > 0.99 \iff u < 1 - \sqrt{0.99}.$$

Therefore the strict accuracy condition is

$$\frac{x}{a} > \frac{1}{\sqrt{1 - \sqrt{0.99}}} \simeq 14.1,$$

$$\frac{x}{d} > 7.06, \quad d = 2a.$$

Interpretation and independent checks

The dipole approximation underestimates the exterior axial field. At $x = 10a$ the error is 1.99%; at $x = 10d = 20a$ it is 0.499%. The threshold is an infimum: equality gives exactly 1%.

Chapter support: Problem A8; Solution A8, Example 2.41.

Uniform-field motion: keep the charge and field signed



Take $+y$ upward, $\mathbf{E} = E_y \hat{\mathbf{j}}$, and entry conditions $x = 0$, $y = y_0$, $\mathbf{v} = v_0 \hat{\mathbf{i}}$.
For an external uniform field,

$$ma_y = qE_y, \quad x = v_0 t, \quad y = y_0 + \frac{qE_y}{2m} t^2.$$

Eliminate time using $t = x/v_0$:

$$y(x) = y_0 + \frac{qE_y}{2mv_0^2} x^2.$$

Direction and model

The sign of qE_y determines curvature. The path is parabolic although the field lines are straight. Assume nonrelativistic motion, no collisions or other forces, and a field constant during passage.

Check: qE_y/m is acceleration and $(x/v_0)^2$ is time squared, giving a length.

Chapter support: §2.8; signed transverse motion and trajectory.

The plates set the exit state; the drift is straight

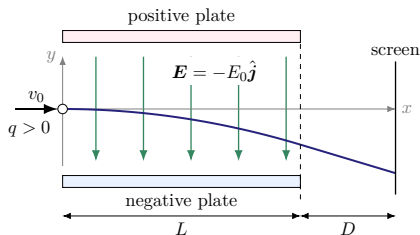


Figure 2.7: A positive particle in a downward uniform field: a parabolic path between the plates followed by straight field-free drift. The ideal trajectory clears the plates. A negative particle with the same entry conditions bends upward. Dimensions and curvature are schematic.

For entry on the midplane, the exit time is $t_L = L/v_0$ and

$$y_L = \frac{qE_y L^2}{2mv_0^2}, \quad v_{y,L} = \frac{qE_y L}{mv_0}.$$

The figure uses $q > 0$ and $E_y < 0$; the next example uses $E_y > 0$.

Chapter support: §2.8; Figure 2.7.

Example 2.13: complete ion-deflector problem

Example 2.13 — complete statement

Singly charged positive ions of mass $4.0 \times 10^{-26} \text{ kg}$ enter horizontally at $8.0 \times 10^4 \text{ m s}^{-1}$. The plates have length $L = 0.12 \text{ m}$ and gap $g = 8.0 \text{ mm}$. A screen lies $D = 0.20 \text{ m}$ beyond the exit. What upward field component produces a screen displacement of 4.0 mm , and does the beam clear the plates?

Explicit model and coordinate convention

Take upward as $+y$, $q = +e = 1.602 \times 10^{-19} \text{ C}$, and $y_{\text{screen}} = +0.0040 \text{ m}$. The beam enters on the midplane with zero transverse velocity. Treat ions as nonrelativistic point particles; neglect fringing, beam width, space charge and other forces.

The force acts over L only; the subsequent distance D is field-free. The gap g is a geometric clearance dimension, not gravitational acceleration.

Chapter support: Example 2.13; §2.8 downstream-screen model.

Example 2.13: infer the field from the screen

During the drift, $v_{y,L}$ remains constant. Thus

$$\begin{aligned} y_{\text{screen}} &= y_L + v_{y,L} \frac{D}{v_0} \\ &= \frac{qE_y L^2}{2mv_0^2} + \frac{qE_y L D}{mv_0^2} = \frac{qE_y L}{mv_0^2} \left(D + \frac{L}{2} \right). \end{aligned}$$

For $q = e$ and an upward screen displacement,

$$\begin{aligned} E_y &= \frac{m y_{\text{screen}} v_0^2}{e L (D + L/2)} \\ &= \frac{(4.0 \times 10^{-26})(0.0040)(8.0 \times 10^4)^2}{(1.602 \times 10^{-19})(0.12)(0.26)} \\ &\simeq +2.0 \times 10^2 \text{ N/C}. \end{aligned}$$

Both q and y_{screen} are positive, so the field is upward. Using $L + D$ in the parabolic law would apply acceleration through the drift incorrectly.

Chapter support: Example 2.13; screen-deflection equation, §2.8.

Example 2.13: clearance, angle and validity



The exit displacement and subsequent drift contribution are

$$y_L = y_{\text{screen}} \frac{L/2}{D + L/2} = 0.92 \text{ mm}, \quad y_{\text{drift}} = 4.0 - 0.92 = 3.08 \text{ mm}.$$

Since $0 < y_L < g/2 = 4.0 \text{ mm}$ and the in-field path increases monotonically, the ideal central ray clears both plates.

$$\tan \phi = \frac{v_{y,L}}{v_0} = \frac{y_{\text{screen}}}{D + L/2} = 0.0154, \quad \phi \simeq 0.88^\circ.$$

Here $v_{y,L} \simeq 1.23 \times 10^3 \text{ m/s}$. The exit speed is $8.00 \times 10^4 \text{ m/s} \simeq 2.67 \times 10^{-4} c$, safely nonrelativistic. The collision bound is

$$|E_y| < \frac{mgv_0^2}{|q|L^2} \simeq 8.9 \times 10^2 \text{ N/C} \quad (g \text{ is the plate gap}).$$

Finite beam radius, entry offset and velocity spread consume clearance; this bound is a model result, not an equipment safety rating.

Chapter support: Example 2.13.

Answered checks: field direction and detector readings

Check 1: reverse the responding charge

For the same field, mass and entry speed, what changes when q changes sign?
Both a_y and all deflections reverse; the external field remains unchanged.

Check 2: compare the screen with the plate exit

Why is the screen displacement larger? For $D > 0$, the acquired exit transverse velocity produces $y_{\text{drift}} = v_{y,L} D / v_0$. A larger reading does not mean that the field acts in the drift.

Check 3: change the entry speed

What happens to deflection when v_0 rises by 1% at fixed q/m , field and geometry? Since $y \propto v_0^{-2}$, the displacement decreases by approximately 2%.

Chapter support: §2.8.

Continuous sources: integrate charge and vector direction



Density replaces a large discrete source by a macroscopic distribution:

$$dq = \lambda d\ell, \quad dq = \sigma dA, \quad dq = \rho dV.$$

The units are respectively C/m, C/m² and C/m³. At a fixed observation point $\mathbf{r}$,

$$\mathbf{R} = \mathbf{r} - \mathbf{r}', \quad \boxed{\mathbf{E}(\mathbf{r}) = k_e \int \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} dq}.$$

A reliable integration procedure

Choose the source coordinate, express dq and $\mathbf{R}$, cancel components using the *signed distribution's* symmetry, then integrate the surviving components with explicit bounds.

The distribution is prescribed. A uniform insulating source is a useful model; a conductor generally redistributes charge. Scalar magnitudes cannot be added when directions differ.

Chapter support: §2.9.

Ring geometry: equal distances, different directions

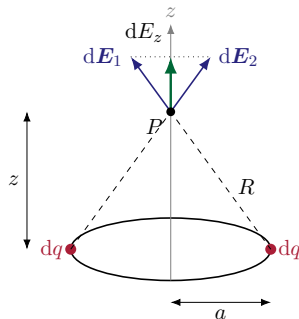


Figure 2.8: Opposite equal elements of a uniformly charged ring. The horizontal components cancel and the axial components add. The labelled dE_z is the axial component of one element; the pair contributes $2dE_z$. The ring is drawn in perspective.

$$dE_z = \frac{k_e dq}{R^2} \frac{z}{R}, \quad R = \sqrt{a^2 + z^2}.$$

Chapter support: §2.9.1; Figure 2.8.

Integrate the ring: sign, centre and distant field



For a uniform ring, choose azimuth φ and $dq = \lambda a d\varphi = (Q/2\pi) d\varphi$. Then

$$\begin{aligned} E_z &= \frac{k_e z}{(a^2 + z^2)^{3/2}} \int_0^{2\pi} \frac{Q}{2\pi} d\varphi \\ &= \boxed{\frac{k_e Q z}{(a^2 + z^2)^{3/2}}}, \quad \mathbf{E} = E_z \hat{\mathbf{k}}. \end{aligned}$$

Checks for the fixed source Q, a

At $z = 0$, $E_z = 0$ by symmetry. Near the centre, $E_z \simeq k_e Q z / a^3$. Far from the ring, $E_z \simeq k_e Q z / |z|^3$: the point-charge field on either side.

The sign reverses across the ring plane; the axial coordinate z is not integrated. A point-charge model fails near the centre despite having the same total charge.

Chapter support: §2.9.1; Example 2.14.

The signed ring profile has a zero and two extrema

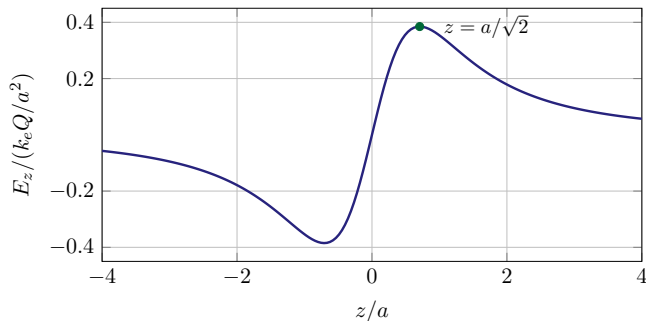


Figure 2.9: Signed axial field of a uniformly charged positive ring. The dimensionless plot uses the exact field; its maximum for $z > 0$ occurs at $a/\sqrt{2}$. The negative side follows by reflection and a sign change.

With $u = z/a$, $E_z = (k_e Q / a^2) f(u)$ and

$$f(u) = \frac{u}{(1 + u^2)^{3/2}}, \quad f'(u) = \frac{1 - 2u^2}{(1 + u^2)^{5/2}}.$$

Chapter support: §2.9.1; Figure 2.9.

Finite rod: project before integrating

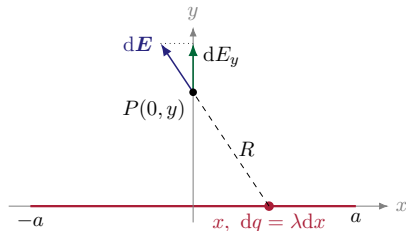


Figure 2.10: Field contribution from a positive element of a rod on its perpendicular bisector. Pairing the element at x with one at $-x$ cancels the horizontal component. The axial coordinate is a source variable, whereas y remains fixed in the integral.

For $-a \leq x \leq a$, $dq = \lambda dx$ and $\mathbf{R} = -x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$. Opposite elements cancel E_x ; the surviving element is

$$dE_y = k_e \lambda y \frac{dx}{(x^2 + y^2)^{3/2}}.$$

Chapter support: §2.9.2; Figure 2.10.

Finite rod: evaluate the endpoints and test the limits



For the perpendicular-bisector point $(0, y)$, $y \neq 0$,

$$\begin{aligned} E_y &= k_e \lambda y \int_{-a}^a \frac{dx}{(x^2 + y^2)^{3/2}} \\ &= k_e \lambda y \left[\frac{x}{y^2 \sqrt{x^2 + y^2}} \right]_{-a}^a \\ &= \boxed{\frac{2k_e \lambda a}{y \sqrt{a^2 + y^2}}}, \quad E_x = 0. \end{aligned}$$

Two limits with different fixed quantities

For $a \gg |y|$ at fixed λ , $E_y \rightarrow 2k_e \lambda / y$, the infinite-line result. For $|y| \gg a$ with the finite source fixed, $E_y \simeq k_e Q y / |y|^3$, where $Q = 2a\lambda$.

The sign of y reverses the component. For positive charge the field points away from the rod.

Chapter support: §2.9.2.

Example 2.16: a finite rod is not an infinite line



Example 2.16 — complete statement

A rod of length 0.60 m carries total charge $Q = 18 \text{ nC}$ uniformly. Find the field at a point 0.20 m from the centre on the perpendicular bisector.

Take the observation side as $+y$. Here $a = 0.30 \text{ m}$ and $\lambda = Q/(2a) = 3.0 \times 10^{-8} \text{ C/m}$.

$$E_y = \frac{2(8.99 \times 10^9)(3.0 \times 10^{-8})(0.30)}{(0.20)\sqrt{0.30^2 + 0.20^2}} \simeq +2.2 \times 10^3 \text{ N/C}.$$

The field is perpendicular to and away from the positive rod. The infinite-line value $2k_e\lambda/y \simeq 2.7 \times 10^3 \text{ N/C}$ is larger: it adds source charge beyond both endpoints.

$$\frac{E_{\text{finite}}}{E_{\text{infinite}}} = \frac{a}{\sqrt{a^2 + y^2}} = 0.832.$$

Chapter support: Example 2.16.

Disk geometry: build the source from annuli

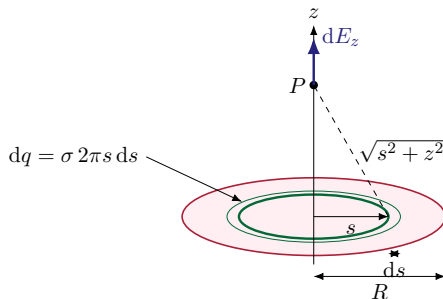


Figure 2.13: A disk decomposed into concentric annuli. Each annulus contributes the axial field of a ring. The radial width is exaggerated; the ellipse is a perspective view of a circular disk.

A thin annulus has $dA = 2\pi s ds$; apply the ring result:

$$dE_z = \frac{k_e z dq}{(s^2 + z^2)^{3/2}}, \quad dq = 2\pi\sigma s ds.$$

Chapter support: §2.9.6–2.9.7; Figure 2.13.

Disk integral: the axis and the one-sided limits

At fixed $z > 0$, integrate the annular source radius $0 \leq s \leq R$:

$$\begin{aligned} E_z &= 2\pi k_e \sigma z \int_0^R \frac{s \, ds}{(s^2 + z^2)^{3/2}} \\ &= 2\pi k_e \sigma z \left[-\frac{1}{\sqrt{s^2 + z^2}} \right]_0^R \\ &= \boxed{\frac{\sigma}{2\epsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}} \right)}. \end{aligned}$$

Below the disk the sign reverses. At its centre the one-sided limits are $E_z(0^\pm) = \pm\sigma/(2\epsilon_0)$; the ideal surface has no continuous field value.

What stays fixed?

$R \gg z$ at fixed σ gives the sheet value. For $z \gg R$ with the disk fixed, $E_z \simeq k_e Q/z^2$, $Q = \pi R^2 \sigma$. Increasing R at fixed Q instead dilutes the density.

Chapter support: §2.9.6–2.9.7.

Example 2.21: quantify the sheet approximation

Example 2.21 — complete statement

At height $z > 0$, require the exact uniform-disk field to differ from the sheet value $E_{\text{sheet}} = \sigma/(2\epsilon_0)$ by less than 5% of E_{sheet} . Find a condition on R/z .

For a nonzero prescribed uniform density, the omitted fraction is

$$\frac{E_{\text{sheet}} - E_{\text{disk}}}{E_{\text{sheet}}} = \frac{z}{\sqrt{z^2 + R^2}} < 0.05.$$

Dividing by $z > 0$ and squaring gives

$$1 + (R/z)^2 > 0.05^{-2}, \quad \boxed{R/z > \sqrt{399} \simeq 20}.$$

At $z = 2 \text{ mm}$ the threshold radius is about 4 cm . The finite-disk field is smaller than the sheet value. This axial criterion concerns prescribed uniform charge; it does not measure general fringing or solve a conducting-electrode problem.

Chapter support: Example 2.21; §2.9.7.

Choose the source model and its validity domain



Ring	Axial symmetry cancels transverse components; $E_z = 0$ at its centre and reverses across its plane.
Finite rod	The bisector symmetry removes E_x . An observation point away from the bisector generally retains both components.
Disk	Annular decomposition gives an axial field and one-sided surface limits for a prescribed uniform density.

Further chapter reading

§2.9.3: the general rod, signed endpoint angles and Example 2.17. §2.9.4: an exterior axial rod and Example 2.18. §2.9.5: arc components and the complete quarter-circle Example 2.19.

Geometry alone is insufficient: the signed density must have the symmetry used. The chosen integration formula computes a known distribution, not an unknown conductor's charge.

Chapter support: §2.9: [detailed ring/bisector-rod/disk teaching](#).

A rigid dipole: zero force, finite torque

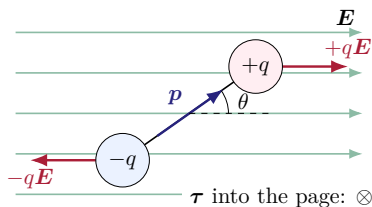


Figure 2.15: A rigid dipole in a horizontal external field. The angle θ is measured from $\mathbf{E}$ to $\mathbf{p}$. Equal and opposite forces give zero net force and a clockwise torque, into the page, tending to reduce θ .

Chapter support: §2.10; Figure 2.15.

For fixed separation $\mathbf{d}$ from $-q$ to $+q$, $\mathbf{p} = q\mathbf{d}$.

$$\mathbf{F}_+ = q\mathbf{E}, \quad \mathbf{F}_- = -q\mathbf{E}.$$

Thus $\mathbf{F}_{\text{net}} = 0$. About the midpoint,

$$\begin{aligned} \boldsymbol{\tau} &= \frac{\mathbf{d}}{2} \times q\mathbf{E} + \left(-\frac{\mathbf{d}}{2}\right) \times (-q\mathbf{E}) \\ &= \mathbf{p} \times \mathbf{E}, \quad |\boldsymbol{\tau}| = pE \sin \theta. \end{aligned}$$

The *external* field is uniform. A dipole's initial translational velocity persists; zero net force does not mean rest.

Rotational work gives the dipole energy

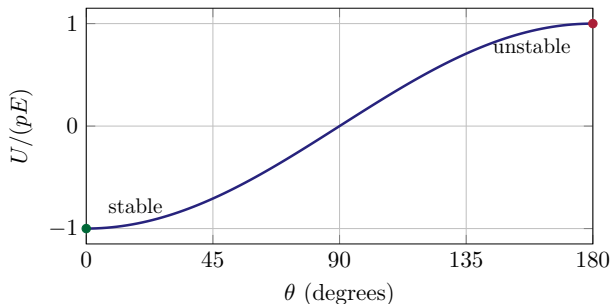


Figure 2.14: Interaction energy of a rigid permanent dipole in a uniform field. The curve is calculated from $U/(pE) = -\cos \theta$. Its stationary points have opposite stability.

With signed torque $\tau_\theta = -pE \sin \theta$, integrating $dU = -\tau_\theta d\theta$ gives $U = -pE \cos \theta$, with $U(\pi/2) = 0$. This permanent-dipole interaction energy has stable alignment and unstable antialignment; an undamped dipole can oscillate rather than settle.

Chapter support: §2.10; Figure 2.14; mechanical-work derivation.

Student practice A13 — complete problem



A rigid dipole of moment p rotates from 120° to 30° in a uniform field of magnitude E . Find ΔU and the work by the field. What is the external work for a quasistatic rotation? What additional term is needed if rotational kinetic energy changes?

Energy bookkeeping. Keep p and E fixed. The initial and final angles are measured from the external field:

$$U_i = -pE \cos 120^\circ, \quad U_f = -pE \cos 30^\circ.$$

The field's rotational work changes the interaction energy with the opposite sign:

$$W_{\text{field}} = \int_{\theta_i}^{\theta_f} \tau_\theta d\theta = -\Delta U.$$

The work–energy theorem retains the external agent explicitly:

$$W_{\text{ext}} + W_{\text{field}} = \Delta K_{\text{rot}}.$$

Chapter support: A13, §2.16; Solution A13, Example 2.43, §2.17.

A13 — all four answers and their signs



Since $\cos 120^\circ = -1/2$ and $\cos 30^\circ = \sqrt{3}/2$,

$$\Delta U = -\frac{1 + \sqrt{3}}{2} pE \simeq -1.366 pE,$$

$$W_{\text{field}} = -\Delta U = +\frac{1 + \sqrt{3}}{2} pE.$$

Quasistatic rotation: $\Delta K_{\text{rot}} \simeq 0$, hence

$$W_{\text{ext}} = \Delta U = -\frac{1 + \sqrt{3}}{2} pE.$$

The agent removes energy while restraining field-driven rotation.

Changing rotational kinetic energy:

$$W_{\text{ext}} = \Delta K_{\text{rot}} + \Delta U.$$

All terms have units $\text{C m} \times \text{N/C} = \text{J}$. With no external work, $\Delta K_{\text{rot}} = -\Delta U > 0$, providing an independent sign check.

Chapter support: A13; Example 2.43.

A field probe: infer E from mechanical balance

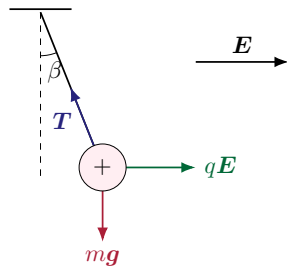


Figure 2.16: Inferring a horizontal field from the equilibrium of a positive charged bead. The force diagram is schematic; arrow lengths are not a calibrated force scale. The angle is measured from the vertical.

Chapter support: §2.11; Figure 2.16.

For a known positive bead charge, β is measured from the vertical:

$$T \cos \beta = mg,$$

$$T \sin \beta = qE,$$

$$E = \frac{mg \tan \beta}{q}.$$

Assume equilibrium, stable charge, a small bead and negligible source disturbance.

Distinguish: calibration and reading uncertainties from physical probe disturbance. Repetition does not remove a common bias.

Linear drag: acceleration becomes terminal drift



Take y along the chosen field axis; E is its signed component. For constant E , linear drag $b > 0$, and $v(0) = y(0) = 0$,

$$m\dot{v} = qE - bv, \quad \tau = \frac{m}{b}, \quad v_d = \frac{qE}{b}.$$

The integrating factor $e^{t/\tau}$ gives

$$\frac{d}{dt} \left(e^{t/\tau} v \right) = \frac{qE}{m} e^{t/\tau} \Rightarrow v(t) = v_d (1 - e^{-t/\tau}).$$

Integrating the velocity gives the exact displacement:

$$y(t) = v_d \left[t - \tau(1 - e^{-t/\tau}) \right].$$

For $t \ll \tau$, $y \simeq qEt^2/(2m)$: constant acceleration. For $t \gg \tau$, $y \simeq v_d(t - \tau) \simeq v_d t$ when $\tau/t \ll 1$.

At terminal speed, electric force is balanced by drag. Negative charges drift opposite to $\mathbf{E}$. This collisional model differs from a collision-free beam.

Chapter support: §2.12; Example 2.24.

Compute physical vectors before plotting directions

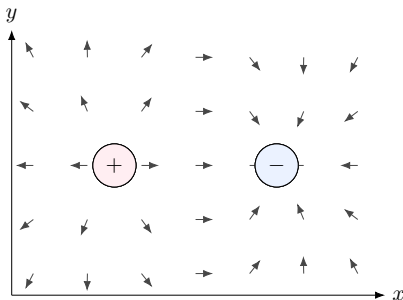


Figure 2.17: A normalized arrow plot computed from Coulomb superposition for a dipole pair of point charges. Each arrow shows the direction of the computed field at a grid point. Arrow lengths are equal by construction and carry no magnitude information. Coordinates are in arbitrary length units.

Chapter support: §2.13; Figure 2.17; analytic validation in §2.15.9.

Compute signed SI components:

$$\mathbf{E} = k_e \sum_i q_i \frac{\mathbf{R}_i}{R_i^3},$$

where $\mathbf{R}_i = \mathbf{r} - \mathbf{r}_i$ and $R_i = |\mathbf{R}_i|$.

Keep $|\mathbf{E}|$; normalize only at regular nonzero fields.

Check units, signs, symmetry and the dipole's r^{-3} far field.

Scaling all charges by 10^3 scales magnitudes by 10^3 , preserving directions.

Core synthesis, reading and the next field method



Specify signed sources and observation point; sum or integrate vectors; choose the response law; check units, direction, limits and accuracy.

Keep field lines distinct from trajectories, normalized directions from magnitudes, and collision-free acceleration from collisional drift.

Reading pointers from the chapter bibliography

Halliday–Resnick–Walker, *Fundamentals of Physics*, Vol. 2, 12th ed.: Ch. 22, “Electric Fields.”

Serway–Jewett, *Physics for Scientists and Engineers with Modern Physics*, 9th ed.: §§23.4–23.7.

Ling–Sanny–Moebs, *University Physics*, Vol. 2, OpenStax: §§5.4–5.7.

The full books were not supplied or independently consulted.

Next: Chapter 3 develops flux and Gauss’s law; Chapter 4 develops scalar electric potential.

Chapter support: §2.14; §2.18.

Advanced: source moments organize the far field



Integrate over sources $\mathbf{r}'$ at fixed observation $\mathbf{r}$:

$$\mathbf{E}(\mathbf{r}) = k_e \int \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} dq.$$

For source size $\ell_s \ll r$, expand to first order in ℓ_s/r using $(1 + u)^{-3/2} = 1 - 3u/2 + O(u^2)$. Define

$$Q_{\text{tot}} = \int dq, \quad \mathbf{p} = \int \mathbf{r}' dq, \quad \hat{\mathbf{n}} = \frac{\mathbf{r}}{r}.$$

$$\mathbf{E} \simeq \frac{k_e Q_{\text{tot}}}{r^2} \hat{\mathbf{n}} + \frac{k_e}{r^3} [3(\mathbf{p} \cdot \hat{\mathbf{n}}) \hat{\mathbf{n}} - \mathbf{p}].$$

Neutrality removes the monopole term. The remainder is $O(k_e Q_{\text{abs}} \ell_s^2 / r^4)$, with $Q_{\text{abs}} = \int |dq|$: an *absolute* error scale, not a small relative error after cancellation.

For a neutral source, the axial field is parallel to $\mathbf{p}$ with magnitude $2k_e p / r^3$; the equatorial field is $-k_e \mathbf{p} / r^3$.

Chapter support: §2.15.1.

Advanced Example 2.26 — oblique dipole direction



A compact neutral source has dipole moment $\mathbf{p} = p\hat{\mathbf{z}}$. Find the field direction at $\theta = 60^\circ$ in the far zone, expressing it in the local radial and angular basis.

$\hat{\mathbf{r}}$ points outward; $\hat{\boldsymbol{\theta}}$ points toward increasing polar angle measured from $+z$. Resolve $\hat{\mathbf{z}} = \cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}}$:

$$E_r = \frac{2k_e p \cos\theta}{r^3}, \quad E_\theta = \frac{k_e p \sin\theta}{r^3}.$$

At 60° ,

$$\mathbf{E} = \frac{k_e p}{r^3} \left(\hat{\mathbf{r}} + \frac{\sqrt{3}}{2} \hat{\boldsymbol{\theta}} \right), \quad |\mathbf{E}| = \frac{\sqrt{7} k_e p}{2r^3}.$$

$$\psi = \tan^{-1} \frac{E_\theta}{E_r} = \tan^{-1}(\sqrt{3}/2) = 40.9^\circ.$$

The tilt is toward increasing θ ; the field is generally neither radial nor parallel to the dipole axis. At 0° and 90° , the expressions recover the axial and equatorial fields. At fixed angle, both components scale as r^{-3} .

Chapter support: Example 2.26, §2.15.1.

Advanced: permanent and induced dipoles differ



In a smooth field varying slowly over the dipole size,

$$\begin{aligned}\mathbf{F} &= q [\mathbf{E}(\mathbf{r} + \mathbf{d}/2) - \mathbf{E}(\mathbf{r} - \mathbf{d}/2)] \\ &\simeq (\mathbf{p} \cdot \nabla) \mathbf{E}, \quad F_i \simeq \sum_j p_j \frac{\partial E_i}{\partial x_j}.\end{aligned}$$

For a fixed permanent moment, reversing its orientation reverses the gradient force. Neutrality alone does not specify its direction.

For isotropic *linear induced* response, building the moment costs internal energy. Minimizing the full energy gives

$$U(p) = \frac{p^2}{2\alpha} - pE, \quad \frac{\partial U}{\partial p} = \frac{p}{\alpha} - E = 0 \quad \Rightarrow \quad p = \alpha E.$$

$$U_{\text{eff}} = -\frac{1}{2}\alpha E^2, \quad \mathbf{F} = -\nabla U_{\text{eff}} = \frac{1}{2}\alpha \nabla(E^2).$$

Here $\nabla f = (\partial_x f, \partial_y f, \partial_z f)$. The half is polarization cost. Assume quasistatic linear response, constant α , a small particle and negligible source disturbance.

Chapter support: §2.15.2.

Advanced Example 2.29 — force near a charge



A small isotropic particle of polarizability $\alpha > 0$ is a distance r from a fixed point charge Q . Ignore the particle's influence on the source and use its leading induced-dipole response. Find its radial force.

The prescribed external charge gives

$$\mathbf{E} = \frac{k_e Q}{r^2} \hat{\mathbf{r}}, \quad E^2 = \frac{k_e^2 Q^2}{r^4}.$$

Using the induced energy,

$$\mathbf{F} = \frac{\alpha}{2} \frac{d}{dr} \left(\frac{k_e^2 Q^2}{r^4} \right) \hat{\mathbf{r}} = \boxed{-\frac{2\alpha k_e^2 Q^2}{r^5} \hat{\mathbf{r}}}.$$

The force is inward for either sign of Q . Its r^{-5} dependence combines the induced moment (r^{-2}) with the gradient (r^{-3}). Energy check: $U_{\text{eff}} = -\alpha k_e^2 Q^2 / (2r^4)$, so $-U'_{\text{eff}} = -2\alpha k_e^2 Q^2 / r^5$, confirming sign and factor. Require object size $\ll r$, linear polarization and negligible source disturbance.

Chapter support: Example 2.29, §2.15.2.

Advanced: prescribed coaxial uniform disks



The radius is R : lower $+\sigma$ at $z = -g/2$, upper $-\sigma$ at $z = +g/2$, with $\sigma > 0$. Set $E_\infty = \sigma/\epsilon_0$.

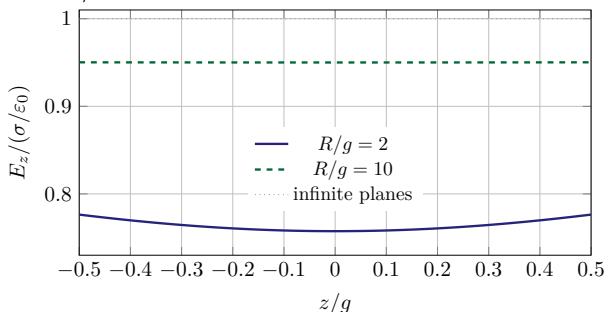


Figure 2.18: Exact on-axis field between opposite uniform disks. The endpoints show limits taken from inside the gap; ideal charged surfaces have discontinuous normal field. Increasing R/g reduces the finite-size deficit and flattens the profile.

Chapter support: §2.15.6; Figure 2.18.

Advanced Example 2.33 — two different errors



Use the model with $R = 0.10$ m and $g = 0.010$ m. Compare the centre field with the infinite-plane value and with the axial field immediately inside either disk.

For the gap, $d_{\pm} = g/2 \pm z > 0$; superpose the two disk fields:

$$\frac{E_z(z)}{E_{\infty}} = 1 - \frac{1}{2} \left[\frac{d_+}{\sqrt{R^2 + d_+^2}} + \frac{d_-}{\sqrt{R^2 + d_-^2}} \right].$$

Field point	(d_+, d_-) [m]	E_z/E_{∞}
Centre	(0.005, 0.005)	0.95006238
Upper inside-limit	$(0.010, 0^+)$	0.95024814

$$(1 - 0.95006238) \times 100\% = 4.994\% \quad (E_{\infty} \text{ reference}),$$

$$(0.95024814/0.95006238 - 1) \times 100\% = 0.01955\% \quad (E_z(0) \text{ reference}).$$

Both inner limits agree by symmetry. Flatness and magnitude accuracy differ; prescribed uniform sheets exclude conductor charge redistribution.

Chapter support: Example 2.33, §2.15.6.

Advanced 2.34 — method and complete problem



Use $h = 2a/N$, midpoint x_j and source weight $\Delta q_j = \lambda(x_j)h$. For the uniform rod, keep $P = (0, a)$ and total charge $2a\lambda$ fixed:

$$\mathbf{E}_N = k_e \sum_j \Delta q_j \frac{\mathbf{r} - x_j \hat{\mathbf{i}}}{|\mathbf{r} - x_j \hat{\mathbf{i}}|^3}.$$

Example 2.34 — complete statement

For a rod extending from $-a$ to a , evaluate the bisector field at $y = a$ with midpoint quadrature. Normalize by the exact field $E_{\text{exact}} = \sqrt{2} k_e \lambda / a$. The dimensionless sum is

$$\frac{E_N}{k_e \lambda / a} = \frac{2}{N} \sum_{j=1}^N \frac{1}{[1 + u_j^2]^{3/2}}, \quad u_j = -1 + \left(j - \frac{1}{2}\right) \frac{2}{N}. \quad (1)$$

The normalized coordinates are $u_j = x_j/a$. Apply the midpoint sum to this smooth integrand; the next slide calculates and checks the convergence.

Chapter support: Example 2.34, §2.15.7; exact rod integral from §2.9.2.

Advanced Example 2.34 — recomputed convergence



For the fixed uniform rod and $P = (0, a)$, compute the displayed midpoint sum and define $\delta_N = (E_N - E_{\text{exact}})/E_{\text{exact}}$.

The successive error ratios are

$$\frac{\delta_N}{\delta_{2N}} = 3.94, 3.99, 4.00, 4.00.$$

N	E_N/E_{exact}	Error (%)
4	1.00768040	0.76804
8	1.00194841	0.19484
16	1.00048800	0.04880
32	1.00012205	0.01221
64	1.00003052	0.00305

Thus the data approach second-order error $O(h^2)$.

Near a source, require $h \ll y$ to resolve the integrand before relying on that regime. No geometry or charge changes between rows.

Exact $E_x = 0$ tests symmetry, not magnitude accuracy. Masking excludes singular points; softening changes the Coulomb kernel and therefore the model.

Chapter support: Example 2.34, §2.15.7.

Advanced challenge 2.37 — infer the two charges



Unknown charges q_L and q_R lie at $x = -a$ and $x = +a$. At $P = (0, y)$ with $a, y > 0$, suppose both field components are measured. Determine the two charges.

Numerical continuation from the chapter: $a = 0.050 \text{ m}$, $y = 0.10 \text{ m}$, $E_x = 965 \text{ N/C}$, $E_y = 643 \text{ N/C}$; $k_e = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$.

Known geometry gives source-to-point vectors

$$\mathbf{R}_L = a\hat{\mathbf{i}} + y\hat{\mathbf{j}}, \quad \mathbf{R}_R = -a\hat{\mathbf{i}} + y\hat{\mathbf{j}}, \quad R = \sqrt{a^2 + y^2}.$$

Thus superposition gives

$$E_x = \frac{k_e a}{R^3}(q_L - q_R), \quad E_y = \frac{k_e y}{R^3}(q_L + q_R).$$

The horizontal component measures the *difference*; the vertical measures the *sum*. The next slide solves both.

Chapter support: Example 2.37, §2.15.10.

Example 2.37 — inversion and identifiability



Add and subtract the measured combinations:

$$q_L = \frac{R^3}{2k_e} \left(\frac{E_y}{y} + \frac{E_x}{a} \right),$$
$$q_R = \frac{R^3}{2k_e} \left(\frac{E_y}{y} - \frac{E_x}{a} \right).$$

Use $R = 0.1118034$ m and SI data:

$$q_L = \frac{(0.1118034)^3}{2(8.99 \times 10^9)} (6430 + 19300) \simeq +2.00 \text{ nC},$$
$$q_R = \frac{(0.1118034)^3}{2(8.99 \times 10^9)} (6430 - 19300) \simeq -1.00 \text{ nC}.$$

Check: rounded charges give $(E_x, E_y) = (964.9, 643.3) \text{ N/C}$, consistent with the data.

E_y alone gives only the sum. At $y = 0$ inversion fails. Far away, sum and difference scale as y^{-2} and y^{-3} : source moments.

Chapter support: Example 2.37, §2.15.10.

