

Direct-Current Circuits

Physics II - Chapter 7

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Why do we need a circuit model?



Electric fields explain how charges respond locally. A circuit model connects that physics to the voltages and currents we can predict at component terminals.

A battery maintains energy supply, resistors dissipate energy, and a capacitor stores it. Connecting them creates a network whose behaviour follows conservation laws.

Conceptual roadmap

Charge conservation determines how currents split. Energy per charge determines voltage balances. Element laws close the equations. Switching introduces a changing capacitor state and a relaxation time.

These tools predict battery sag, sensor loading and the shape of a measured charging curve.

Chapter support: §7.1 and Preface.

Learning route and prerequisites



- 1 **Model the connections.** Identify nodes, current references and voltage polarities.
- 2 **Solve steady networks.** Combine element laws with charge and energy conservation.
- 3 **Include the observer.** Treat each meter as an additional circuit element.
- 4 **Follow a state through switching.** Use capacitor continuity, a differential equation and limiting values.

Prerequisites: current as charge flow, potential as energy per charge, Ohm's law, capacitance and energy conservation. We will refresh these relations before using them.

Mathematics: linear equations, exponentials, derivatives and natural logarithms. Advanced pages add design inequalities and discrete numerical updates.

Chapter support: §7.1.1; core §§7.2–7.17; selected §7.21 enrichment.

Learning goals



By the end of the core lecture, you should be able to:

- distinguish emf from terminal voltage and infer a simple source model from load tests;
- reduce resistor networks and recover individual currents and powers;
- write independent Kirchhoff equations, interpret signed currents and check energy conservation;
- predict the disturbance caused by a finite-resistance meter;
- derive RC responses, determine initial and final states, and check timing and energy;
- infer a relaxation time from voltage data and state the measurement assumptions.

The advanced block extends these outcomes to terminal equivalents, quantitative design constraints and numerical fidelity.

Chapter support: Chapter learning goals, §7.1.

Circuit quantities and element laws



Choose a positive branch direction before writing a current. Voltage is a difference between two specified terminals.

$$\begin{aligned} I &= \frac{dq}{dt} & [I] &= \text{A} = \text{C/s}, \\ V_a - V_b &= Ri & [R] &= \Omega, \\ q &= CV_C & [C] &= \text{F} = \text{C/V}. \end{aligned}$$

For the resistor equation, i enters terminal a . For the capacitor, q is the charge on the plate defining positive V_C .

$$P_R = VI = I^2 R = \frac{V^2}{R}, \quad U_C = \frac{1}{2} CV_C^2.$$

Resistor power becomes heat. Capacitor energy resides in the electric field. These prior relations will also provide checks on circuit solutions.

Chapter support: Conventions and notation; §7.1.1; prerequisite energy refresh.

Connectivity defines the circuit



A **node** is an entire ideal conducting connection. All its points share one potential. A **branch** carries one reference current between nodes. A **loop** is a closed path.

An essential node joins at least three branches. A mesh is a loop enclosing no smaller loop in a planar network.

An answered topology check

Two distant points joined by an uninterrupted ideal wire are one node. Moving their positions on the page changes no circuit equation. Crossing lines connect only when the connection is specified.

In the lumped model, wires have negligible resistance and propagation delay. The diagram records connections and element laws, rather than component dimensions.

Chapter support: §7.2, §7.2.1 and §7.2.2.

Reference choices and model limits



Adding the same constant V_0 to every node potential changes no voltage difference and therefore no resistor current:

$$(V_a + V_0) - (V_b + V_0) = V_a - V_b.$$

Choosing ground sets a reference. Joining two separate nodes with a wire changes the physical circuit.

An **open** branch has zero current but can have finite voltage. An ideal **short** has zero voltage, with current set by the surrounding network.

$$\frac{\ell}{v_{\text{sig}}} \ll T_{\text{variation}}.$$

Here ℓ is circuit size and v_{sig} the signal speed. The fastest relevant edge sets $T_{\text{variation}}$. Stray inductance and capacitance must also be negligible at the intended accuracy.

Chapter support: §7.2.2; Eq. for the lumped-model propagation-time check.

Emf supplies energy per charge



A source uses a process such as chemical work to move charge internally toward higher potential. Its emf is

$$\mathcal{E} = \frac{W_{\text{source}}}{q}, \quad [\mathcal{E}] = \text{V} = \text{J/C}.$$

An ideal source maintains its specified terminal voltage. A simple real-source model adds a series internal resistance r .

$$V_{\text{terminal}} = \mathcal{E} - rI_{\text{out}}.$$

The signed current I_{out} leaves the positive terminal. Delivery has $I_{\text{out}} > 0$ and lowers terminal voltage. Absorption has $I_{\text{out}} < 0$ and raises it above the emf in this model.

Emf is measured in volts despite the historical word “force.”

Chapter support: §7.3; emf definition and signed terminal-voltage relation.

The source boundary and terminal polarity

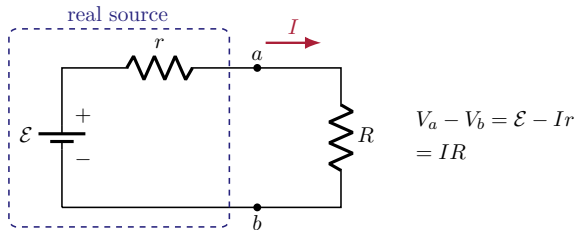


Figure 7.1: A real source model with explicitly identified external terminals a and b . Conventional current leaves a during delivery. The dashed boundary includes the ideal emf and internal resistance, not the load.

$$\mathcal{E} = Ir + IR, \quad V_a - V_b = IR.$$

The internal drop belongs to the source. The load receives the remaining energy per charge.

Chapter support: Figure 7.1; §7.3.

Example 7.2: a loaded battery



Example 7.2.

A battery has emf $\mathcal{E} = 12.0 \text{ V}$ and internal resistance $r = 0.20 \Omega$. It is connected to a load that draws $I = 3.0 \text{ A}$. Find the terminal voltage and the power dissipated inside the battery.

Delivery model: I leaves the positive terminal, so the internal voltage drop is subtracted.

$$V_{\text{terminal}} = 12.0 - (3.0)(0.20) = 11.4 \text{ V},$$

$$P_r = (3.0)^2(0.20) = 1.8 \text{ W}.$$

Energy check: source power is $I\mathcal{E} = 36 \text{ W}$ and load power is $IV_{\text{terminal}} = 34.2 \text{ W}$. Their difference is 1.8 W .

The 0.60 V internal drop explains the reduction in terminal voltage.

Chapter support: Example 7.2; independent arithmetic and power balance.

Inferring a source model from load tests



If emf and internal resistance are approximately constant over the tested range, terminal voltage is affine in delivered current:

$$V = \mathcal{E} - rl.$$

Two load tests remove the intercept when subtracted:

$$r = -\frac{V_2 - V_1}{I_2 - I_1}, \quad \mathcal{E} = V_1 + rI_1.$$

Thus a voltage–current slope has units $V/A = \Omega$. An external load R then gives

$$I = \frac{\mathcal{E}}{R + r}, \quad V = IR.$$

Several finite loads can reveal curvature or drift. Heating and state of charge can change the model parameters. A short-circuit test is unnecessary.

Chapter support: §7.3.1 and §7.4.

Problem 7.10: source characterization



Model: ideal wires, ohmic resistors and constant sources unless stated otherwise. Values specify teaching models.

Problem 7.10.

A source gives 11.8 V at 0.50 A and 11.2 V at 2.00 A . Infer $\mathcal{E}$ and r . Predict the current and terminal voltage with a $5.6\ \Omega$ load, and discuss one limitation of extrapolating this model.

Solution strategy. Define positive current out of the source. Subtract the two voltage equations to find the slope, then recover the intercept. The load shares its current with the internal resistance. Use their sum when predicting that current, and distinguish an operating-range fit from a universal battery law.

Chapter support: Problem 7.10; solution on slide 14.

Problem 7.10: parameters and prediction



The load-test equations are $11.8 = \mathcal{E} - 0.50r$ and $11.2 = \mathcal{E} - 2.00r$, in SI units.

$$r = \frac{11.8 - 11.2}{2.00 - 0.50} = 0.400 \, \Omega,$$

$$\mathcal{E} = 11.8 + (0.50)(0.400) = 12.0 \, \text{V},$$

$$I = \frac{12.0}{5.6 + 0.400} = 2.00 \, \text{A}, \quad V = I(5.6) = 11.2 \, \text{V}.$$

Checks: the prediction reproduces the second test point. Source power is 24.0 W, load power 22.4 W, and internal heat rate 1.6 W.

Limitation: heating or a changed state of charge can alter r and $\mathcal{E}$. Extrapolation to much larger current needs further evidence.

Chapter support: Problem 7.10; §7.23 source-characterization solution.

One current around a single loop



With no junction, steady charge flow has the same current through every element. Choose current and traversal directions, then add energy changes per charge.

$$+\mathcal{E} - Ir - IR = 0 \quad \Rightarrow \quad I = \frac{\mathcal{E}}{R + r}.$$

The terminal voltage follows by two consistent routes:

$$V_{\text{terminal}} = IR = \mathcal{E} - Ir.$$

Multiplying the loop equation by I checks energy transfer per second:

$$I\mathcal{E} = I^2r + I^2R.$$

For fixed emf, increasing either resistance lowers current. Taking $r \rightarrow 0$ recovers the ideal-source result for finite positive R .

Chapter support: §7.4 and power-balance reasoning in §7.6.

Series and parallel resistance



Series: one current passes through every resistor. Voltage drops add:

$$V = I \sum_j R_j \implies R_{\text{eq}} = \sum_j R_j.$$

Parallel: every branch joins the same two nodes. Branch currents add at their common voltage:

$$I = V \sum_j \frac{1}{R_j} \implies \frac{1}{R_{\text{eq}}} = \sum_j \frac{1}{R_j}.$$

For two positive resistances, $R_{\text{eq}} = R_1 R_2 / (R_1 + R_2)$ in parallel. This is smaller than either branch resistance because both paths carry current.

The rules follow connectivity. A component's position on the drawing does not establish a series or parallel connection.

Chapter support: §7.5.1 and §7.5.2.

Common current or common voltage

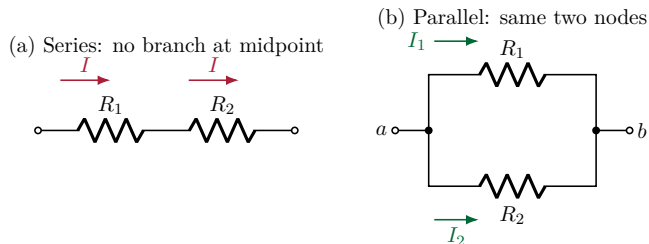


Figure 7.2: Connectivity determines series and parallel combinations. In (a), the current is common. In (b), both resistors connect to the same nodes a and b , so their voltage $V_a - V_b$ is common.

Series groups divide voltage. Parallel groups divide current. Redrawing the same nodes preserves these relations.

Chapter support: Figure 7.2; §7.5.

Division rules and terminal equivalence



For series resistors across fixed voltage V , the common current gives

$$V_j = V \frac{R_j}{\sum_k R_k}.$$

For parallel resistors carrying total current I , use conductance $G_j = 1/R_j$:

$$I_j = I \frac{G_j}{\sum_k G_k}.$$

The smaller parallel resistance carries the larger current. These rules follow from Ohm's law and conservation.

An equivalent resistance preserves the voltage–current relation at the external terminals. It preserves total power there, but individual branch currents and heating must be recovered from the original topology.

Chapter support: §7.5.3 and §7.5.4.

Example 7.5: reduce and reconstruct



Example 7.5.

A 12 V ideal source is connected to $R_1 = 6.0\ \Omega$ in series with a parallel combination of $R_2 = 12\ \Omega$ and $R_3 = 4.0\ \Omega$. Find the source current and the power in each resistor.

First reduce the parallel pair, then place that equivalent in series with R_1 :

$$\begin{aligned} R_{23} &= \frac{12 \times 4}{12 + 4} = 3.0\ \Omega, & R_{\text{eq}} &= 6.0 + 3.0 = 9.0\ \Omega, \\ I &= \frac{12}{9} = \frac{4}{3}\ \text{A}, & V_1 &= 6I = 8.0\ \text{V}, & V_{23} &= 4.0\ \text{V}, \\ I_2 &= \frac{4}{12} = \frac{1}{3}\ \text{A}, & I_3 &= \frac{4}{4} = 1.00\ \text{A}. \end{aligned}$$

Both parallel branches share 4.0 V. Their currents sum to the source current. The next slide completes the requested powers.

Chapter support: Example 7.5; power solution on slide 20.

Example 7.5: power in each resistor



Retain exact currents until the final rounding. Each resistor absorbs positive power regardless of the reference direction.

$$P_1 = 6 \left(\frac{4}{3} \right)^2 = \frac{32}{3} \text{ W} \simeq 10.7 \text{ W},$$

$$P_2 = 12 \left(\frac{1}{3} \right)^2 = \frac{4}{3} \text{ W} \simeq 1.33 \text{ W},$$

$$P_3 = 4(1)^2 = 4.0 \text{ W}.$$

$$P_1 + P_2 + P_3 = 16.0 \text{ W} = 12 \left(\frac{4}{3} \right) \text{ W}.$$

This exact power balance checks the reduction and reconstruction. The terminal equivalent alone would hide where these 16.0 W become heat.

Chapter support: Example 7.5; conservation check with unrounded values.

Example 7.6: adding a parallel load



Example 7.6.

A source with $\mathcal{E} = 12\text{ V}$ and $r = 1.0\ \Omega$ initially drives one $11\ \Omega$ resistor. The current is 1.0 A and the terminal voltage is 11 V . An identical second resistor is now connected in parallel.

Teaching task: determine the new total current, terminal voltage and current in each load. Compare the original branch before and after connection. The source holds its emf fixed. Its terminal voltage can change because the total current through r changes.

For an ideal voltage source, adding a parallel load leaves the original branch voltage unchanged. Here both loads share the source's internal voltage drop.

Chapter support: Example 7.6; complete worked comparison on slide 22.

Example 7.6: the original current falls



The two identical loads have parallel resistance $11/2 = 5.5 \Omega$.

$$I_{\text{total}} = \frac{12}{1.0 + 5.5} = \frac{24}{13} \text{ A} \simeq 1.846 \text{ A},$$

$$V = 12 - I_{\text{total}}(1.0) = \frac{132}{13} \text{ V} \simeq 10.154 \text{ V},$$

$$I_{\text{each}} = \frac{V}{11} = \frac{12}{13} \text{ A} \simeq 0.923 \text{ A}.$$

The original branch falls from 1.0 A by $(1 - 12/13) \times 100\% = 7.69\%$, while total current rises.

Checks: $2I_{\text{each}} = I_{\text{total}}$. As $r \rightarrow 0$, each load retains $12/11 \text{ A}$, so this reduction disappears.

Chapter support: Example 7.6; exact fractions and limiting-source check.

Signed power and energy accounting



Label a voltage $v = V_a - V_b$, and let signed current i enter terminal a . The passive sign convention defines

$$p_{\text{abs}} = vi.$$

Positive absorbed power means energy enters the element. Negative absorbed power means delivery. A source can do either.

$$v = Ri \implies p_R = Ri^2 \geq 0.$$

In a steady network, the sum of all signed absorbed powers is zero. During a transient, storage changes must also enter the balance.

For constant power over duration Δt , transferred energy is $E = P\Delta t$. Power is measured in watts, energy in joules. Component heating therefore needs both a power calculation and an operating-time model.

Chapter support: §7.6 and §7.14.

Practice 1: Problem 7.14, power bounds



For a voltage-driven resistor, $P = V^2/R$ increases as voltage rises or resistance falls. A specified derating rule places a bound on this computed power.

Model: ideal wires, ohmic resistors and constant sources unless stated otherwise. Values specify teaching models.

Problem 7.14.

A $1.0\text{ k}\Omega$ resistor with tolerance $\pm 5\%$ is connected to a supply bounded by $15.0\text{ V} \pm 2\%$. Find the worst-case power. Under a stated design rule allowing at most 50% of rated power, is a 0.50 W part adequate? Do not interpret this rule as a universal component specification.

Attempt: choose the two adverse parameter limits before calculating. Keep a tolerance bound distinct from a standard uncertainty.

Chapter support: §7.6.1; Problem 7.14; complete answer on slide 25.

Problem 7.14: worst-case heating



The adverse limits have opposite signs: maximum voltage and minimum resistance.

$$V_{\max} = 15.0(1.02) = 15.3 \text{ V},$$

$$R_{\min} = 1000(0.95) = 950 \Omega,$$

$$P_{\max} = \frac{15.3^2}{950} = 0.2464 \text{ W}.$$

The stated 50% rule requires a rating of at least $2P_{\max} = 0.4928 \text{ W}$. The 0.50 W part meets this mathematical bound by a small margin.

Check and interpretation: $0.2464 < 0.250 \text{ W}$. This conclusion uses the supplied bounds and rule. Actual thermal conditions and component ratings require the relevant device specifications.

Chapter support: Problem 7.14; §7.23 worst-case solution.

Kirchhoff's junction rule



Charge continuity for a small region enclosing a node gives

$$\frac{dQ_{\text{node}}}{dt} = \sum I_{\text{in}} - \sum I_{\text{out}}.$$

With negligible charge storage in the ideal node, entering and leaving currents balance. Capacitor branch currents must still be included during transients.

Example 7.11: answered current check.

At a node, currents $I_1 = 2.0 \text{ A}$ and $I_2 = 0.70 \text{ A}$ enter. A current $I_3 = 1.1 \text{ A}$ leaves, and another current I_4 is unknown. If I_4 is defined as leaving, find I_4 .

$$I_4 = I_1 + I_2 - I_3 = 2.0 + 0.70 - 1.1 = 1.6 \text{ A}.$$

The positive result confirms the leaving reference. Substitution gives 2.70 A entering and leaving.

Chapter support: §7.7; Example 7.11.

Kirchhoff's loop rule and signs



A closed walk returns to its initial potential. The signed voltage changes therefore sum to zero in the present lumped circuit model.

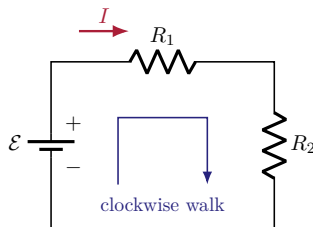
$$\sum_{\text{loop}} \Delta V = 0.$$

- Across a resistor, traversal with its chosen current gives $-IR$. Traversal against it gives $+IR$.
- Across a source, traversal from $-$ to $+$ gives $+\mathcal{E}$. The reverse gives $-\mathcal{E}$.
- Across a capacitor, traversal from negative to positive plate gives $+q/C$. The reverse gives $-q/C$.

Current reference and traversal are separate choices. A changing magnetic flux requires an induced-emf term, developed in later chapters.

Chapter support: §7.8 and §7.8.1.

One consistent voltage walk



up through source: $+\mathcal{E}$
right through R_1 : $-IR_1$
down through R_2 : $-IR_2$

Figure 7.3: The assumed current and the chosen loop traversal are separate choices. This clockwise walk gives $\mathcal{E} - IR_1 - IR_2 = 0$. Reversing the walk reverses every term. Reversing the walk changes every term:

$$-\mathcal{E} + IR_1 + IR_2 = 0.$$

This is the same equation multiplied by -1 , so the solved current is unchanged.

Chapter support: Figure 7.3; §7.8.

Example 7.12: an unknown current direction



Example 7.12.

A loop contains an ideal 10 V source opposing an ideal 4 V source and a total resistance of $3.0\ \Omega$. The assumed current direction passes from negative to positive through the 4 V source and from positive to negative through the 10 V source. Find the current.

Model: ideal steady sources and a positive total resistance. Retain the reference direction stated in the problem.

Traversing along that reference gives a rise through the smaller source, a drop through the larger source and a signed resistor drop. These signs follow the chosen walk before the actual current is known.

Chapter support: Example 7.12; signed solution and energy interpretation on slide 30.

Example 7.12: interpreting the negative sign



$$+4.0 - 10.0 - 3.0I = 0 \implies I = -2.0 \text{ A.}$$

The actual current is 2.0 A opposite to the reference. The net emf magnitude 6.0 V divided by 3.0Ω independently gives this magnitude.

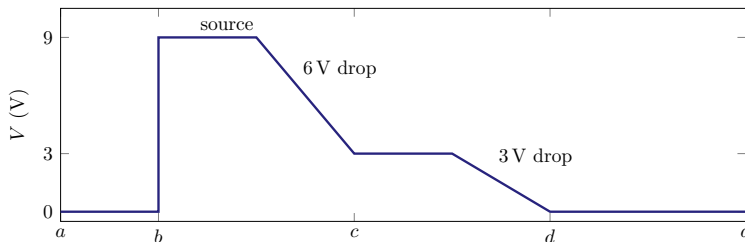
Energy interpretation. Actual current leaves the positive terminal of the 10 V source and enters the positive terminal of the 4 V source.

$$p_{10} = -20 \text{ W}, \quad p_4 = +8 \text{ W}, \quad p_R = (2.0)^2(3.0) = 12 \text{ W}.$$

Their absorbed-power sum is $-20 + 8 + 12 = 0$. The smaller source absorbs energy. A negative solved current is useful physical information.

Chapter support: Example 7.12; independently developed signed-power check.

Potential along a path



successive positions along a clockwise walk

Figure 7.4: A voltage walk around a 9 V source and series resistors 6 kΩ and 3 kΩ carrying 1 mA. The horizontal coordinate orders elements; it is not a spatial scale. Ideal wires are level segments. The walk returns to the initial potential.

For an open path, $V_{\text{end}} - V_{\text{start}} = \sum \Delta V$. For a closed path, this difference is zero. A source rise balances resistor drops without making the horizontal coordinate a time axis.

Chapter support: Figure 7.4; §7.8.2.

A shared branch in a two-loop network

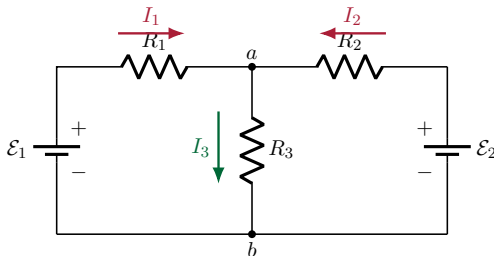


Figure 7.5: Three branch-current references in a two-loop network: I_1 and I_2 enter the top node, and I_3 leaves downward. Each source has its positive terminal above its negative terminal. These arrows are retained even if a solved current is negative.

$$I_1 + I_2 = I_3.$$

Use the single branch current I_3 in both walks through the central resistor. It is the same physical element in each loop.

Chapter support: Figure 7.5; §7.9.

Example 7.14: a complete two-loop calculation



Example 7.14.

In Figure 7.5, let $\mathcal{E}_1 = 12\text{ V}$, $\mathcal{E}_2 = 6\text{ V}$, $R_1 = 4\ \Omega$, $R_2 = 3\ \Omega$ and $R_3 = 6\ \Omega$. Use the indicated reference directions to find all three currents and check energy conservation.

Restored figure context (slide 32): both source positive terminals are above their negative terminals. I_1 and I_2 point toward top node a , and I_3 points from a down to bottom node b .

Both sources are ideal and all resistors are ohmic. Set $V_b = 0$. Three unknown branch currents require three independent equations: one junction equation and two loop equations.

Chapter support: Example 7.14; Figure 7.5; solution on slides 34–36.

Example 7.14: independent equations



The junction and left-loop equations give

$$I_3 = I_1 + I_2, \quad 12 - 4I_1 - 6I_3 = 0.$$

For the right loop, walk right from a , down through the source, then up through R_3 . Both resistor traversals oppose their reference currents:

$$+3I_2 - 6 + 6I_3 = 0.$$

Eliminating I_3 produces

$$\begin{aligned} 10I_1 + 6I_2 &= 12, \\ 6I_1 + 9I_2 &= 6. \end{aligned}$$

Coefficients in ohms multiply currents in amperes to give volts. Reversing an entire loop walk changes its signs without changing its information.

Chapter support: Example 7.14; §7.9.

Example 7.14: solve and check the voltage



Multiply the first reduced equation by 3 and the second by 2 to eliminate I_2 :

$$18I_1 = 24 \implies I_1 = \frac{4}{3} \text{ A.}$$

Substitution into $6I_1 + 9I_2 = 6$ and the junction equation gives

$$I_2 = -\frac{2}{9} \text{ A}, \quad I_3 = \frac{10}{9} \text{ A.}$$

Three branch relations independently reproduce the node voltage:

$$V_a = 12 - 4I_1 = 6 - 3I_2 = 6I_3 = \frac{20}{3} \text{ V.}$$

Because $V_a > 6 \text{ V}$, current flows from a toward the right source. This confirms the negative I_2 interpretation.

Chapter support: Example 7.14; current and voltage checks.

Example 7.14: a source can absorb power



The left source delivers $12(4/3) = 16$ W. Current enters the positive terminal of the right source, which absorbs $6(2/9) = 4/3$ W.

$$P_1 = 4(4/3)^2 = \frac{64}{9} \text{ W},$$

$$P_2 = 3(2/9)^2 = \frac{4}{27} \text{ W},$$

$$P_3 = 6(10/9)^2 = \frac{200}{27} \text{ W}.$$

Total resistor power is $44/3$ W. The complete balance is

$$16 = \frac{44}{3} + \frac{4}{3} \quad \text{in watts.}$$

An emf symbol specifies a voltage rise. It does not guarantee that the source delivers energy for every network connection.

Chapter support: Example 7.14; exact signed-power verification.

Practice 2: Problem 7.18, two loops

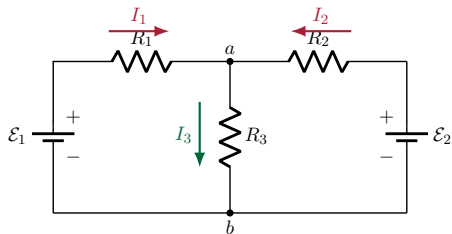


Figure 7.5: Three branch-current references in a two-loop network: I_1 and I_2 enter the top node, and I_3 leaves downward. Each source has its positive terminal above its negative terminal. These arrows are retained even if a solved current is negative.

Model: ideal wires, ohmic resistors and constant sources unless stated otherwise. Values specify teaching models.

Problem 7.18.

In Figure 7.5, use $\mathcal{E}_1 = 10\text{ V}$, $\mathcal{E}_2 = 4\text{ V}$, $R_1 = 2\Omega$, $R_2 = 4\Omega$ and $R_3 = 4\Omega$. Determine all signed currents, $V_a - V_b$, and the complete power balance.

Chapter support: Problem 7.18; unchanged Figure 7.5 context; answer on slide 38.

Problem 7.18: currents and power balance



Use $V = V_a - V_b$ and the preserved reference arrows. The branch relations and junction equation are

$$\frac{10 - V}{2} + \frac{4 - V}{4} = \frac{V}{4} \implies V = 6.0 \text{ V}.$$

$$I_1 = 2.0 \text{ A}, \quad I_2 = -0.50 \text{ A}, \quad I_3 = 1.50 \text{ A}.$$

Loop residuals vanish: $10 - 2(2) - 4(1.5) = 0$ and $4 - 4(-0.5) - 4(1.5) = 0$. The 10 V source delivers 20 W. The 4 V source absorbs 2 W. Resistors absorb 8, 1 and 9 W respectively, so $20 = 18 + 2$ in watts.

The node-voltage solution checks the branch-current method using the same topology.

Chapter support: Problem 7.18; §7.23 detailed solution.

A meter changes the circuit



An ammeter goes in **series** so the branch current passes through it. Its ideal resistance is zero. A voltmeter goes in **parallel** to compare two node potentials. Its ideal resistance is infinite.

For a fixed source voltage and original resistance R_0 , ammeter resistance R_A gives

$$I_0 = \frac{V}{R_0}, \quad I_A = \frac{V}{R_0 + R_A}, \quad \frac{I_0 - I_A}{I_0} = \frac{R_A}{R_0 + R_A}.$$

The burden voltage is $I_A R_A$. Disturbance depends on resistance relative to the circuit being measured.

A voltmeter adds a branch that draws current. Its loading depends on the output resistance of the driving network. More displayed digits do not remove either physical disturbance.

Chapter support: §7.10 and §7.10.1.

Which current does the ammeter measure?

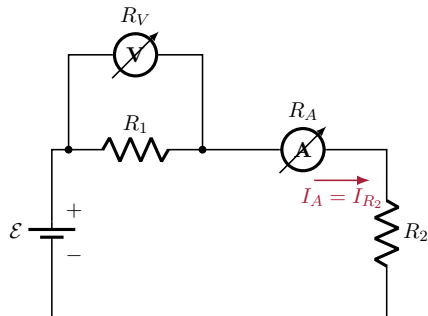


Figure 7.6: The voltmeter bridges the terminals of R_1 ; its branch contains the meter, with no bypass wire. The ammeter is in series with R_2 and measures I_{R_2} . With finite voltmeter resistance, this current is the sum of the currents through R_1 and the voltmeter, not just I_{R_1} .

The voltmeter is across R_1 in this figure. Its finite current joins the current through R_1 before both pass through the ammeter and R_2 .

Chapter support: Figure 7.6; §7.10. The next example measures a different divider output.

Example 7.16: voltmeter loading



Example 7.16.

A voltage divider consists of $R_1 = 100\text{ k}\Omega$ in series with $R_2 = 100\text{ k}\Omega$ across an ideal 10.0 V source. The intended output is across R_2 . First find the unloaded output. Then find the output when a voltmeter of resistance $R_V = 100\text{ k}\Omega$ is connected across R_2 .

Model and geometry: the meter bridges the lower resistor R_2 . It is a parallel branch, while R_1 remains the upper series resistor. First find the unobserved output. Then replace the lower arm by $R_2 \parallel R_V$ to find the voltage of the changed circuit.

Chapter support: Example 7.16; complete solution on slide 42.

Example 7.16: the loaded output



Unloaded, the equal resistors give

$$V_0 = 10.0 \frac{100}{100 + 100} = 5.0 \text{ V.}$$

With resistances in kilohms, the lower arm and measured voltage become

$$R_{2V} = \frac{100 \times 100}{100 + 100} = 50 \text{ k}\Omega, \quad V_M = 10.0 \frac{50}{100 + 50} = \frac{10}{3} \text{ V} \simeq 3.33 \text{ V.}$$

Check: source current is $66.67 \mu\text{A}$, split into $33.33 \mu\text{A}$ through each lower branch. Their sum satisfies the junction rule.

The voltage reduction relative to the unloaded value is $1/3$, or 33.3% . It results from changed connectivity with finite input resistance.

Chapter support: Example 7.16; independently developed current and fractional-loading checks.

Four-terminal resistance measurement

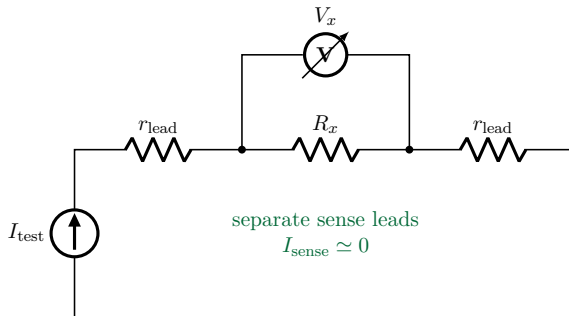


Figure 7.7: Four-terminal resistance measurement. The test current flows through the current leads and R_x ; separate sense leads contact R_x directly. A sufficiently high input resistance makes their voltage drops negligible, so $R_x = V_x / I_{\text{test}}$ excludes the current-lead drops.

Current leads produce drops, but separate sense leads compare the sample's own terminals. With negligible sense current, their resistance contributes negligible voltage error.

Chapter support: Figure 7.7; §7.10.1.

Problem 7.24: lead resistance and sample voltage



Model: ideal wires, ohmic resistors and constant sources unless stated otherwise. Values specify teaching models.

Problem 7.24.

A $0.100\ \Omega$ resistor is measured using $0.500\ \text{A}$. Each current lead has $0.050\ \Omega$. Compare the voltage measured at the supply-side terminals with the voltage measured directly across the resistor using separate ideal sense leads. What resistances would be inferred?

$$V_{\text{outer}} = 0.500(0.050 + 0.100 + 0.050) = 0.100\ \text{V},$$
$$V_x = 0.500(0.100) = 0.0500\ \text{V}.$$

Dividing each voltage by $0.500\ \text{A}$ gives $R_{\text{two wire}} = 0.200\ \Omega$ and $R_x = 0.100\ \Omega$. Direct sense contacts exclude both lead drops.

Chapter support: Problem 7.24; Figure 7.7 on slide 43; §7.23 solution.

A capacitor state through switching



For current entering the positive-reference plate,

$$q = CV_C, \quad I_C = C \dot{V}_C.$$

Charge changes on the plates without steady conduction through the dielectric. Finite current makes V_C continuous at switching.

An initially uncharged capacitor has $V_C(0^+) = 0$. A precharged capacitor retains its initial voltage. At long-time DC equilibrium its branch current is zero.

$$V_C(t) = V_f + (V_i - V_f)e^{-t/\tau}, \quad \tau = R_{\text{seen}}C.$$

For one independent capacitor state and finite positive resistance, determine V_f with the capacitor open. Find R_{seen} after independent voltage sources become shorts and current sources become opens.

Chapter support: §7.11 and §7.15. The following pages derive the simple case.

RC charging: the governing equation



An ideal constant source $\mathcal{E}$ charges an initially uncharged capacitor through finite positive R . Let q denote charge on the plate connected toward the source's positive terminal.

$$I = \dot{q}, \quad \mathcal{E} - IR - \frac{q}{C} = 0.$$

Hence

$$R\dot{q} + \frac{q}{C} = \mathcal{E}, \quad \dot{q} = \frac{C\mathcal{E} - q}{RC}, \quad q(0) = 0.$$

The rate is proportional to the charge still needed to reach equilibrium. As q increases, the resistor voltage and charging current decrease.

At $t = 0^+$, $I = \mathcal{E}/R$. At equilibrium, $q = C\mathcal{E}$ and $I = 0$. The equation must reproduce both limits.

Chapter support: §7.12; charging differential equation.

The exponential charging solution



Separate variables and integrate with the initial condition:

$$\int_0^{q(t)} \frac{dq'}{C\mathcal{E} - q'} = \int_0^t \frac{dt'}{RC} \implies \ln\left(1 - \frac{q}{C\mathcal{E}}\right) = -\frac{t}{RC}.$$

Exponentiation gives

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau}), \quad \tau = RC,$$
$$V_C(t) = \mathcal{E}(1 - e^{-t/\tau}), \quad I(t) = \frac{\mathcal{E}}{R}e^{-t/\tau}.$$

Checks: differentiation satisfies the original equation and $q(0) = 0$. Also $[RC] = \Omega F = s$.

At $t = \tau$, charge reaches $1 - e^{-1} \simeq 0.632$ of its final value and current retains $e^{-1} \simeq 0.368$ of its initial value.

Chapter support: §7.12.1; independent differential-equation and dimensional checks.

Charging circuit, voltage and current

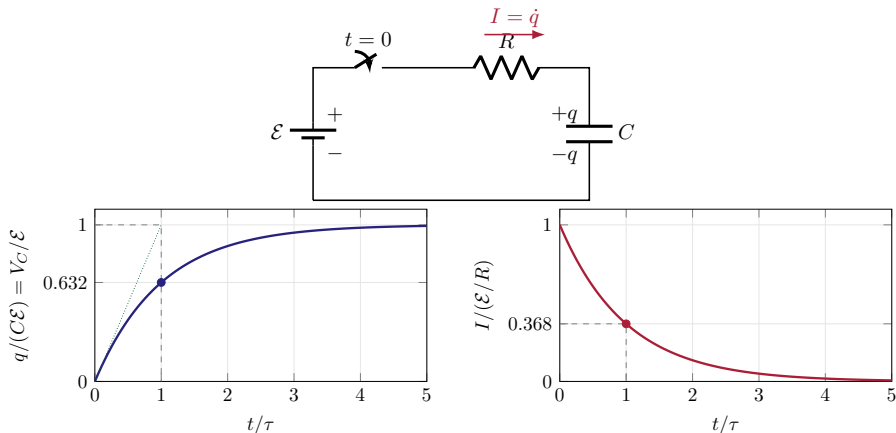


Figure 7.8: An initially uncharged capacitor begins charging when the switch closes. Separate quantitative axes show the normalized charge/voltage and current. The dotted initial tangent in the left panel reaches the final-value level at $t = \tau$, while the actual charge reaches only $1 - e^{-1}$.

Chapter support: Figure 7.8, unchanged circuit and both quantitative panels.

Example 7.18: one time constant



Example 7.18.

A $10\ \mu\text{F}$ capacitor is charged through a $200\ \text{k}\Omega$ resistor from an ideal $9.0\ \text{V}$ source. Find the time constant, the capacitor voltage at one time constant and the current at one time constant.

$$\tau = (200 \times 10^3)(10 \times 10^{-6}) = 2.0\ \text{s},$$

$$V_C(\tau) = 9.0(1 - e^{-1}) = 5.69\ \text{V},$$

$$I_0 = \frac{9.0}{200 \times 10^3} = 45\ \mu\text{A}, \quad I(\tau) = 45e^{-1} = 16.6\ \mu\text{A}.$$

Loop check: $V_R = 9.0e^{-1} = 3.31\ \text{V}$, and $V_R + V_C = 9.0\ \text{V}$. One time constant gives a characteristic fraction, rather than complete charging.

Chapter support: Example 7.18; independent numeric and loop checks.

RC discharge and current references



Let $q > 0$ be the remaining charge on the initially positive plate. Current in the physical discharge direction is $I_d = -\dot{q}$.

$$I_d R = \frac{q}{C} \implies \dot{q} = -\frac{q}{RC}.$$

Integrating from $q(0) = Q_0$ gives $\ln(q/Q_0) = -t/\tau$, hence

$$\begin{aligned} q &= Q_0 e^{-t/\tau}, & V_C &= V_0 e^{-t/\tau}, \\ I_d &= \frac{V_0}{R} e^{-t/\tau}, & I_q &= \dot{q} = -\frac{V_0}{R} e^{-t/\tau}. \end{aligned}$$

Both current expressions describe the same flow using opposite references. Voltage and charge retain their initial sign in this passive discharge model.

Chapter support: §7.13; discharge solution and sign conventions.

Voltage decay and energy decay

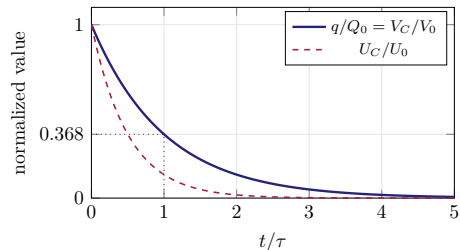
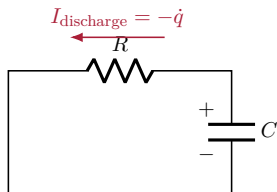


Figure 7.9: A charged capacitor discharges through R . The current arrow follows the physical discharge direction for $q > 0$. Voltage and charge decay as $e^{-t/\tau}$, but stored energy decays faster, as $e^{-2t/\tau}$.

Because $U_C = CV_C^2/2$, its normalized decay is $e^{-2t/\tau}$. Its characteristic decay time is $\tau/2$, although the voltage decay time is τ .

Chapter support: Figure 7.9; §7.13 and §7.14.

Example 7.19: reaching a discharge threshold



Example 7.19.

A capacitor initially at 5.0 V discharges through a resistor with time constant $\tau = 0.80$ s. How long does it take for the voltage to fall to 1.0 V?

With the zero-offset discharge law,

$$\frac{V_C}{V_0} = e^{-t/\tau}, \quad t = -\tau \ln \left(\frac{V_C}{V_0} \right).$$

Thus

$$t = -0.80 \ln(1.0/5.0) = 1.29 \text{ s}.$$

The logarithm acts on a dimensionless ratio. After one time constant the voltage is $5.0e^{-1} = 1.84$ V, so reaching 1.0 V must take longer than 0.80 s.

Chapter support: Example 7.19; logarithm and limiting-value check.

Where the RC charging energy goes



For charging from zero toward a fixed ideal source voltage,

$$W_s = \int_0^{C\mathcal{E}} \mathcal{E} \, dq = C\mathcal{E}^2, \quad U_f = \frac{1}{2} C\mathcal{E}^2.$$

The resistor receives the difference. An independent power integral gives

$$W_R = \int_0^\infty \frac{\mathcal{E}^2}{R} e^{-2t/(RC)} dt = \frac{\mathcal{E}^2}{R} \frac{RC}{2} = \frac{1}{2} C\mathcal{E}^2.$$

Resistance controls duration and peak current, but cancels from the integrated heat in this finite- R model.

For discharge from V_0 with no source, the same integral gives $W_R = CV_0^2/2$, equal to the initial capacitor energy. The half-energy charging result depends on the stated charging protocol.

Chapter support: §7.14; independent integral and conservation checks.

Example 7.20: energy supplied and stored



Example 7.20.

A $100\ \mu\text{F}$ capacitor is charged from 0 to 12 V through a resistor. Find the energy supplied by the ideal source, the final capacitor energy and the energy dissipated in the resistor.

Convert $C = 100 \times 10^{-6}\ \text{F}$. The complete fixed-voltage charging process gives

$$W_s = (100 \times 10^{-6})(12)^2 = 0.0144\ \text{J},$$

$$U_f = \frac{1}{2}(100 \times 10^{-6})(12)^2 = 0.0072\ \text{J},$$

$$W_R = W_s - U_f = 0.0072\ \text{J}.$$

Check: $0.0144 = 0.0072 + 0.0072$ in joules. A larger finite resistance lowers peak current and slows charging without changing these integrated energies.

Chapter support: Example 7.20; fixed-source, initially uncharged model.

A precharged capacitor can return energy



For a finite- R step from initial V_i to source voltage V_f , the net charge supplied is $C(V_f - V_i)$:

$$W_s = CV_f(V_f - V_i), \quad \Delta U_C = \frac{1}{2}C(V_f^2 - V_i^2),$$
$$W_R = W_s - \Delta U_C = \frac{1}{2}C(V_f - V_i)^2 \geq 0.$$

Model: ideal wires, ohmic resistors and constant sources unless stated otherwise. Values specify teaching models.

Problem 7.28.

A $20\ \mu\text{F}$ capacitor starts at $8.0\ \text{V}$ and is connected with the same polarity to an ideal $3.0\ \text{V}$ source through $50\ \text{k}\Omega$. Find $V_C(t)$ and the signed current into its positive plate. Find the total resistor heat, capacitor energy change and signed source work.

Chapter support: §7.14 general-step energy; Problem 7.28; answer on slide 56.

Problem 7.28: signed current and source work



Continuity gives $V_i = 8.0 \text{ V}$, the final source sets $V_f = 3.0 \text{ V}$, and $\tau = (50 \text{ k}\Omega)(20 \mu\text{F}) = 1.0 \text{ s}$.

$$V_C(t) = 3.0 + 5.0e^{-t/(1.0\text{s})} \text{ V},$$
$$I_C(t) = C\dot{V}_C = -100e^{-t/(1.0\text{s})} \mu\text{A}.$$

$$W_R = 250 \mu\text{J}, \quad \Delta U_C = -550 \mu\text{J}, \quad W_s = -300 \mu\text{J}.$$

The negative current flows out of the capacitor's positive plate and into the source's positive terminal. The capacitor loses $550 \mu\text{J}$, of which $300 \mu\text{J}$ reaches the source and $250 \mu\text{J}$ becomes heat.

Checks: $(3 - V_C)/R = I_C$ and $W_s = \Delta U_C + W_R$.

Chapter support: Problem 7.28; §7.23 energy-return solution.

Example 7.21: switching onto a divider

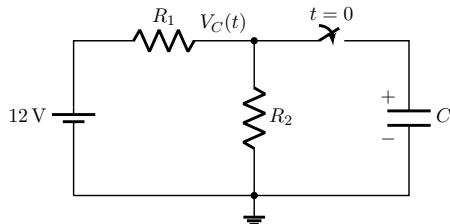


Figure 7.10: After the switch closes, the capacitor is across R_2 . The initial capacitor voltage is specified by its prior preparation. Both R_1 and R_2 influence the relaxation time; the nearest resistor alone does not determine it.

Example 7.21.

An ideal 12 V supply feeds $R_1 = 3.0 \text{ k}\Omega$ and $R_2 = 6.0 \text{ k}\Omega$ in series. At $t = 0$, a $100 \mu\text{F}$ capacitor with initial voltage 2.0 V is connected across R_2 , with its positive voltage reference at the divider node.

Teaching task: find initial branch currents, final voltage, time constant and $V_C(0.300 \text{ s})$.

Chapter support: Example 7.21; Figure 7.10; solution on slide 58.

Example 7.21: the resistance seen by C



At connection, continuity gives $V_C = 2.0 \text{ V}$. In milliamperes,

$$I_1 = \frac{12 - 2}{3} = 3.333, \quad I_2 = \frac{2}{6} = 0.333, \quad I_C = I_1 - I_2 = 3.000.$$

At equilibrium the capacitor branch is open, so $V_f = 12(6/9) = 8.0 \text{ V}$. With the source shorted, both resistors connect across the capacitor:

$$R_{\text{seen}} = 3 \parallel 6 = 2.0 \text{ k}\Omega, \quad \tau = 0.200 \text{ s}.$$

$$V_C(t) = 8.0 - 6.0e^{-t/(0.200 \text{ s})} \text{ V}, \quad V_C(0.300 \text{ s}) = 6.661 \text{ V}.$$

Direct check: $C \dot{V}_C = (12 - V_C)/R_1 - V_C/R_2$ reduces to $0.200 \text{ s } \dot{V}_C + V_C = 8.0 \text{ V}$. Differentiation gives $I_C = 3.000e^{-t/\tau} \text{ mA}$.

Chapter support: Example 7.21; node-equation and initial-current checks.

Example 7.22: measuring a relaxation time



A zero-offset discharge gives $\ln[V_C(t)/V_0] = -t/\tau$. Its dimensionless voltage ratio can determine a decay time.

Example 7.22.

During discharge, a capacitor voltage is 4.0 V at $t = 0$ and 1.5 V at $t = 3.0$ s. Estimate the time constant.

$$\frac{1.5}{4.0} = e^{-3.0/\tau} \implies \tau = -\frac{3.0}{\ln(1.5/4.0)} = 3.06 \text{ s.}$$

Check: $4.0e^{-3.0/3.0586} \simeq 1.5$ V. A fit to multiple readings tests the model more effectively than two points.

Subtract an independently established offset before taking logs. Finite probe resistance and capacitance can change the measured time constant. Late-time log points amplify additive voltage noise.

Chapter support: Example 7.22; §7.16 and §7.16.1. Synthetic fit example 7.23 is reading.

A conservation method for circuit problems



- 1 State the model and node connections. Choose references that remain fixed through the algebra.
- 2 Combine conservation with element laws. Use independent equations and recover internal branch quantities after reduction.
- 3 For switching, preserve initial capacitor voltage, find the final state and determine the resistance seen by its terminals.
- 4 Check residuals, signs, units, energy and limiting cases. Include the meter when its loading is significant.

Reading: the chapter's full practice set, offset-aware RC fitting and alternative node/mesh methods.

The next ten slides deepen terminal models and numerical dynamics. Later induction and AC chapters add magnetic storage and frequency-dependent response to the conservation framework.

Chapter support: §§7.18–7.20; §7.21.19; §7.22.

Advanced: a network viewed through its port

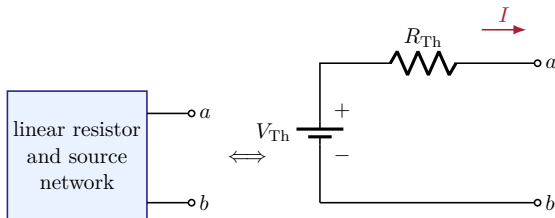


Figure 7.11: Thevenin equivalence preserves the terminal relation $V_a - V_b = V_{Th} - IR_{Th}$, with I positive out of terminal a . The open port is shown explicitly. Internal currents and dissipation need not be reproduced by the equivalent network.

A linear network with a well-defined finite voltage–current port relation has $V = V_{Th} - IR_{Th}$. Here V_{Th} is the open-circuit voltage and I leaves terminal a .

Chapter support: §7.21.1; Figure 7.11; advanced instruction begins here.

Example 7.25: a Thevenin divider equivalent



Setting independent sources to zero removes the constant term in the linear port response. A zero-voltage source becomes a short, while its resistances remain.

Example 7.25.

A 12 V ideal source feeds a divider with $R_1 = 3.0 \text{ k}\Omega$ from the source to the output node and $R_2 = 6.0 \text{ k}\Omega$ from the output node to ground. Find the Thevenin equivalent seen from the output node to ground.

$$V_{\text{Th}} = 12 \frac{6}{3 + 6} = 8.0 \text{ V}, \quad R_{\text{Th}} = 3 \parallel 6 = 2.0 \text{ k}\Omega.$$

Open-circuit division fixes V_{Th} . After source deactivation both resistors join the output to ground, giving R_{Th} .

Check: for a $6.0 \text{ k}\Omega$ test load, both the original loaded divider and its equivalent give 6.0 V. Terminal equivalence preserves the port response, rather than the internal power distribution.

Chapter support: §7.21.1; Example 7.25; independent finite-load check.

Example 7.26: a loading bound



The port model makes loading an explicit error. For positive input resistance R_M , $V_M = V_{Th} R_M / (R_{Th} + R_M)$.

Example 7.26.

A sensor has $V_{Th} = 2.5 \text{ V}$ and $R_{Th} = 20 \text{ k}\Omega$. Choose a minimum meter input resistance so the measured DC voltage is within 1% of its open-circuit value.

$$\epsilon = \frac{V_{Th} - V_M}{V_{Th}} = \frac{R_{Th}}{R_{Th} + R_M} \leq 0.01 \quad \implies \quad R_M \geq 99 R_{Th} = 1.98 \text{ M}\Omega.$$

Boundary check: at $R_M = 1.98 \text{ M}\Omega$, the reading is 2.475 V , exactly 1% below 2.5 V . Larger positive R_M reduces this error monotonically.

The bound controls loading alone. Calibration, offset and noise require additional measurement information.

Chapter support: Example 7.26; quantitative inequality and boundary check.

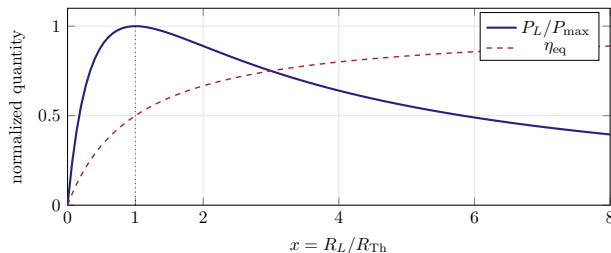
Maximum power and equivalent efficiency

Hold V_{Th} and finite $R_{Th} > 0$ fixed. For a resistive load,

$$P_L = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2}, \quad \frac{dP_L}{dR_L} = \frac{V_{Th}^2 (R_{Th} - R_L)}{(R_{Th} + R_L)^3}.$$

The derivative changes from positive to negative at $R_L = R_{Th}$, giving $P_{max} = V_{Th}^2 / (4R_{Th})$.

$$\eta_{eq} = \frac{P_L}{P_L + I^2 R_{Th}} = \frac{R_L}{R_{Th} + R_L}.$$



Chapter support: §7.21.4; Figure 7.12. Its complete caption continues on slide 65.

Problem 7.36: matching or efficient transfer



Figure 7.12: For $R_{Th} > 0$, maximum load power occurs at $R_L = R_{Th}$. The efficiency η_{eq} refers to the simple equivalent source and series resistance. It rises toward unity as the load resistance grows, while the absolute load power eventually falls toward zero.

Figure context: the unchanged graph is on slide 64. Maximum power has $\eta_{eq} = 1/2$; larger R_L improves this ratio while power eventually falls.

Model: ideal wires, ohmic resistors and constant sources unless stated otherwise. Values specify teaching models.

Problem 7.36: advanced design challenge.

★ For $V_{Th} = 6\text{ V}$ and $R_{Th} = 50\ \Omega$, find the load and power at maximum power transfer. Find the smallest load resistance that gives 90% efficiency in the equivalent series-source model. Calculate its power and explain why a general original network may have a different internal efficiency.

Chapter support: Problem 7.36; unchanged Figure 7.12; complete solution on slide 66.

Problem 7.36: two different design targets



For $V_{Th} = 6\text{ V}$ and $R_{Th} = 50\ \Omega$, maximum power occurs at $R_L = 50\ \Omega$:

$$P_{\max} = \frac{6^2}{4(50)} = 0.180\text{ W}.$$

For equivalent efficiency at least 0.90,

$$\frac{R_L}{50 + R_L} \geq 0.90 \implies R_L \geq 450\ \Omega.$$

At the smallest allowed load, $I = 6/500 = 12\text{ mA}$ and $P_L = (0.012)^2(450) = 0.0648\text{ W}$. Equivalent internal heat is $P_r = (0.012)^2(50) = 0.0072\text{ W}$. Thus $\eta_{\text{eq}} = P_L/(P_L + P_r) = 0.90$.

The original network can dissipate power internally even with its port open. Port equivalence fixes delivered load power, but does not determine that original internal loss or its overall efficiency.

Chapter support: Problem 7.36; source-equivalence limitation from §7.21.4.

Numerical relaxation: stability and fidelity



For normalized discharge $\dot{x} = -x/\tau$, define $h = \Delta t/\tau > 0$. Exact, explicit Euler and backward Euler updates are

$$x_{n+1} = e^{-h}x_n, \quad (1 - h)x_n, \quad \frac{x_n}{1 + h}.$$

Explicit Euler freezes the slope at the old state. Backward Euler uses $(x_{n+1} - x_n)/\Delta t = -x_{n+1}/\tau$ and solves for the new state.

Explicit Euler is stable in magnitude for $0 < h < 2$, and preserves sign monotonically for $0 < h < 1$. At $h = 1$ it reaches zero prematurely. At $h = 2$ it alternates without decay, and for $h > 2$ it grows.

Backward Euler preserves sign and decays for every $h > 0$, but large steps still distort the relaxation time. Stability and physical accuracy require separate checks.

Chapter support: §7.17 and §7.21.15; exact exponential benchmark.

Problem 7.34: an integration challenge



Model: ideal wires, ohmic resistors and constant sources unless stated otherwise. Values specify teaching models.

Problem 7.34: attempt, then compare.

For normalized discharge $x' = -x/\tau$, use explicit Euler with $\Delta t = 0.5\tau$, 1.5τ and 2.5τ . Compute the first four values from $x_0 = 1$. Distinguish accuracy, monotonicity and stability, and compare with backward Euler.

Method: for each fixed h , multiply the previous value by $1 - h$ four times. For backward Euler use $1/(1 + h)$ instead. Compare with $x_n = e^{-nh}$ at the same sample times.

Interpret sign preservation, error relative to the analytic solution, and long-time growth separately. A decaying squared voltage can hide a sign-changing voltage.

Chapter support: Problem 7.34; complete numerical tables and interpretation on slides 69–70.

Problem 7.34: explicit Euler outcomes



With $x_0 = 1$, repeated multiplication gives:

h	x_1	x_2	x_3	x_4
0.5	0.5	0.25	0.125	0.0625
1.5	-0.5	0.25	-0.125	0.0625
2.5	-1.5	2.25	-3.375	5.0625

At $h = 0.5$, the state is monotone and stable, but $x_4 = 0.0625$ is far below the exact $e^{-2} = 0.135335$.

At $h = 1.5$, magnitude decays but the sign alternates, contrary to the passive discharge. At $h = 2.5$, magnitude grows and the method is unstable.

The update factor supplies an independent check on each table and on its long-time behaviour.

Chapter support: Problem 7.34; repeated updates and exact-solution comparison.

Problem 7.34: backward Euler and energy



Backward Euler gives $x_n = (1 + h)^{-n}$:

h	x_1	x_2	x_3	x_4
0.5	0.666667	0.444444	0.296296	0.197531
1.5	0.400000	0.160000	0.064000	0.025600
2.5	0.285714	0.081633	0.023324	0.006664

Every row decays with the correct sign. At $h = 0.5$, the exact fourth value is 0.135335, so backward Euler is also inaccurate at this step size.

For fixed capacitance, $U_n/U_0 = x_n^2$. Energy multiplies by $(1 - h)^2$ for explicit Euler and $(1 + h)^{-2}$ for backward Euler. Energy decay alone misses the sign alternation at $h = 1.5$. Reduce the step and compare state and energy with the exact response.

Chapter support: Problem 7.34; §7.21.15; advanced challenge completed.

