

Electric Current, Resistance, and Power

Physics II - Chapter 6

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Charge transport and energy transfer



Electrostatics describes charge distributions and potential differences. Operating devices also require a rate of charge transport and an account of the energy transferred.

The chapter's physical questions

How much signed charge crosses a surface? Which field maintains its drift?
How do material and geometry set the current? Where does electrical energy go?

Current density makes transport local. Carrier drift explains its microscopic origin. A constitutive law then connects fields to terminal resistance, and power connects transport to heating or useful output.

Chapter 6: §6.1; §6.2

Learning route and prerequisites



- ① Count signed charge, then integrate current over time.
- ② Resolve the normal component of current density and connect it to carrier drift.
- ③ Combine material response with contact geometry to derive resistance.
- ④ Account for terminal power, temperature changes and measurement errors.

Prerequisite tools

Use charge signs, directed area, dot products, elementary integration and $\mathbf{E} = -\nabla V$. Keep SI conversions explicit, especially $1 \text{ mm}^2 = 10^{-6} \text{ m}^2$.

The advanced block adds divergence, a one-dimensional resistance integral, thermal linearization and thermistor sensitivity.

Chapter 6: §6.1

Learning goals



- Compute signed transported charge and current-density flux, with the correct surface orientation.
- Derive the drift-current relation and distinguish drift from signal propagation.
- Derive $R = \rho L/A$ under explicit material and contact assumptions.
- Distinguish linear resistance, operating-point resistance and differential response.
- Calculate absorbed power and explain heating, conversion, cable loss and sensor bias.
- Interpret how temperature and the sensing arrangement affect a measured resistance.

Advanced goals: integrate nonuniform resistance, test thermal stability and infer temperature with a calibrated thermistor model.

Chapter 6: §6.1

Steady transport in a resistive conductor



In electrostatic equilibrium a conductor has no current and zero internal electric field. A resistive wire carrying steady current has a nonzero driving field.

$$\frac{dQ_{\text{segment}}}{dt} = I_{\text{in}} - I_{\text{out}} = 0.$$

The equal currents express charge conservation. They can remain equal even when the carriers scatter and the wire receives electrical energy.

Steady does not mean static

Macroscopic quantities are time independent at a fixed location. Individual carriers continue their microscopic motion with a small average drift.

A nearly neutral wire can pass a large cumulative charge. Transported charge and charge stored in the wire are different quantities.

Chapter 6: §6.2

Signed current and transported charge



Choose a counting surface and a positive normal. Let $q(t)$ be the cumulative signed charge transported through it.

$$I = \frac{dq}{dt}, \quad \Delta q = \int_{t_1}^{t_2} I(t) dt.$$

Current has units $A = C s^{-1}$. The signed area under an I - t graph is net transported charge.

Constant current is a special case

If I stays constant, $\Delta q = I \Delta t$. Otherwise use the integral or a justified average current, $I_{av} = \Delta q / \Delta t$.

Negative carriers crossing against the positive normal contribute positive current. Counting particles per second requires multiplication by their signed charge and crossing direction.

Chapter 6: §6.3

Conventional current and electron drift

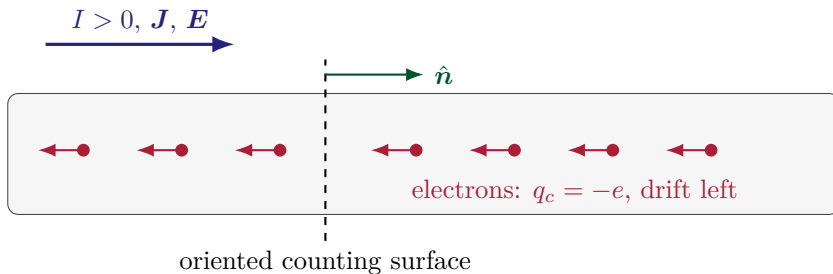


Figure 6.1: Sign conventions in a resistive metal. The reference normal, conventional current, current density and driving field point right. Electrons drift left, so $n(-e)\mathbf{v}_d$ points right. The dots are a schematic carrier representation.

For electrons drifting left, both their charge and axial velocity are negative. Their product gives rightward conventional current.

Chapter 6: Figure 6.1, manuscript PDF p. 8

Example 6.1: a decreasing sensor current



A sensor draws a current

$$I(t) = I_0 \left(1 - \frac{t}{T}\right) \quad (1)$$

for $0 \leq t \leq T$, where $I_0 = 40 \text{ mA}$ and $T = 5.0 \text{ s}$. Find the total charge transported.

Solution. The current is time dependent, so

$$\Delta q = I_0 \left[t - \frac{t^2}{2T} \right]_0^T = \frac{I_0 T}{2} = \frac{(0.040 \text{ A})(5.0 \text{ s})}{2} = \boxed{0.10 \text{ C}}.$$

Check. The graph is a triangle of base T and height I_0 . Its area equals the result. The average current is 20 mA , and $I_{\text{av}} T = 0.10 \text{ C}$.

Chapter 6: Example 6.1

Practice: Problem 11, a current ramp



A current is $I(t) = (0.60 \text{ A s}^{-1})t$ for $0 \leq t \leq 4.0 \text{ s}$. Find transported charge, average current and the equivalent net number of elementary charges. See Solution S2.

Inherited chapter convention

Use $e = 1.602 \times 10^{-19} \text{ C}$. The current is positive with respect to the chosen surface normal.

Try the integral first. Then use its result to obtain the average current and the equivalent net count. The complete solution follows on slide 11.

Chapter 6: Problem 11

Problem 11: complete ramp solution



For the stated ramp, integrate from 0 to 4.0 s:

$$\Delta q = \int_0^{4.0 \text{ s}} (0.60 \text{ A s}^{-1}) t \, dt = \frac{1}{2} (0.60) (4.0)^2 \text{ A s} = \boxed{4.8 \text{ C}}.$$

$$I_{\text{av}} = \frac{4.8 \text{ C}}{4.0 \text{ s}} = \boxed{1.2 \text{ A}}, \quad N_{\text{net}} = \frac{4.8}{1.602 \times 10^{-19}} = \boxed{3.0 \times 10^{19}}.$$

Check. The final current is 2.4 A. A triangular graph has mean height 1.2 A, consistent with the integral. The count is an equivalent net excess of elementary-charge crossings. It does not count every random microscopic crossing in both directions.

Chapter 6: Problem 11

Current density as an oriented flux



The current density $\mathbf{J}$ specifies the local conventional charge flow. Its units are A m^{-2} .

$$I = \int_S \mathbf{J} \cdot d\mathbf{A}, \quad d\mathbf{A} = \hat{\mathbf{n}} dA.$$

For a uniform field and a flat patch of area A ,

$$I = JA \cos \theta,$$

where θ is the angle between $\mathbf{J}$ and the positive normal. Only the normal component crosses the surface.

A local description retains geometry

For uniform normal flow, $I = JA$. For tangential flow, $I = 0$. For a nonuniform profile, integrate the actual area element.

Chapter 6: §6.4

Uniform current through a wire cross-section

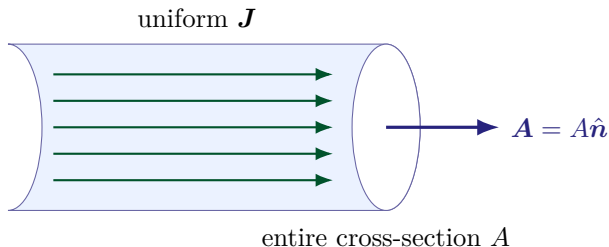


Figure 6.3: For uniform flow normal to a cross-section, $I = JA$. The arrow represents the area vector of the entire end face. A local patch would instead have the differential vector $d\mathbf{A}$.

The total current is JA for this uniform model. The large area arrow describes the whole face; an integration patch carries $d\mathbf{A}$.

Chapter 6: Figure 6.3, manuscript PDF p. 10

Example 6.5: a nonuniform strip



A conducting strip has width w and thickness b . Treat the following current-density profile as prescribed; for example, conductivity may vary across the width in a uniform axial field. Across its width coordinate $x \in [0, w]$, the current density is

$$J(x) = J_0 \left(1 + \frac{x}{w} \right) \quad (2)$$

and is everywhere perpendicular to the cross-section. Find the total current.

Model and integration element

The profile varies with width coordinate x . A thin strip has area $dA = b dx$, and its normal follows the prescribed current direction.

Integrate over $0 \leq x \leq w$. The solution follows on slide 15.

Chapter 6: Example 6.5

Example 6.5: flux integral and average density



Because flow is normal to the cross-section, the dot product contributes the scalar $J(x)$:

$$\begin{aligned} I &= bJ_0 \int_0^w \left(1 + \frac{x}{w}\right) dx \\ &= bJ_0 \left[x + \frac{x^2}{2w} \right]_0^w = \boxed{\frac{3}{2} J_0 bw}. \end{aligned}$$

Independent check. A linear profile rises from J_0 to $2J_0$. Its width average is $3J_0/2$, so $I = J_{\text{av}}(bw)$ gives the same result. Units are $(\text{A m}^{-2})(\text{m}^2) = \text{A}$.

For a reversed surface normal, the signed result would reverse.

Chapter 6: Example 6.5

A complete tilted cut through a wire

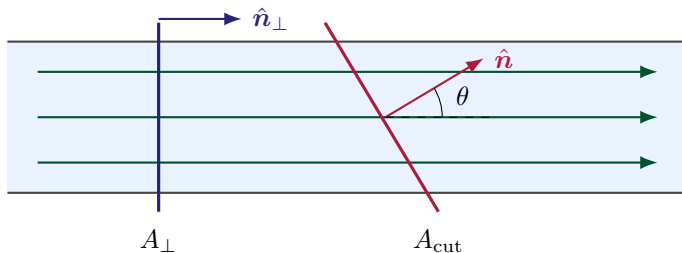


Figure 6.4: Side view of two complete cuts through a uniform wire. The tilted surface has a larger area. Its normal makes angle θ with the current density, so both surfaces carry the same flux I .

$$A_{\text{cut}} \cos \theta = A_{\perp}, \quad I = JA_{\text{cut}} \cos \theta = JA_{\perp}.$$

Tilting a complete cut enlarges its area. Tilting a fixed-area patch keeps its area fixed and changes its flux.

Chapter 6: Figure 6.4, manuscript PDF p. 12

Problem 2: the sixty-degree cut



A perpendicular cut through a wire has area $A_{\perp}$ and carries current I . A second complete cut has its normal at 60° to the wire axis. Find its area and signed current. Contrast this with tilting a fixed-area patch in an infinite uniform current density. See Solution S1.

Solution. The normal has a positive axial component:

$$A_{\text{cut}} = \frac{A_{\perp}}{\cos 60^{\circ}} = \boxed{2A_{\perp}}, \quad I_{\text{cut}} = J(2A_{\perp})^{\frac{1}{2}} = \boxed{I}.$$

For a fixed-area patch initially normal to $\mathbf{J}$, the same tilt gives $I' = JA \cos 60^{\circ} = I_{\text{patch}}/2$. **Check.** Reversing the complete-cut normal gives $-I$. The area compensation applies to complete cuts, while the patch comparison holds area fixed.

Chapter 6: Problem 2

Annular weighting for an axial current profile

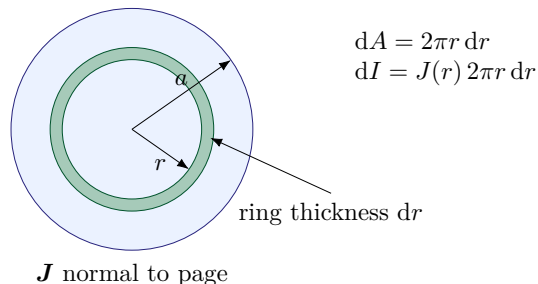


Figure 6.5: Annular integration element for an axially directed, radially varying current density. The finite-width colored ring illustrates an infinitesimal element; no radial current is implied.

$$I = 2\pi \int_0^a J(r) r dr.$$

The factor r weights the area of each ring. This is axial flow through a disk, rather than radial flow through a cylinder's side.

Chapter 6: Figure 6.5, manuscript PDF p. 13

Example 6.6: a prescribed circular profile



Suppose $J(r) = J_0(1 - r^2/a^2)$ along the axis of a circular sample. Find its total current and the fraction carried within radius $a/2$.

Stated lecture model

Let $J_0 > 0$ and choose the normal along the axis. Treat the profile as prescribed, for example by radially varying conductivity in a uniform axial field. A homogeneous, isothermal DC metal with full-face contacts would instead have uniform J .

Use $dA = 2\pi r dr$ for both the whole disk and the inner disk. The solution is on slide 20.

Chapter 6: Example 6.6

Example 6.6: the inner-disk current fraction



$$I = 2\pi J_0 \left[\frac{r^2}{2} - \frac{r^4}{4a^2} \right]_0^a = \boxed{\frac{\pi J_0 a^2}{2}},$$

$$I_{\text{inner}} = 2\pi J_0 \left[\frac{r^2}{2} - \frac{r^4}{4a^2} \right]_0^{a/2} = \frac{7\pi J_0 a^2}{32}.$$

$$\boxed{\frac{I_{\text{inner}}}{I} = \frac{7}{16} = 0.4375}, \quad J_{\text{av}} = \frac{I}{\pi a^2} = \frac{J_0}{2}.$$

Check and interpretation. The inner disk has area fraction $1/4$. Its larger current fraction is consistent with a profile concentrated near the centre. Differentiating the primitive recovers $J(r)r$.

Chapter 6: Example 6.6

Carrier drift and the current-density vector



For number density n , carrier charge q_c and drift speed v_d , the effective volume crossing area A in Δt is $Av_d\Delta t$.

$$|\Delta q| = n|q_c|Av_d\Delta t, \quad |I| = n|q_c|Av_d.$$

The signed vector relation is

$$\mathbf{J} = nq_c\mathbf{v}_d.$$

For electrons, $q_c = -e$ and $\mathbf{J}$ opposes the electron drift. Random microscopic crossings cancel in the net average.

Drift and signal propagation

Drift is a small directional bias of the carriers. A change in the electromagnetic field establishes a circuit response much faster than a carrier drifts around the circuit.

Chapter 6: §6.5

Example 6.7: electron drift speed



A copper wire carries $I = 5.0$ A. Its cross-sectional area is $A = 2.5 \text{ mm}^2$, and the density of conduction electrons is approximately $n = 8.5 \times 10^{28} \text{ m}^{-3}$. Estimate the electron drift speed.

Solution. Use $e = 1.602 \times 10^{-19} \text{ C}$ and $A = 2.5 \times 10^{-6} \text{ m}^2$:

$$\begin{aligned} v_d &= \frac{I}{neA} = \frac{5.0}{(8.5 \times 10^{28})(1.602 \times 10^{-19})(2.5 \times 10^{-6})} \\ &= \boxed{1.5 \times 10^{-4} \text{ m s}^{-1}}. \end{aligned}$$

This is approximately 0.15 mm s^{-1} , opposite to conventional current. **Check.**

Substitution into $neAv_d$ recovers 5.0 A before rounding. Doubling the area at fixed current would halve the drift speed.

Chapter 6: Example 6.7

Carrier density and mixed charge transport



An elementary metal model assumes z conducting electrons per atom:

$$n = z \frac{\rho_{\text{mass}}}{M} N_A.$$

Here ρ_{mass} is mass density, M is molar mass and N_A is Avogadro's constant. Electrical resistivity ρ is a different quantity.

$$\mathbf{J} = \sum_s n_s q_s \mathbf{v}_{d,s}.$$

Charge neutrality constrains $\sum_s n_s q_s$. Current instead depends on the charge-weighted velocities.

Answered reasoning check

Equal positive and negative charge densities moving with the same velocity give zero net current. Opposite charges drifting in opposite directions can reinforce the current while remaining nearly neutral.

Resistance, resistivity and conductance



For terminals a, b , choose current entering a and define the voltage drop $\Delta V = V_a - V_b$.

$$R_{\text{static}} = \frac{\Delta V}{I} \quad (I \neq 0), \quad G = \frac{1}{R}, \quad \sigma = \frac{1}{\rho}.$$

Resistance has units $\Omega = \text{V A}^{-1}$, conductance has units $\text{S} = \Omega^{-1}$, resistivity has units $\Omega \text{ m}$, and conductivity has units S m^{-1} .

Geometry must be specified

R is a two-terminal sample property. ρ is a material response under stated conditions. A uniform ohmic bar with suitable contacts has $R = \rho L/A$; the next slides derive why.

At $I = 0$ the ratio is undefined. A linear resistance can instead be determined from the limiting slope.

Chapter 6: §6.6

The uniform-bar field solution



Take a homogeneous isotropic bar of length L and area A , with full equipotential end contacts and insulated sides. Let $V_a > V_b$.

$$V(x) = V_a - \frac{V_a - V_b}{L}x, \quad E_x = -\frac{dV}{dx} = \frac{\Delta V}{L}.$$

The local ohmic model $\mathbf{J} = \sigma \mathbf{E}$ gives a uniform axial flow and zero sidewall current:

$$I = JA = \sigma \frac{\Delta V}{L} A, \quad \boxed{R = \frac{\Delta V}{I} = \frac{L}{\sigma A} = \rho \frac{L}{A}}.$$

The potential meets both end values. Constant J_x also gives zero net charge accumulation. The result uses material response and boundary conditions together.

Chapter 6: §6.7

Potential slope, field and current in the bar

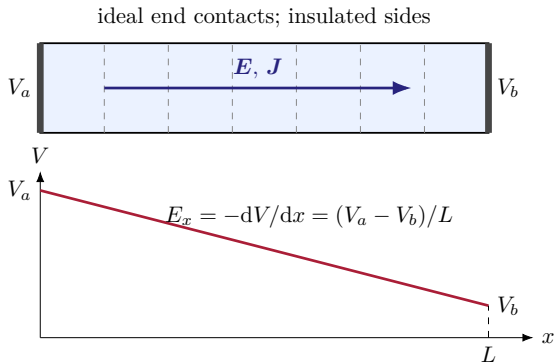


Figure 6.8: The uniform-bar solution. Dashed vertical lines in the bar represent equally spaced equipotentials. The potential decreases linearly, while $\mathbf{E}$ and $\mathbf{J}$ point right. The lower graph allows an arbitrary potential reference; only the terminal difference matters.

Chapter 6: Figure 6.8, manuscript PDF p. 21

Geometry scaling and contact placement



For fixed material state and one-dimensional uniform transport,

$$R \propto L, \quad R \propto A^{-1}.$$

Doubling length doubles resistance. Doubling area halves it. Scaling every length by a factor s changes L by s and A by s^2 , so $R' = R/s$.

Contacts define the path

Resistance is measured between specified terminals. Full opposite-face contacts can give nearly parallel current paths. Small contacts can cause spreading and current crowding, so an arbitrary L/A need not represent the measurement.

For a fixed-thickness film, $R = (\rho/t)(L/w)$. The dimensionless ratio L/w counts squares; bulk resistivity need not remain valid in an extremely thin film.

Chapter 6: §6.6

Bus-bar geometry and full-face contacts

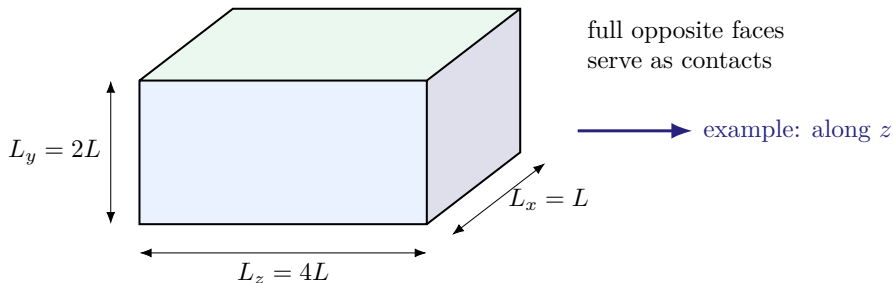


Figure 6.6: A rectangular bus bar with dimensions L , $2L$ and $4L$, drawn schematically. Each resistance calculation uses the dimension along the current as path length and the product of the other two as cross-section.

Use the dimension along the current for the path length. Multiply the other two dimensions to obtain the contacted cross-section.

Chapter 6: Figure 6.6, manuscript PDF p. 17

Example 6.10: resistance along three axes



A rectangular copper block has dimensions $L_x = L$, $L_y = 2L$ and $L_z = 4L$. The resistivity is ρ . Large conducting plates are attached to two opposite faces so that current flows uniformly along one coordinate direction at a time. Find the resistance for current along x , y and z .

Solution. Full-face contacts justify $R_i = \rho L_i / A_i$:

direction	L_i	A_i	R_i
x	L	$8L^2$	$\rho/(8L)$
y	$2L$	$4L^2$	$\rho/(2L)$
z	$4L$	$2L^2$	$2\rho/L$

Check. The ratio $R_x : R_y : R_z = 1 : 4 : 16$ follows from changing both length and area. Every result has units $(\Omega \text{ m})/\text{m} = \Omega$.

Chapter 6: Example 6.10

Ohm's law and the operating regime



An isotropic ohmic material obeys $\mathbf{J} = \sigma \mathbf{E}$ in a specified linear regime. A corresponding linear two-terminal resistor obeys $\Delta V = IR$ with constant R .

Constitutive response

Ohm's law describes material response. Charge conservation constrains transport, while energy conservation constrains transfers. They remain separate physical ingredients.

At fixed geometry and temperature, doubling voltage in the uniform-bar model doubles E , J , v_d and I . If the device heats, its state can change during a sweep. A metal can be locally ohmic at each fixed temperature while its self-heated terminal curve is nonlinear. Curvature alone does not identify its cause.

Chapter 6: §6.7

Ohmic and nonlinear terminal curves

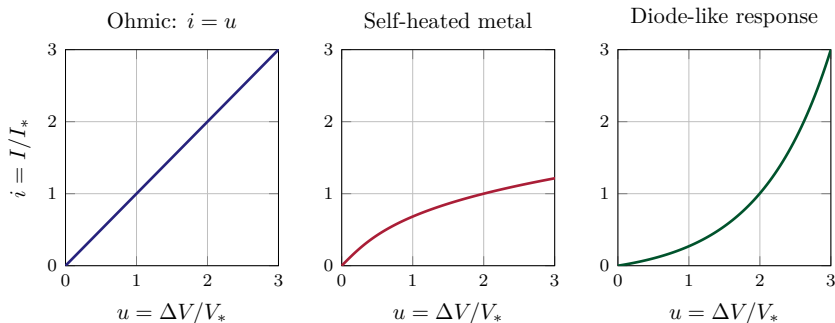


Figure 6.7: Dimensionless illustrative models, not measured device data. The middle curve is parametrized by $u = i(1 + i^2)$, a phenomenological low-heating approximation with increasing static resistance. Its I - V slope decreases as the sample heats. The right curve is $i = 3(\exp u - 1)/(\exp 3 - 1)$ and illustrates increasing forward conductance; it is not a calibration of a particular diode. Each panel may use different positive scales I_* , V_* .

Chapter 6: Figure 6.7, manuscript PDF p. 19

Scattering, mobility and the Drude estimate



The mean drift obeys a drive-and-relaxation model:

$$m \frac{d\mathbf{v}_d}{dt} = q_c \mathbf{E} - \frac{m\mathbf{v}_d}{\tau}.$$

Here m is carrier mass and τ is the effective momentum-relaxation time. In the settled state,

$$\mathbf{v}_d = \frac{q_c \tau}{m} \mathbf{E}, \quad \mathbf{J} = \frac{n q_c^2 \tau}{m} \mathbf{E}, \quad \boxed{\rho = \frac{m}{n e^2 \tau}} \quad \text{for electrons.}$$

Mobility is $\mu = |\mathbf{v}_d|/|\mathbf{E}|$ with units $\text{m}^2\text{V}^{-1}\text{s}^{-1}$; a one-carrier model gives $\sigma = n|q_c|\mu$. The model explains useful trends, but band structure and quantum scattering are needed for a full theory of metals. Steady drift balances ongoing force and relaxation.

Chapter 6: §6.8

Example 6.15: infer a relaxation time



Use the Drude estimate to find the effective scattering time for copper with $\rho = 1.68 \times 10^{-8} \Omega \text{ m}$ and $n = 8.5 \times 10^{28} \text{ m}^{-3}$. Use $m = 9.11 \times 10^{-31} \text{ kg}$ and $e = 1.602 \times 10^{-19} \text{ C}$.

Solution. Rearranging the material model gives

$$\tau = \frac{m}{ne^2\rho} = \frac{9.11 \times 10^{-31}}{(8.5 \times 10^{28})(1.602 \times 10^{-19})^2(1.68 \times 10^{-8})} = \boxed{2.5 \times 10^{-14} \text{ s}}.$$

Check. Insert the unrounded time into $ne^2\tau/m$: it recovers $\sigma = 1/\rho \simeq 5.95 \times 10^7 \text{ S m}^{-1}$. This is an effective momentum-relaxation estimate. It explains the small mean drift under a continuing electric force; it is not a complete microscopic collision model.

Chapter 6: Example 6.15

Terminal power and the passive sign convention



Choose I entering terminal a and leaving terminal b , with $\Delta V = V_a - V_b$. The signed transported charge is $dq = I dt$.

$$dW = (V_a - V_b)dq, \quad \boxed{P_{\text{absorbed}} = I\Delta V}.$$

Power has units $\text{A V} = \text{W} = \text{J s}^{-1}$. Positive P means absorption; negative P means electrical delivery.

Energy destination depends on the element

A resistor converts absorbed electrical energy into internal energy. A motor can also transfer useful mechanical energy, and a charging cell can store energy.

The voltage drop $V_a - V_b$ differs in sign from the potential change along the path, $V_b - V_a$.

Chapter 6: §6.9

Charge passes through; energy enters the resistor

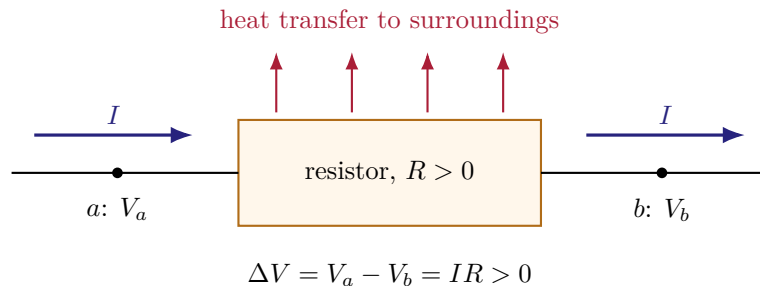


Figure 6.9: Passive sign convention for a resistor. Equal currents enter and leave, while absorbed electrical power is $I(V_a - V_b)$. At steady temperature the heat loss equals that power; during warming some energy increases the resistor's internal energy.

Charge transport continues through both terminals. At constant temperature, electrical input balances outward heat transfer; during warming, some input increases internal energy.

Chapter 6: Figure 6.9, manuscript PDF p. 24

Joule heating and source constraints



For a passive resistive state with $\Delta V = IR$,

$$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R} \geq 0, \quad W = \int_{t_1}^{t_2} P(t) dt.$$

Answered source-control comparison

Doubling R doubles power at fixed I , because $P = I^2 R$. At fixed voltage it halves power, because $P = V^2/R$. An unspecified circuit does not identify either constraint.

For a nonlinear resistive element, the operating-point ratio $R_{\text{static}} = V/I$ reproduces total VI . Its differential resistance generally does not. For constant R , average heating is $R\langle I^2 \rangle$, which can be positive even if $\langle I \rangle = 0$.

Chapter 6: §6.9

Example 6.3: the complete bipolar pulse

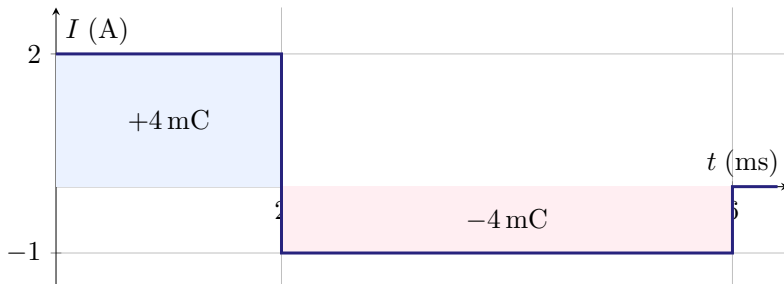


Figure 6.2: An idealized bipolar pulse: equal positive and negative signed areas imply zero net transported charge. Resistive heating depends on I^2 and adds during both intervals. Abrupt edges idealize finite switching times.

The pulse in Fig. 6.2 passes through a 5.0Ω resistor whose temperature remains approximately constant. Find net transported charge and dissipated energy.

Chapter 6: Figure 6.2, manuscript PDF p. 9; Example 6.3

Example 6.3: charge and energy are different integrals



The two durations are 0.0020 s and 0.0040 s:

$$\Delta q = (2.0)(0.0020) + (-1.0)(0.0040) = 4.0 \text{ mC} - 4.0 \text{ mC} = \boxed{0 \text{ C}}.$$

The fixed-temperature resistor has $R = 5.0 \Omega$:

$$W = R \int I^2 dt = 5.0[2.0^2(0.0020) + (-1.0)^2(0.0040)] = \boxed{0.060 \text{ J}}.$$

Check. The two powers are 20 W and 5.0 W, contributing 0.040 J and 0.020 J. Their sum confirms the integral. Current and voltage reverse together, so the resistor absorbs energy in both intervals. Zero net transport does not imply zero heating.

Chapter 6: Example 6.3

Voltage drop in a supply-and-return cable



A device at one-way distance L requires an outgoing and return conductor. With equal conductor areas A , the full resistive path is $2L$.

$$R_c = \frac{2\rho L}{A}, \quad \Delta V_c = IR_c, \quad P_c = I^2 R_c = I\Delta V_c.$$

A maximum pair voltage drop sets

$$R_c \leq \frac{\Delta V_{\max}}{I}, \quad A \geq \frac{2\rho LI}{\Delta V_{\max}}.$$

Voltage-drop model

The load specifies the current in this calculation. Temperature, contacts and heat removal are additional inputs for a thermal calculation; area from a voltage-drop bound alone is not a current rating.

Chapter 6: §6.13

Practice: Problem 23, a cable-area bound



A load 15 m from a source draws 10 A. Identical outgoing and return copper conductors must together drop at most 0.20 V. Find minimum area of each and pair loss at that area. Identify a missing input for a temperature-rise calculation. See Solution S7.

Inherited chapter assumptions

Use copper $\rho = 1.7 \times 10^{-8} \Omega \text{ m}$, fixed temperature and negligible contact resistance. Each conductor has the same unknown area.

Determine the pair's allowed resistance first. Then obtain area, loss and the information needed to estimate temperature. The solution is on slide 41.

Chapter 6: Problem 23

Problem 23: area, loss and thermal limits



The voltage-drop bound requires

$$R_{c,\max} = \frac{0.20 \text{ V}}{10 \text{ A}} = 0.020 \Omega.$$

The full path is $2L = 30 \text{ m}$, so each conductor needs

$$A_{\min} = \frac{\rho(2L)}{R_{c,\max}} = \frac{(1.7 \times 10^{-8})(30)}{0.020} = \boxed{2.55 \times 10^{-5} \text{ m}^2 = 25.5 \text{ mm}^2}.$$

At this bound, $P_c = I\Delta V_c = \boxed{2.0 \text{ W}}$. **Checks and missing input.** $I^2 R_c = 2.0 \text{ W}$ confirms the loss. A temperature rise also needs a heat-transfer model, including thermal coupling and the installation environment. Omitting the return conductor would halve the predicted area incorrectly.

Chapter 6: Problem 23

Electrical input and useful mechanical output



Choose the whole motor as the system. In steady operation its electrical input can supply mechanical output and several loss mechanisms:

$$P_{\text{in}} = VI = P_{\text{mech}} + P_{\text{loss}}, \quad \eta = \frac{P_{\text{mech}}}{P_{\text{in}}}.$$

If the winding resistance R_w is known, its Joule-heating rate is

$$P_w = I^2 R_w.$$

The terminal ratio has a limited meaning

The running motor's V/I is an operating-point ratio. Using it in $I^2(V/I)$ gives the entire electrical input IV , rather than isolating winding heat.

The remaining loss can involve friction, magnetic processes and other mechanisms. Terminal data alone do not separate them.

Chapter 6: §6.9

Problem 16: motor power accounting



A motor receives 24 V and 2.0 A and delivers 36 W of mechanical power. Find total loss and efficiency. If its measured winding resistance is $1.0\ \Omega$, separate winding heating from other loss. Why is $24/2\ \Omega$ not the winding resistance? See Solution S15.

System and state

Treat the entire motor as operating steadily. Use the separately measured winding resistance for its heating, then compare with the total loss.

Keep four outputs distinct: total loss, efficiency, winding heat and other loss. The complete calculation follows on slide 44.

Chapter 6: Problem 16

Problem 16: complete motor solution



$$P_{\text{in}} = (24)(2.0) = 48 \text{ W}, \quad P_{\text{loss}} = 48 - 36 = \boxed{12 \text{ W}}, \quad \eta = \frac{36}{48} = \boxed{75\%}.$$

The independently measured $R_w = 1.0 \Omega$ gives

$$P_w = (2.0)^2(1.0) = \boxed{4.0 \text{ W}}, \quad P_{\text{other}} = 12 - 4.0 = \boxed{8.0 \text{ W}}.$$

The terminal ratio is $V/I = 12 \Omega$. Substituting that ratio into $I^2 R$ gives 48 W, the full input, rather than the actual winding heat. **Independent**

check. $36 + 4.0 + 8.0 = 48 \text{ W}$ closes the energy balance. The given data do not divide the remaining loss among individual mechanisms.

Chapter 6: Problem 16

Temperature dependence as a local approximation



At reference temperature T_0 , define

$$\alpha_0 = \frac{1}{\rho_0} \left. \frac{d\rho}{dT} \right|_{T_0}, \quad \rho(T) \simeq \rho_0[1 + \alpha_0(T - T_0)].$$

For fixed geometry, the same fractional law applies to R . The coefficient has units K^{-1} .

State and validity

Many metals have a positive coefficient near room temperature, because increased lattice vibration increases momentum relaxation. Other materials can have different trends as carrier density and mobility change.

Temperature differences have the same numerical size in kelvin and Celsius. The linear law is local; extrapolating it to negative resistivity signals failure of the approximation.

Chapter 6: §6.10

Specified metal and thermistor temperature models

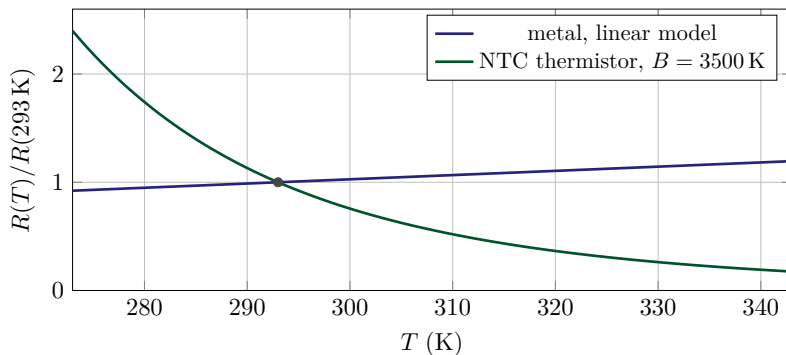


Figure 6.10: Calculated normalized resistance curves at fixed geometry. The metal model uses $\alpha = 3.9 \times 10^{-3} \text{ K}^{-1}$ and the illustrative thermistor model uses $R/R_0 = \exp[B(1/T - 1/T_0)]$ with $T_0 = 293 \text{ K}$. These specified models replace universal-looking material curves; semiconductor temperature trends depend on carrier density and mobility.

Chapter 6: Figure 6.10, manuscript PDF p. 26

Example 6.20: a copper temperature correction



A copper wire has resistance $R_0 = 10.0\ \Omega$ at $T_0 = 20^\circ\text{C}$. Estimate its resistance at $T = 70^\circ\text{C}$ using $\alpha = 3.9 \times 10^{-3}\ \text{K}^{-1}$.

Solution. Fixed geometry lets R scale with ρ . The change is $70 - 20 = 50\ \text{K}$:

$$R = 10.0[1 + (3.9 \times 10^{-3})(50)]\ \Omega = \boxed{11.95\ \Omega}.$$

$$\frac{R - R_0}{R_0} = \alpha \Delta T = 0.195 = \boxed{19.5\%}.$$

Check and interpretation. A positive coefficient and positive temperature change give an increased resistance. At fixed current the heating rises; at fixed voltage it falls. The value is conditional on the linear approximation and negligible expansion.

Chapter 6: Example 6.20

Operating-point ratio and differential resistance



At a nonzero operating point, the static ratio and local slope are

$$R_{\text{static}} = \frac{V_0}{I_0}, \quad r_{\text{diff}} = \left. \frac{dV}{dI} \right|_0.$$

For a small change about that state,

$$\delta V \simeq r_{\text{diff}} \delta I.$$

On an I versus V graph, the secant slope is $1/R_{\text{static}}$ and the tangent slope is $1/r_{\text{diff}}$.

Bias power uses the operating point

$P_0 = V_0 I_0 = I_0^2 R_{\text{static}}$. In general, $V_0 \neq r_{\text{diff}} I_0$, so the differential resistance cannot replace the finite-bias ratio in the power formula.

Specify the thermal state too: a fast perturbation and a slow self-heated sweep can probe different slopes.

Chapter 6: §6.21

Secant and tangent on a nonlinear curve

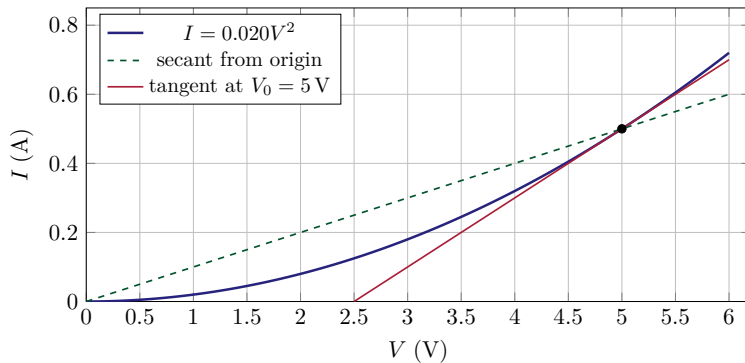


Figure 6.16: Calculated nonlinear response with its secant and tangent. On an I versus V graph the slopes are conductances; resistance is their inverse. The tangent's intercept need not be zero.

For this model at the plotted point, the secant gives 10Ω and the tangent gives 5.0Ω . The tangent's intercept need not vanish.

Chapter 6: Figure 6.16, manuscript PDF p. 45

Problem 22: a nonlinear cubic response



A device obeys $I = aV + bV^3$, where $a = 0.020 \text{ A V}^{-1}$ and $b = 0.0010 \text{ A V}^{-3}$. At $V = 2.0 \text{ V}$, find I , static resistance, differential resistance and total absorbed power. Which resistance belongs in I^2R ? See Solution S16.

Model

Use the stated terminal law at positive V . The coefficients have different units so that both terms contribute amperes. Compute dI/dV before taking its reciprocal.

The solution on slide 51 distinguishes bias power from incremental resistance.

Chapter 6: Problem 22

Problem 22: secant, tangent and total power



$$I = (0.020)(2.0) + (0.0010)(2.0)^3 = \boxed{0.048 \text{ A}}.$$

$$R_{\text{static}} = \frac{2.0}{0.048} = \boxed{41.7 \Omega}.$$

$$\frac{dI}{dV} = a + 3bV^2 = 0.032 \text{ S}, \quad r_{\text{diff}} = \boxed{31.25 \Omega}.$$

The absorbed power is $P = VI = \boxed{0.096 \text{ W}}$. **Which resistance?** The static ratio belongs in I^2R : it gives $(0.048)^2(2/0.048) = 0.096 \text{ W}$. Using the differential value would give 0.072 W , contradicting the terminal work rate. **Check.** Differentiating both terms gives their incremental conductances. The positive derivative and positive bias product are consistent with this passive operating point.

Chapter 6: Problem 22

Semiconductor and superconducting transport



In a simple semiconductor drift model, electrons and holes contribute

$$\mathbf{v}_{d,n} = -\mu_n \mathbf{E}, \quad \mathbf{v}_{d,p} = \mu_p \mathbf{E}, \quad \sigma = e(n\mu_n + p_h\mu_p).$$

A hole has an effective positive carrier response. Doping, temperature and illumination change carrier populations. Density gradients can add diffusion currents.

Superconducting DC states

Nondissipative transport has interdependent temperature, field and current limits. Setting $R = 0$ in $I = V/R$ does not determine persistent current or supply superconductivity's magnetic and quantum physics.

Fixed-temperature low-field linearity is distinct from temperature dependence or self-heated terminal nonlinearity.

Chapter 6: §6.11

Four-wire sensing defines the specimen voltage

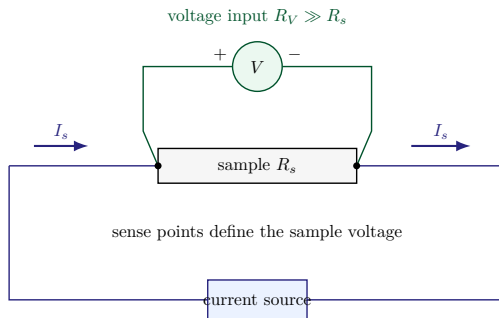


Figure 6.11: Four-wire measurement with a complete excitation loop and a voltmeter inserted between separate sense leads. There is no wire directly joining the sense nodes. Equality of source and sample currents is the approximation that voltmeter loading is negligible.

Two-wire sensing includes lead drops: $R_{\text{two}} \simeq R_s + 2R_\ell$. Kelvin sensing excludes drops outside the selected sense boundaries when voltmeter loading is negligible.

Chapter 6: Figure 6.11, manuscript PDF p. 30

Offsets and tests of a resistance model



A stable linear specimen with constant voltage offset b gives

$$V(+I) = RI + b, \quad V(-I) = -RI + b.$$

Adding and subtracting isolate the two quantities:

$$R = \frac{V(+I) - V(-I)}{2I}, \quad b = \frac{V(+I) + V(-I)}{2}.$$

Reversal cancels a stable offset. It does not remove self-heating, because I^2R remains unchanged. Across a current range, fit $V = RI + b$ and inspect residuals $r_i = V_i - (RI_i + b)$ against measurement uncertainty. Systematic curvature or sweep-rate dependence can reveal an inadequate model or a changing temperature.

Chapter 6: §6.12

Example 6.24: a Kelvin offset measurement



At currents $+0.500\text{ A}$ and -0.500 A , a Kelvin measurement gives $+10.35\text{ mV}$ and -9.65 mV . Find resistance and offset.

Solution. Convert the voltage difference to volts:

$$R = \frac{10.35 - (-9.65)}{2(0.500)}\text{ m}\Omega = \boxed{20.00\text{ m}\Omega},$$
$$b = \frac{10.35 - 9.65}{2}\text{ mV} = \boxed{0.350\text{ mV}}.$$

Independent check. With $R = 0.02000\text{ }\Omega$, $RI + b$ reproduces 10.35 mV at positive current and -9.65 mV at negative current. A positive-current ratio alone gives $20.70\text{ m}\Omega$. Repeat at another current magnitude to test heating, while keeping the offset and specimen state stable.

Chapter 6: Example 6.24

Sensor self-heating and measurement-current choice



A sensor's temperature follows an energy balance. In the small steady-heating approximation, thermal resistance Θ_{th} relates rise to power:

$$\theta \simeq \Theta_{\text{th}} I^2 R_0, \quad \frac{R - R_0}{R_0} \simeq \alpha \Theta_{\text{th}} I^2 R_0.$$

Here Θ_{th} has units K W^{-1} and describes the actual mounting and environment.

Two competing effects

For a current-independent voltage standard uncertainty u_V and approximately exact current, $u_R/R_0 \simeq u_V/(IR_0)$ decreases with I . A positive-coefficient heating bias grows as I^2 .

Noise is a standard uncertainty; heating is a systematic bias. Averaging can reduce some random noise but does not remove the heating shift.

Chapter 6: §6.9; §6.12

Example 6.26: a feasible current window



A sensor has $R_0 = 100\ \Omega$, $\alpha = 0.0040\ \text{K}^{-1}$ and $\Theta_{\text{th}} = 200\ \text{K W}^{-1}$. The readout has $u_V = 10\ \mu\text{V}$. Require the voltage-induced relative standard uncertainty to be at most 0.10% and the self-heating resistance bias to be at most 0.020%.

Model

Use small self-heating about the ambient state and approximately exact current. Convert the two percentages separately: $0.10\% = 0.0010$ and $0.020\% = 0.00020$.

Find the readout lower bound and the heating upper bound. The complete solution follows on slide 58.

Chapter 6: Example 6.26

Example 6.26: bounds and approximation check



The readout constraint gives

$$\frac{u_V}{IR_0} \leq 0.0010 \quad \Rightarrow \quad I \geq \frac{10 \times 10^{-6}}{(0.0010)(100)} = \boxed{0.100 \text{ mA}}.$$

The heating constraint gives

$$I \leq \sqrt{\frac{0.00020}{(0.0040)(200)(100)}} = \boxed{1.58 \text{ mA}}.$$

Thus $\boxed{0.100 \leq I/(\text{mA}) \leq 1.58}$. **Check.** At the upper bound, $\theta = \Theta_{\text{th}} I^2 R_0 = 0.050 \text{ K}$ and $\alpha\theta = 0.00020$, consistent with small heating. Stable offsets and current calibration are still required.

Chapter 6: Example 6.26

Core synthesis: transport, response and energy



$$\Delta q = \int I dt$$

signed transport,

$$I = \int \mathbf{J} \cdot d\mathbf{A}$$

local flux and geometry,

$$\mathbf{J} = \sum_s n_s q_s \mathbf{v}_{d,s}$$

carrier signs and drift,

$$\mathbf{J} = \sigma \mathbf{E}, \quad R = \rho L / A$$

linear response and boundaries,

$$P = I \Delta V, \quad W = \int P dt$$

energy transfer.

Conservation, material response and energy accounting answer different questions. A resistor passes charge through while receiving energy. State and sensing boundaries define the measured resistance. Check units, signs, limits and an independent balance. Keep nonlinear bias power separate from the tangent resistance.

Chapter 6: §6.15; §6.14

Reading and the bridge to DC circuits



Chapter 7 connection

DC circuits combine charge conservation at junctions, potential and energy accounting around loops, and device response laws. Source emf, terminal voltage, source resistance, meter placement and RC transients can then be treated without confusing current with energy.

For further Chapter 6 study, use the complete problem bank and selected detailed solutions. The omitted numerical data-fitting, radial-leakage and distributed-temperature sections provide useful extensions. The manuscript recommends Halliday–Resnick–Walker, Serway–Jewett and OpenStax. These are reading pointers from its bibliography; the current chapter is the source used for this lecture. The final ten slides extend conservation and local response to nonuniform resistance, electrothermal stability and calibrated thermistor readout.

Chapter 6: §6.26

Advanced: local charge conservation



Let ρ_q be charge density, distinct from resistivity. For a fixed volume with outward surface normal,

$$\frac{d}{dt} \int_V \rho_q dV = - \oint_{\partial V} \mathbf{J} \cdot d\mathbf{A}.$$

Divergence measures local net outflow. The divergence theorem and arbitrary volume give

$$\boxed{\partial_t \rho_q + \nabla \cdot \mathbf{J} = 0}, \quad \nabla \cdot \mathbf{J} = 0 \text{ in steady state.}$$

Steady conservation does not require spatially uniform $\mathbf{J}$ or zero charge density. Electric work on carriers gives local power density $p = \mathbf{J} \cdot \mathbf{E}$. For isotropic ohmic dissipation, $p = \rho J^2 \geq 0$, in W m^{-3} .

Chapter 6: §6.17; §6.19

Advanced: resistance of a nonuniform conductor



Steady current is common to all axial slices. With approximately equipotential transverse slices and resistivity $\rho(x)$,

$$J(x) = \frac{I}{A(x)}, \quad dV_{\text{drop}} = \frac{\rho(x)I}{A(x)}dx, \quad \boxed{R = \int_0^L \frac{\rho(x)}{A(x)}dx}.$$

The positive drop is opposite in sign to the potential differential along positive current. A slow taper helps justify the one-dimensional model. **Example**

6.32: complete statement. A conductor of length L has resistivity ρ and cross-sectional area increasing linearly from A_0 at $x = 0$ to $2A_0$ at $x = L$:

$$A(x) = A_0 \left(1 + \frac{x}{L}\right). \quad (3)$$

Find its resistance in a one-dimensional approximation.

Chapter 6: Example 6.32

Example 6.32: integrate the taper and test bounds

Set $u = x/L$ and use the prescribed area:

$$R = \frac{\rho L}{A_0} \int_0^1 \frac{du}{1+u} = \boxed{\frac{\rho L}{A_0} \ln 2}.$$

Bounds. Since $A_0 \leq A(x) \leq 2A_0$,

$$\frac{\rho L}{2A_0} < R < \frac{\rho L}{A_0}; \quad \frac{1}{2} < \ln 2 < 1.$$

Added lecture check. The mean area is $3A_0/2$. Substitution of that mean gives $R_{\text{mean}} = 2\rho L/(3A_0)$, which underestimates the true model resistance by

$$\frac{R - R_{\text{mean}}}{R} = 1 - \frac{2}{3 \ln 2} = \boxed{3.82\%}.$$

The denominator is the exact one-dimensional result. Narrow regions receive stronger reciprocal-area weighting.

Chapter 6: Example 6.32

Advanced: electrothermal equilibrium and stability



Use uniform temperature rise $\theta = T - T_0$, heat capacity $C_{th} > 0$ and whole-device thermal conductance $H > 0$:

$$R(\theta) = R_0(1 + \alpha\theta), \quad C_{th}\dot{\theta} = P_{el}(\theta) - H\theta.$$

The resistance and linear loss laws are stipulated approximations, with $1 + \alpha\theta > 0$.

$$P_{el}(\theta_*) = H\theta_*, \quad C_{th}\delta\dot{\theta} = [P'_{el}(\theta_*) - H]\delta\theta.$$

A first-order Taylor expansion around an equilibrium gives the perturbation equation. A valid root is locally stable when

$$\boxed{P'_{el}(\theta_*) < H}, \quad \tau_{eff} = \frac{C_{th}}{H - P'_{el}(\theta_*)}.$$

Existence of a root, its stability and the physical validity of the model require separate checks.

Chapter 6: §6.20

Advanced: current control and voltage control

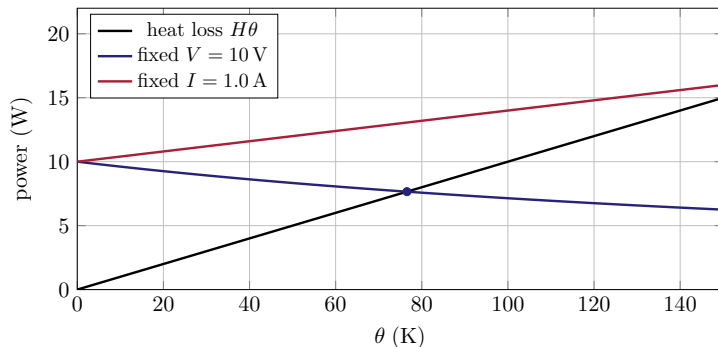


Figure 6.15: Calculated heat-generation and heat-loss curves for $R_0 = 10 \Omega$, $\alpha = 0.004 \text{ K}^{-1}$ and $H = 0.10 \text{ W K}^{-1}$. Both sources supply 10 W at ambient temperature. Voltage control equilibrates at 76.6 K rise; current control equilibrates at 166.7 K rise, just beyond the plotted interval. The linear resistance and cooling laws are stipulated models, not a high-temperature material fit.

Chapter 6: Figure 6.15, manuscript PDF p. 44

Example 6.35: current-driven thermal response



Use the model parameters in Fig. 6.15 with $C_{th} = 2.0 \text{ J K}^{-1}$.

Restored context. $R_0 = 10 \Omega$, $\alpha = 0.004 \text{ K}^{-1}$, $H = 0.10 \text{ W K}^{-1}$; compare fixed $I = 1.0 \text{ A}$ and fixed $V = 10 \text{ V}$ from slide 65. **Current-drive solution.**

Substituting $P = I^2 R_0(1 + \alpha\theta)$ into the energy balance gives

$$\theta_* = \frac{I^2 R_0}{H - \alpha I^2 R_0} = \boxed{166.7 \text{ K}},$$
$$\tau_{\text{eff}} = \frac{2.0}{0.10 - 0.040} = \boxed{33.3 \text{ s}}.$$

Here $\gamma = \alpha I^2 R_0 / H = 0.40 < 1$, so the root is stable. The model threshold is $I_{\text{crit}} = \sqrt{H/(\alpha R_0)} = \boxed{1.58 \text{ A}}$. The voltage-controlled solution and comparison follow on slide 67.

Chapter 6: Example 6.35

Example 6.35: voltage-driven root and comparison



At fixed $V = 10\text{ V}$, balance generation and loss:

$$\frac{V^2}{R_0(1 + \alpha\theta_*)} = H\theta_* \quad \Rightarrow \quad \theta_* = \frac{\sqrt{1 + 4\alpha V^2/(HR_0)} - 1}{2\alpha} = \boxed{76.6\text{ K}}.$$

The negative quadratic root is outside $1 + \alpha\theta > 0$. The physical root gives $R_* = 13.06\ \Omega$ and $P_* = 7.66\text{ W}$.

$$P'_{\text{el}} = -\frac{\alpha V^2}{R_0(1 + \alpha\theta_*)^2} = -0.02344\text{ W K}^{-1},$$

Thus $\tau_{\text{eff}} = 2.0/(0.10 + 0.02344) = \boxed{16.2\text{ s}}$. **Checks.** $P_* = H\theta_*$, and $P'_{\text{el}} < H$ ensures stability. Equal cold power 10 W gives different hot states. These large predicted rises need measured material and cooling data for engineering use.

Chapter 6: Example 6.35

Advanced: thermistor calibration and sensitivity



For an illustrative calibrated NTC model,

$$R(T) = R_0 \exp \left[B \left(\frac{1}{T} - \frac{1}{T_0} \right) \right], \quad T, T_0 > 0.$$

Absolute temperatures are in kelvin and $B > 0$ has units kelvin. Taking logarithms gives

$$\frac{1}{T} = \frac{1}{T_0} + \frac{1}{B} \ln \frac{R}{R_0}, \quad \boxed{\frac{dT}{dR} = -\frac{T^2}{BR}}.$$

With fixed calibration parameters and small resistance readout uncertainty,

$$\boxed{u_T \simeq \frac{T^2}{B} \frac{u_R}{R}}.$$

The logarithm requires a dimensionless ratio. Calibration errors, bath uncertainty and self-heating need their own contributions; this expression isolates the resistance readout term.

Chapter 6: §6.10; §6.21

Advanced challenge: Example 6.36 thermistor readout



Use an illustrative calibrated model with $R_0 = 10.0 \text{ k}\Omega$ at $T_0 = 298.15 \text{ K}$ and $B = 3500 \text{ K}$. A measured resistance is $6.80 \text{ k}\Omega$.

Explicit lecture challenge

Find the temperature in kelvin and Celsius. Then calculate the resistance-readout contribution to temperature standard uncertainty for $u_R/R = 0.005$, keeping B and R_0 fixed.

Use the model only within its calibrated temperature range, and assume self-heating is negligible for this inference. The complete solution is on slide 70.

Chapter 6: Example 6.36

Example 6.36: temperature and readout uncertainty

Use the dimensionless ratio 6.80/10.0:

$$T = \left[\frac{1}{298.15} + \frac{1}{3500} \ln \frac{6.80}{10.0} \right]^{-1} = \boxed{308.3 \text{ K}} \simeq \boxed{35.1^\circ\text{C}}.$$

For the specified small readout uncertainty,

$$u_T = \frac{T^2}{3500} (0.005) = \boxed{0.136 \text{ K}}.$$

Independent checks. The unrounded inferred temperature reproduces 6.80 k Ω in the forward model. Since $R < R_0$ and $B > 0$, $T > T_0$, as expected for an NTC sensor. The uncertainty is only the resistance-readout contribution. Calibration and self-heating can change the total. Model validity, conservation and independent checks remain essential even when a calculation is mathematically precise.

Chapter 6: Example 6.36

