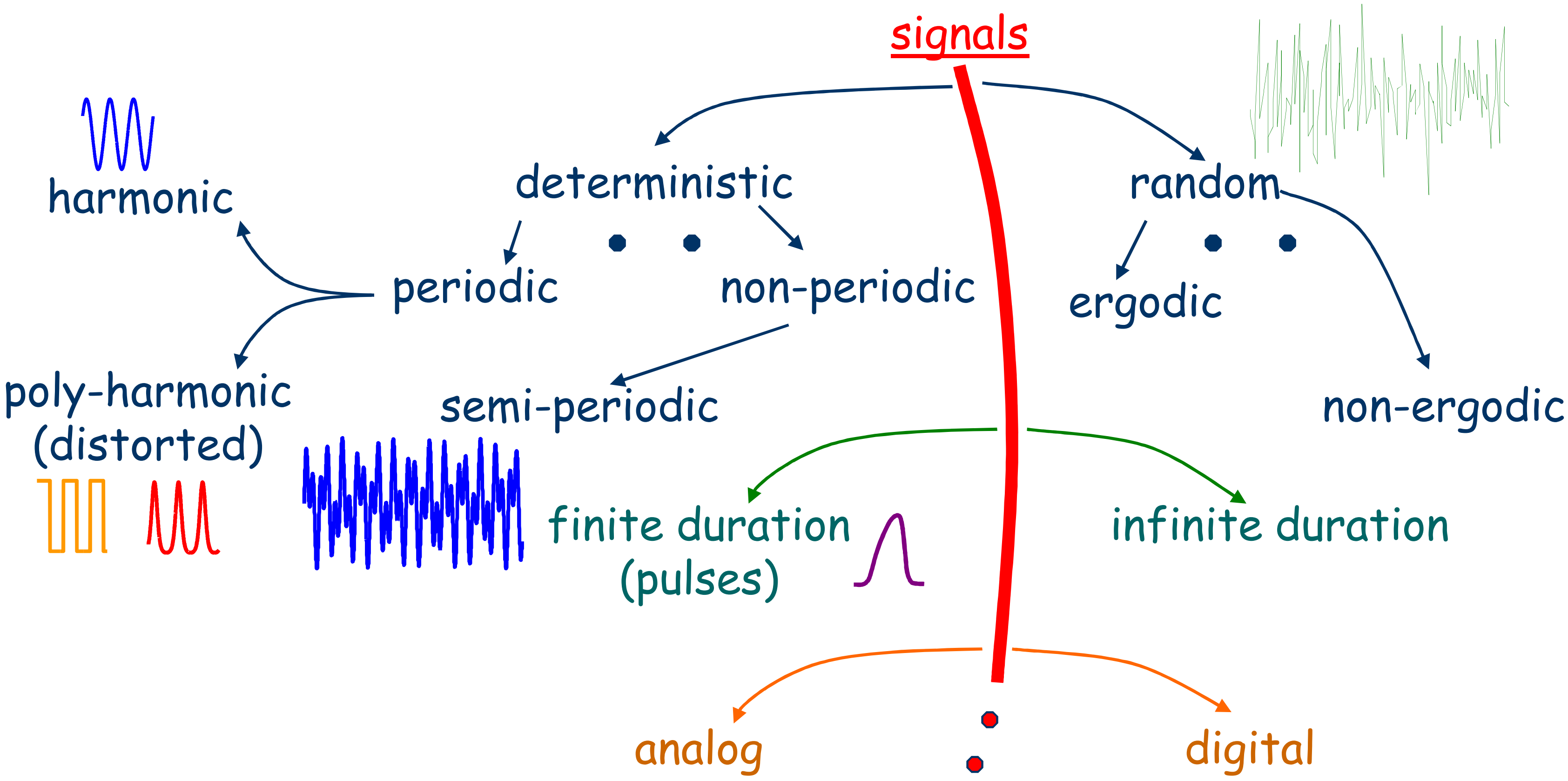
Signals and their parameters

Signal

physical quantity, conveying somehow an information about the state of a physical system or mathematical model used for this purpose

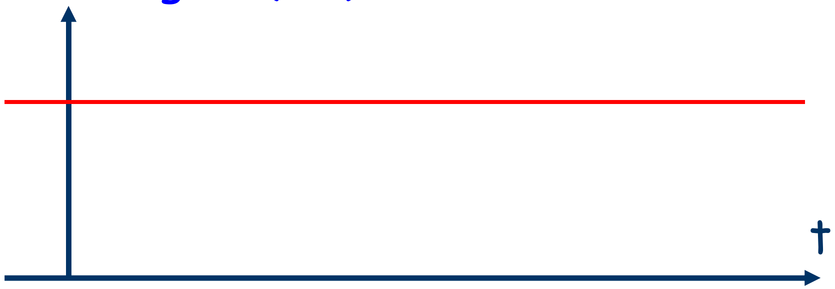


a set of possible meaningful parameters depends on the kind of a signal

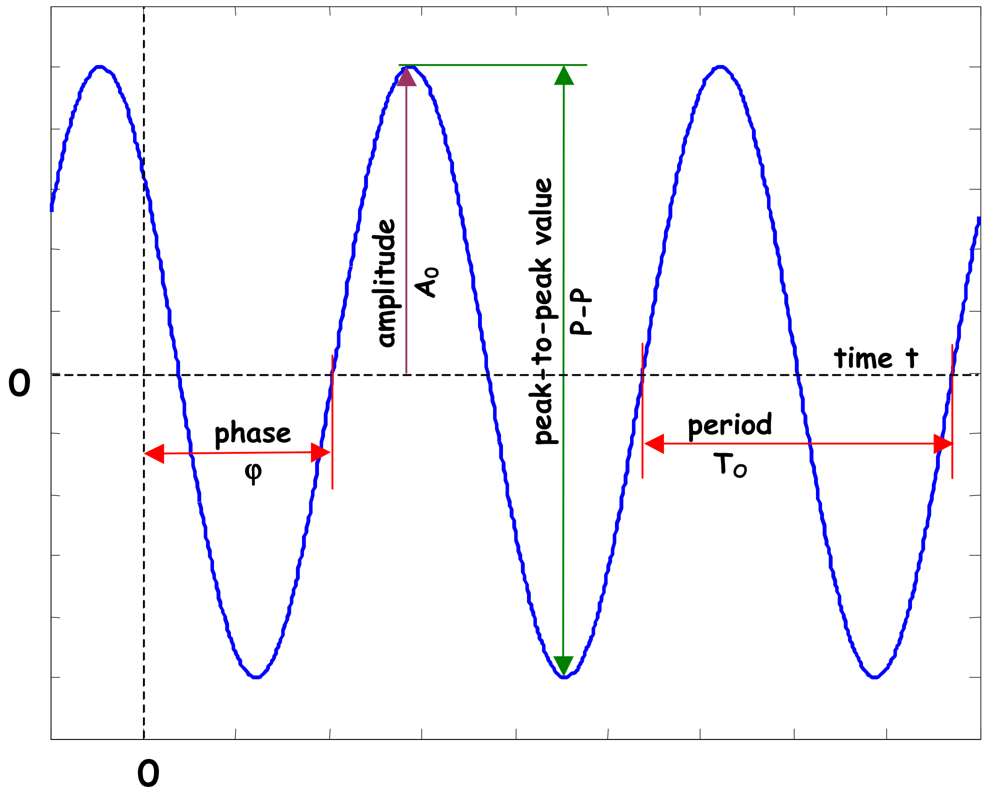
it is generally wise to know what we are going to measure and what we may come across

Basic signals and their parameters

constant signal (DC)



harmonic signal (sinusoidal, AC)

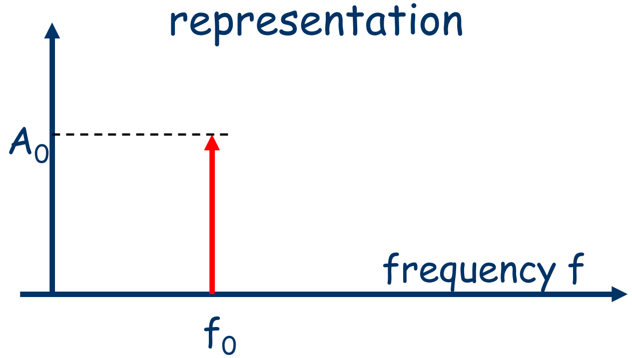


$$y(t) = A_0 \sin(2\pi f_0 t - \varphi)$$

$$T_O = \frac{1}{f_O}$$

$$y(t + nT_O) = y(t)$$

frequency spectrum representation

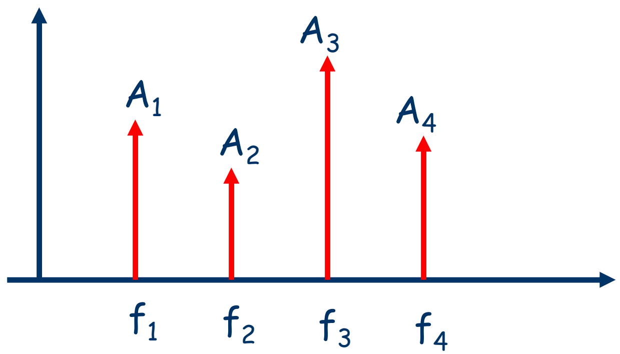


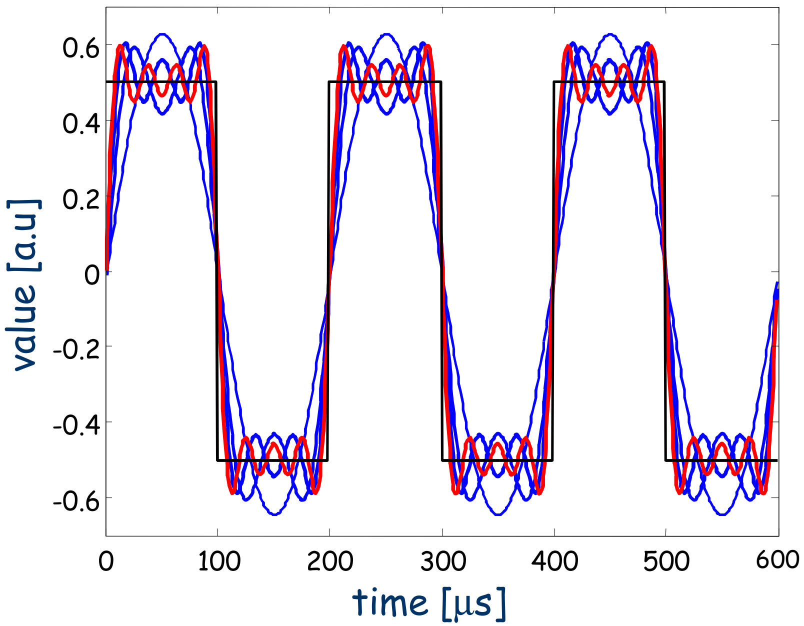
Poly-harmonic signal

time representation

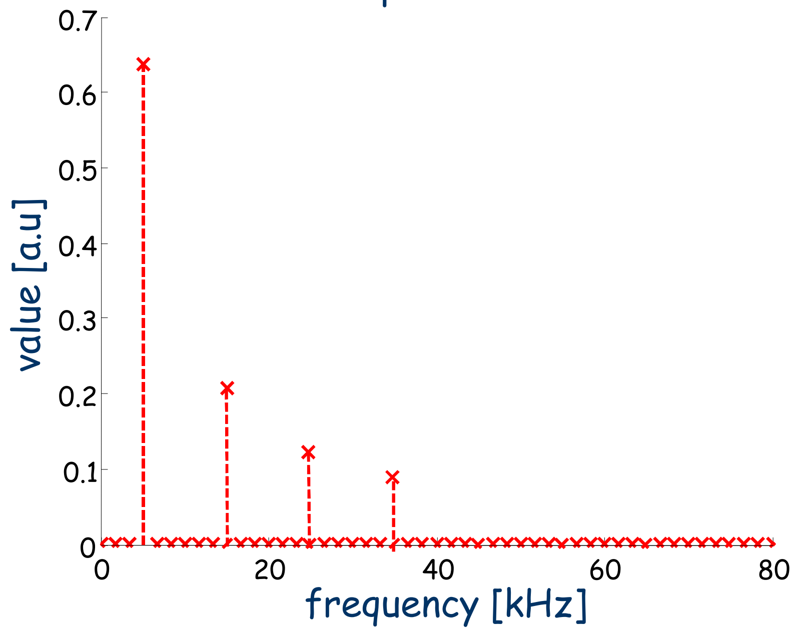
$$y(t) = \sum_n A_n \sin(2\pi \cdot n \cdot f_o \cdot t + \phi_n)$$

spectrum

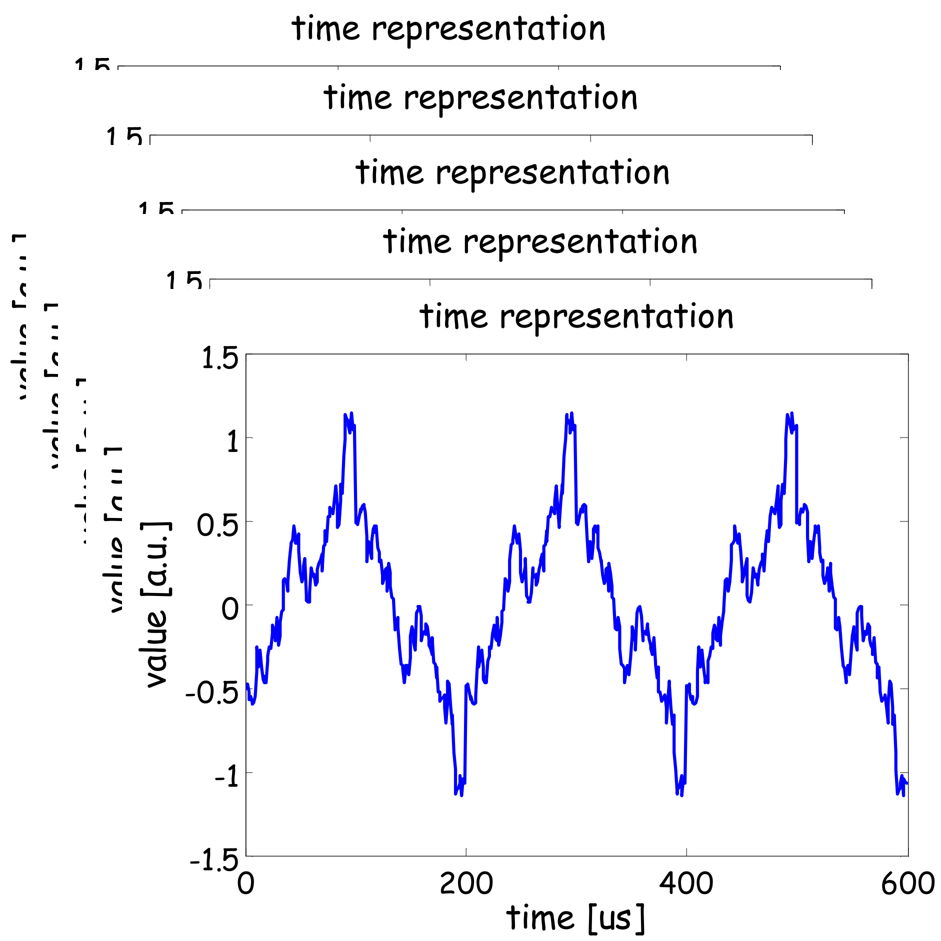


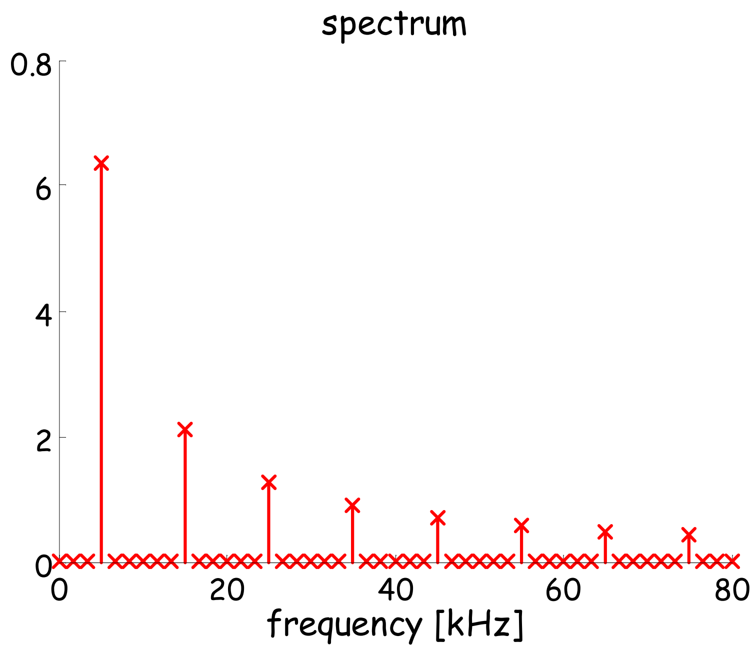


spectrum

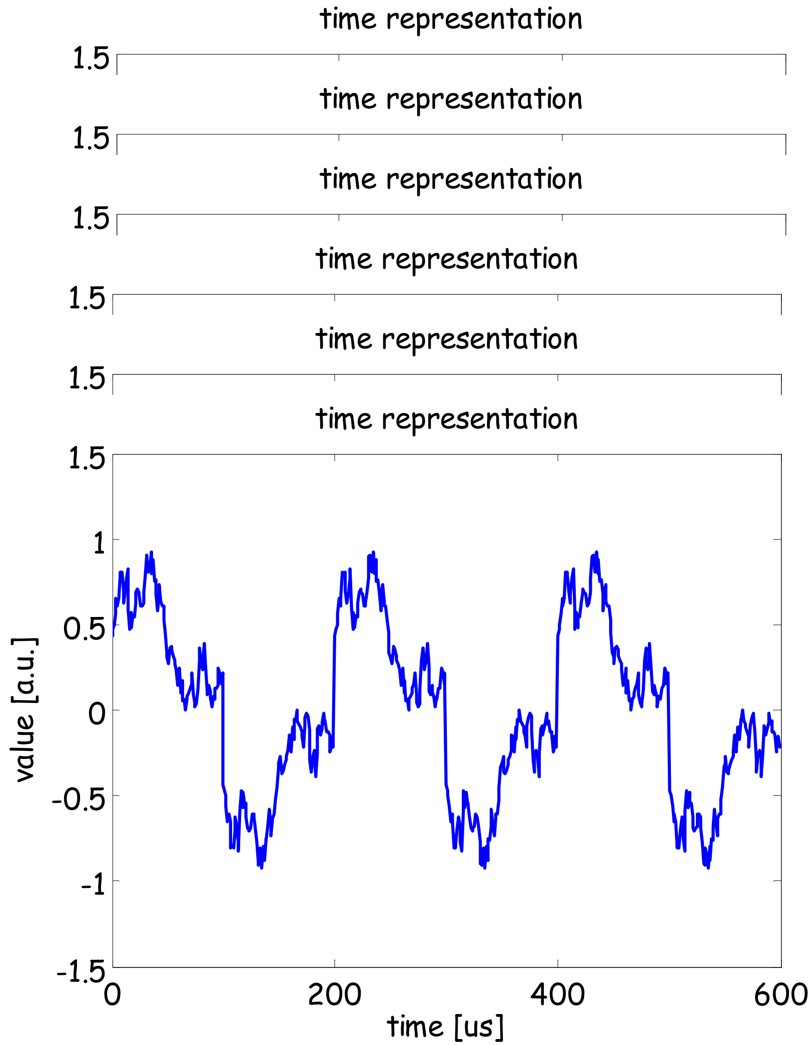


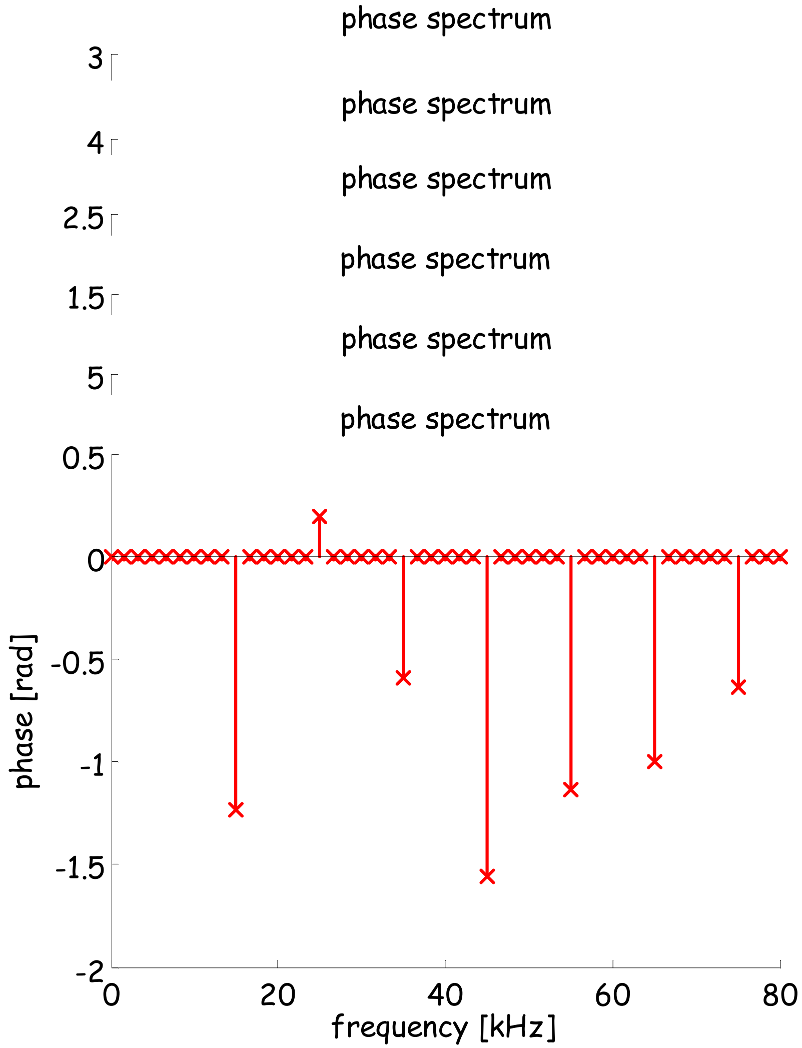
Poly-harmonic signal - cont.





Poly-harmonic signal – cont.





spectrum has two components – amplitude and phase

Basic paramiters

average – AVG (mean value)

periodic signal

$$y_{AV} = \frac{1}{T} \int_{t_0}^{t_0+T} y(t)dt$$

non-periodic signal
random signal

$$y_{AV} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{t_0}^{t_0+\tau} y(t)dt$$

root-mean-square - RMS

$$y_{RMS} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} y^2(t)dt}$$

$$y_{RMS} = \sqrt{\lim_{\tau \to \infty} \frac{1}{\tau} \int_{t_0}^{t_0+\tau} y^2(t)dt}$$

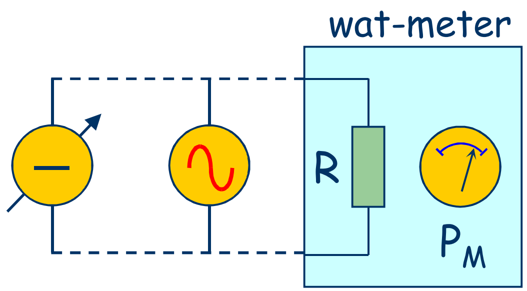
What is the meaning of a RMS value?

the value of constant voltage (or current) dissipating on a resistance R the same amount of electrical power as given AC signal

$$P_{AV} = \frac{1}{T} \int_{t_0}^{t_0+T} u(t) \cdot i(t)dt = \frac{1}{T} \int_{t_0}^{t_0+T} u(t) \cdot \frac{u(t)}{R} dt = \frac{1}{R} \boxed{\frac{1}{T} \int_{t_0}^{t_0+T} u^2(t)dt}$$

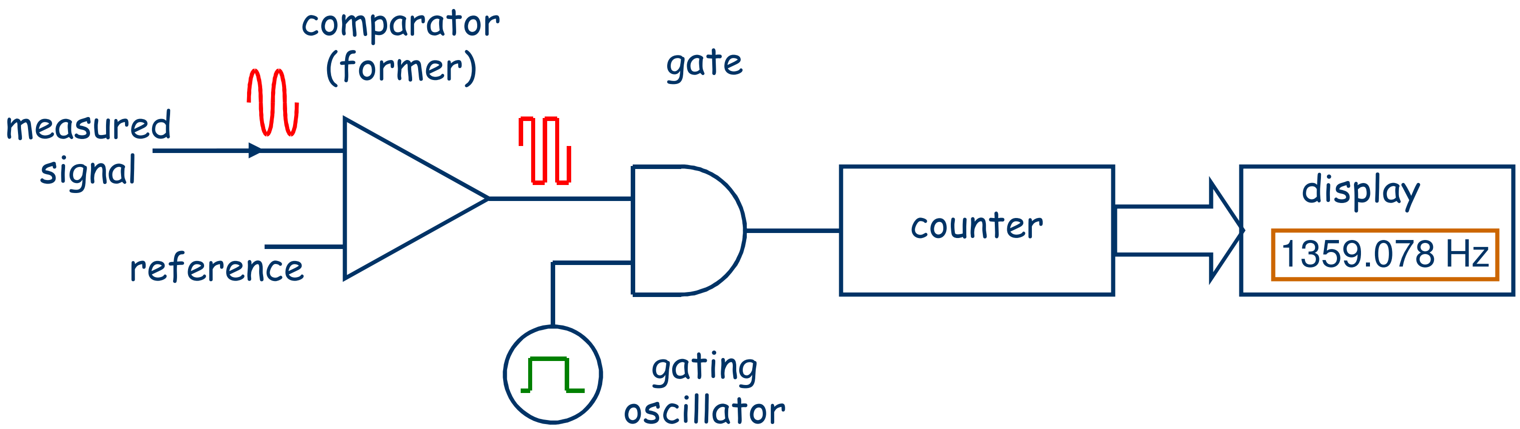
$$P_{AV} = \frac{U_{RMS}^2}{R} = R \cdot I_{RMS}^2; \quad U_{RMS} = \sqrt{R \cdot P_{AV}}$$

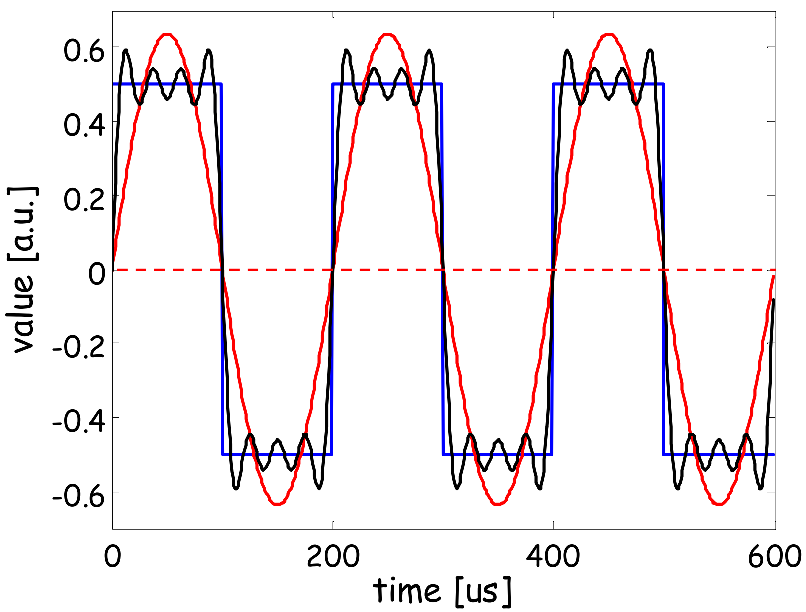
substitution measurement
of the RMS value

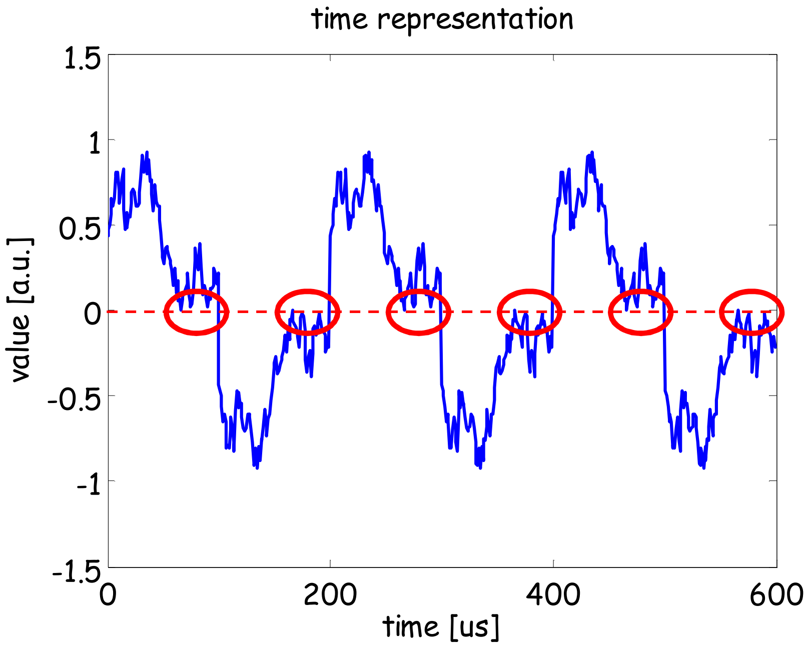


DC-component, AC-component, RMS-AC

Electronic frequency measurement







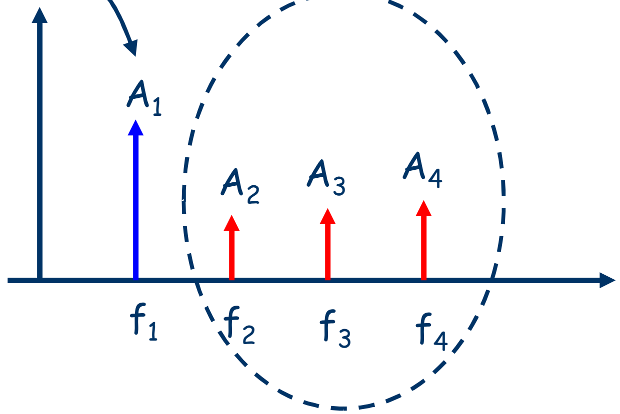
Basic parameters - cont.

harmonic distotrion

$$y(t) = \sum_n A_n \sin(2\pi \cdot f_n \cdot t + \varphi_n)$$

$$h = \sqrt{\frac{\sum_{n=2}^{\infty} A_{nRMS}^2}{\sum_{n=1}^{\infty} A_{nRMS}^2}} = \frac{\sqrt{\sum_{n=2}^{\infty} A_{nRMS}^2}}{y_{RMS}}$$

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} A_{nRMS}^2}}{A_{1RMS}} = \sqrt{\frac{\sum_{n=2}^{\infty} A_{nRMS}^2}{A_{1RMS}^2}}$$

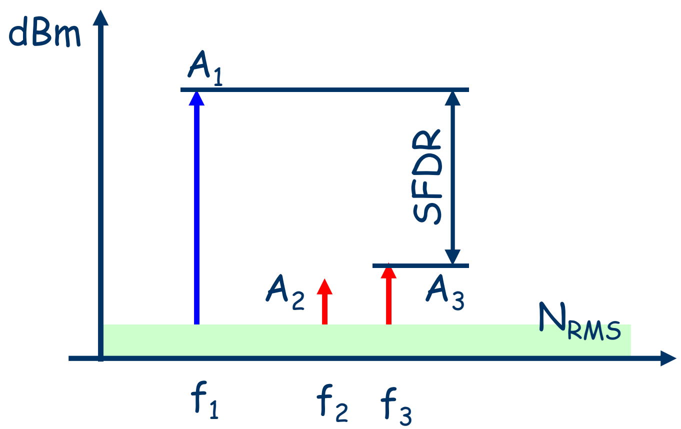


extention for non-harmonic signals:

Total Harmonic Distortion + Noise THD+N

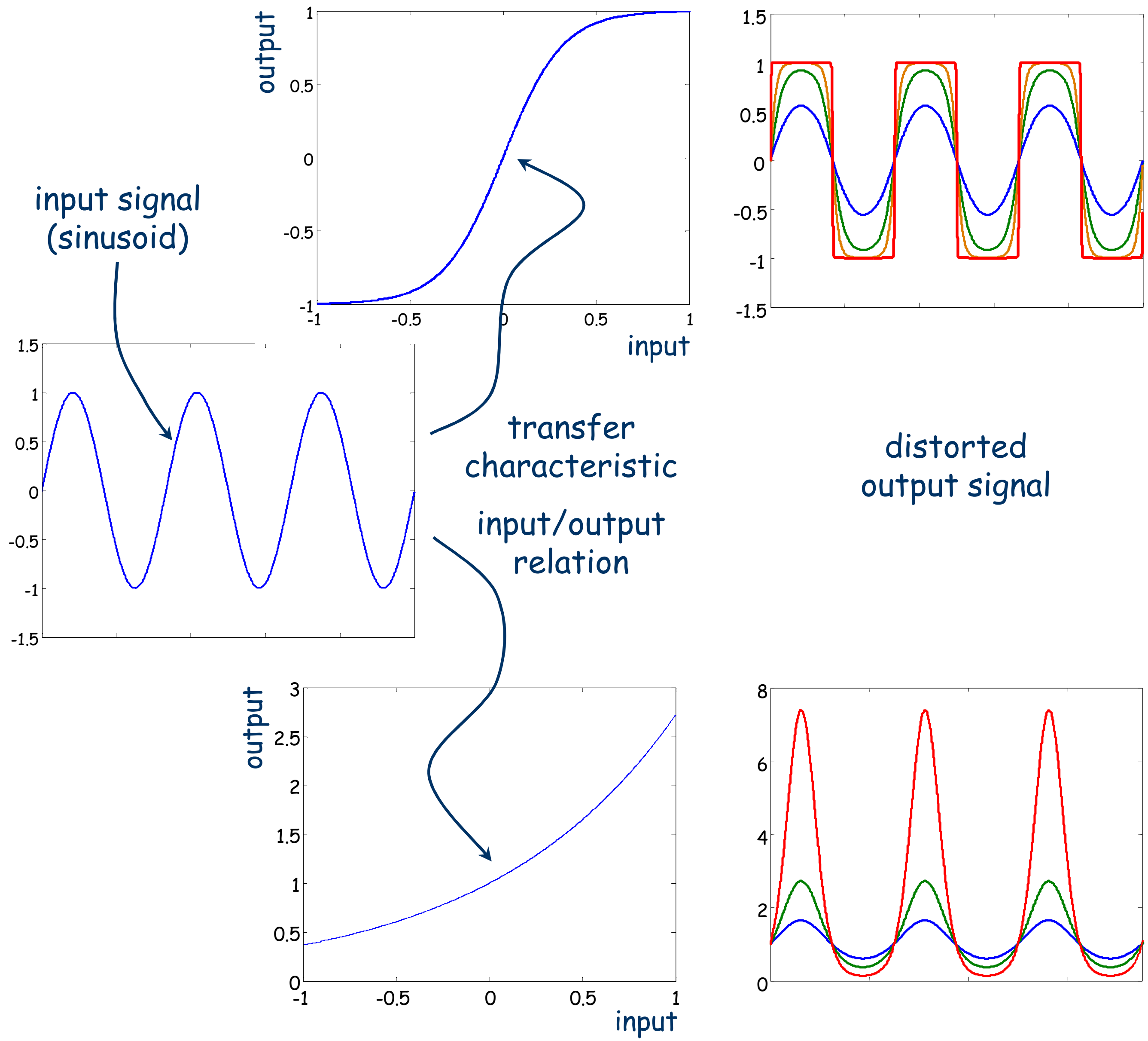
Spurious-Free Dynamic Range SFRD

$$THD + N = \sqrt{\frac{\sum_{n=2}^{\infty} A_{nRMS}^2 + N_{RMS}^2}{A_{1RMS}^2}}$$



$$SFDR[\text{dBc}] = A_1[\text{dBm}] - A_3[\text{dBm}]$$

Non-linear distortion



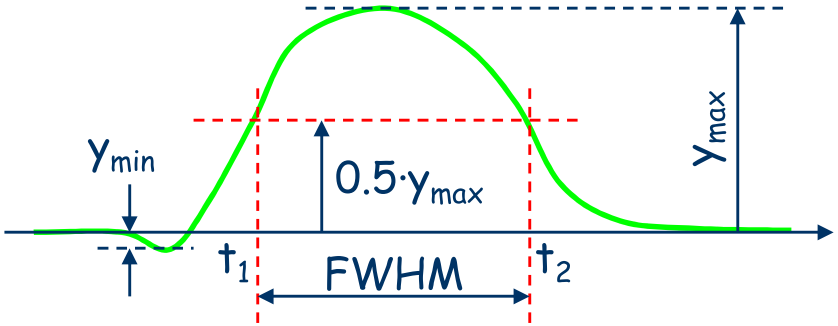
k	h [%]	THD [%]
0.2	3	3
0.5	13	13
1	25	26
10	42	46

$$y = \tanh(k \cdot \pi \cdot x)$$

k	h [%]	THD [%]
0.5	12	13
1	24	24
2	41	45

$$y = \exp(k \cdot x)$$

Pulsed waveforms

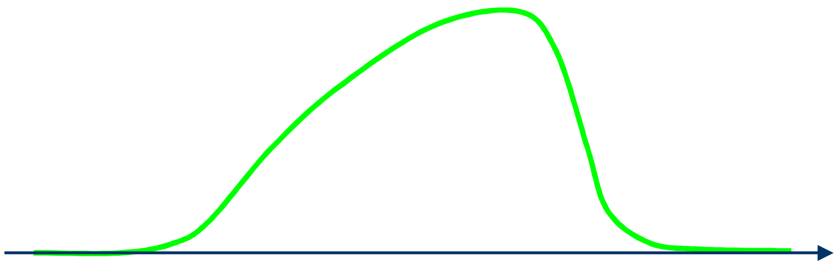


„charge"

$$Q = \int i(t)\,dt$$

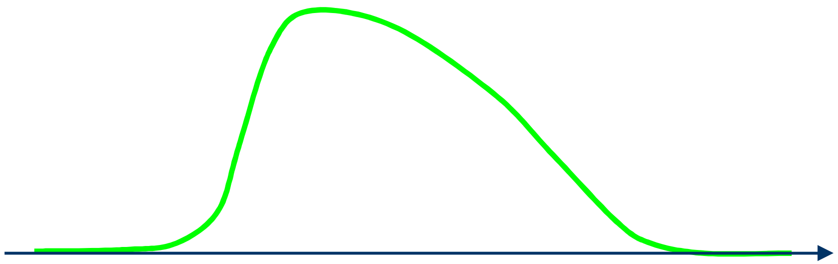
„energy"

$$E = \int y^2(t)\,dt$$



peak value

$$y_{max} = \max|y(t)|$$



peak-peak value

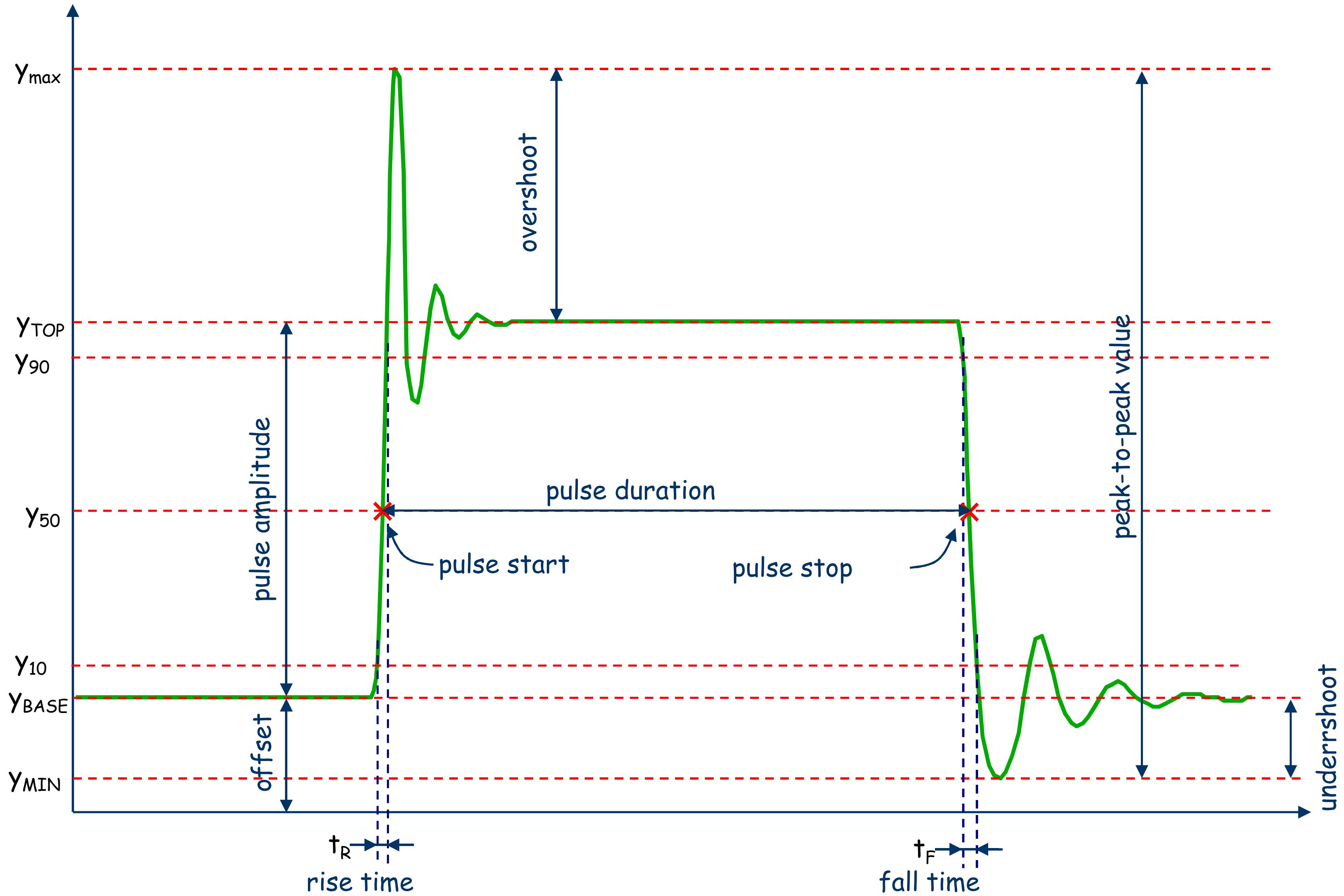
$$y_{p-p} = \max y(t) - \min y(t)$$

FWHM

Full Width at Half Maximum

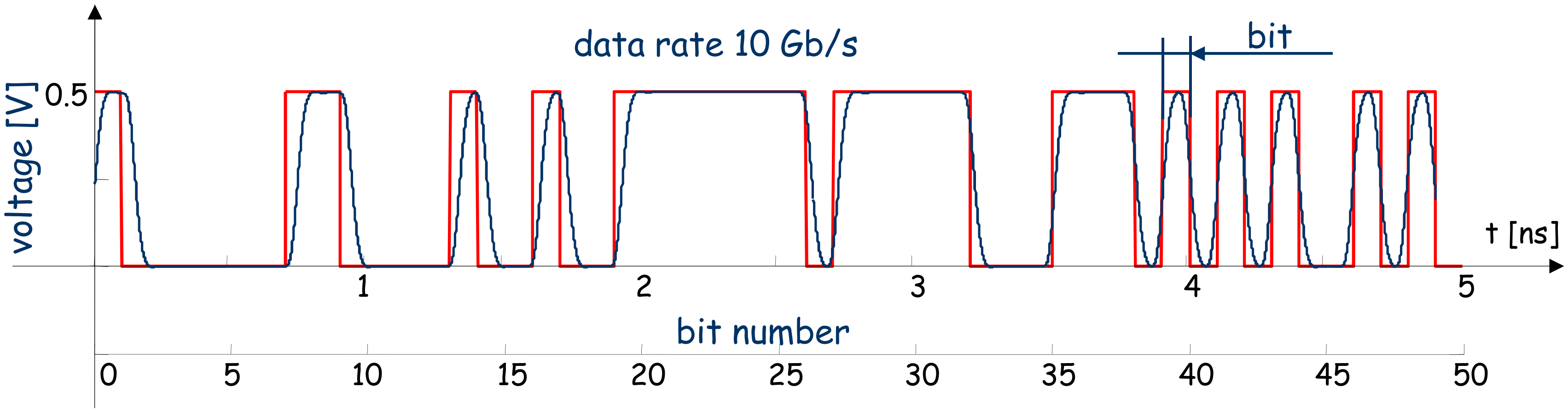
$$t_{FWHM} = t_2 - t_1 \Big|_{y(t_1)=y(t_2)=0.5\,y_{max}}$$

Pulsed waveforms (ANSI/IEEE 194-1977)

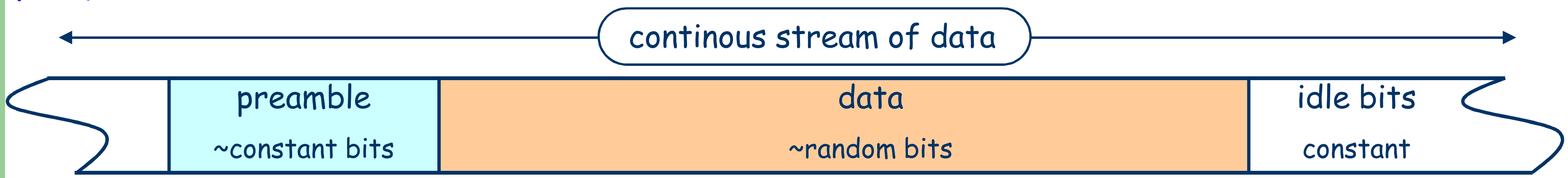


Digital telecommunication signal

binary synchronous signal



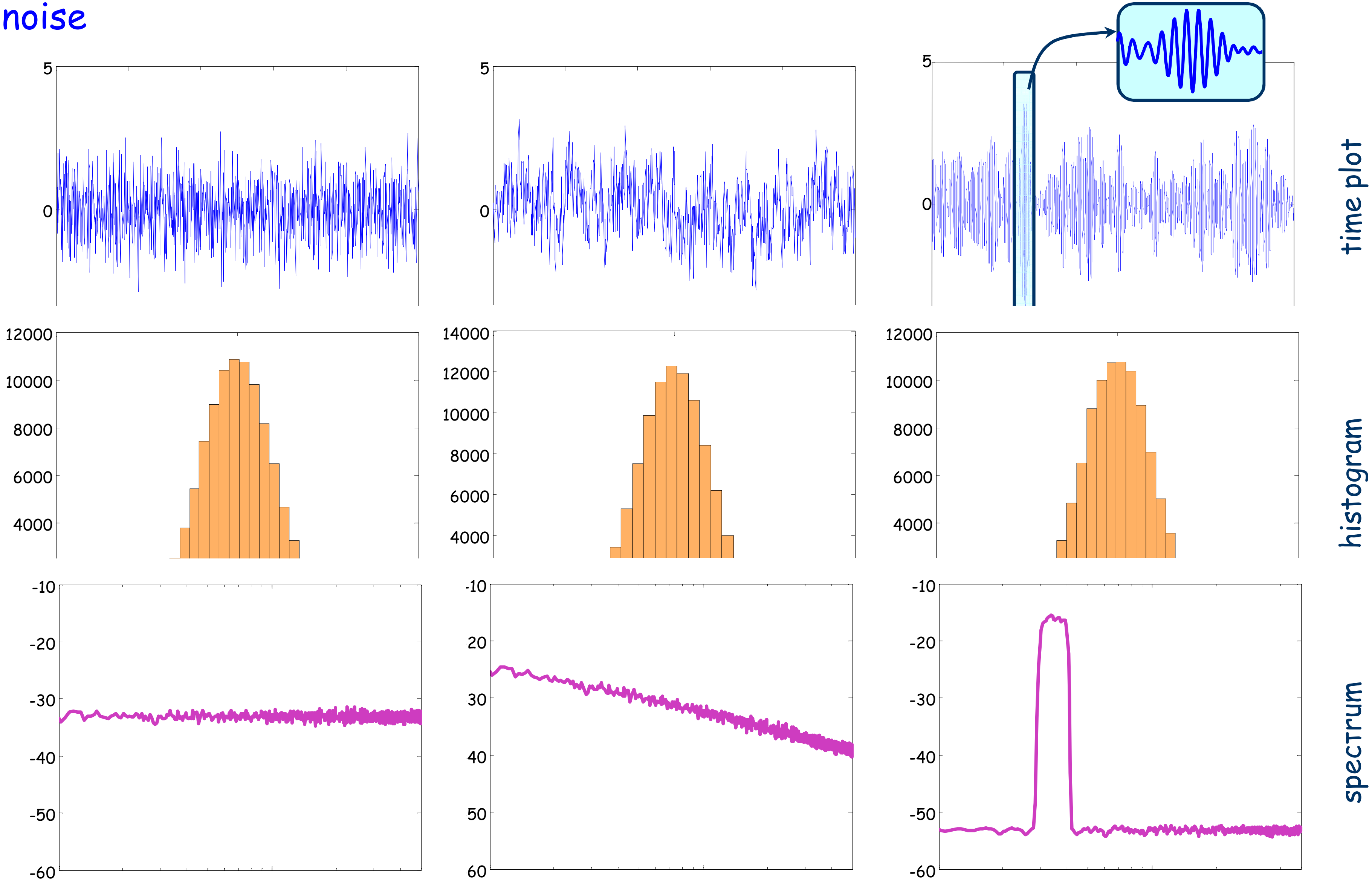
frame



binary asynchronous signal

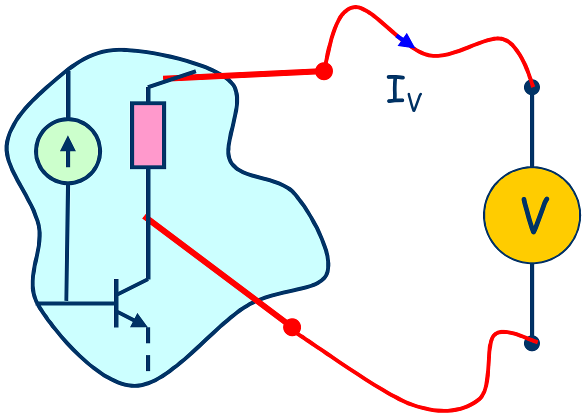
Random signals

noise



Measurement primer – voltage

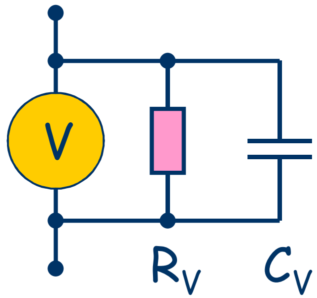
ideal voltmeter



$I_V = 0 \Rightarrow R_V = \infty$

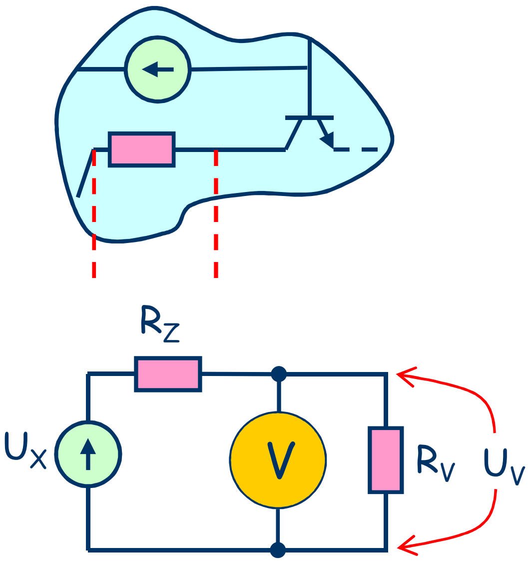
$Z_V = \infty$

real voltmeter



R_V
~ 10 MΩ - 10G Ω
C_V
~ 100 pF – 1 nF

voltage divider effect



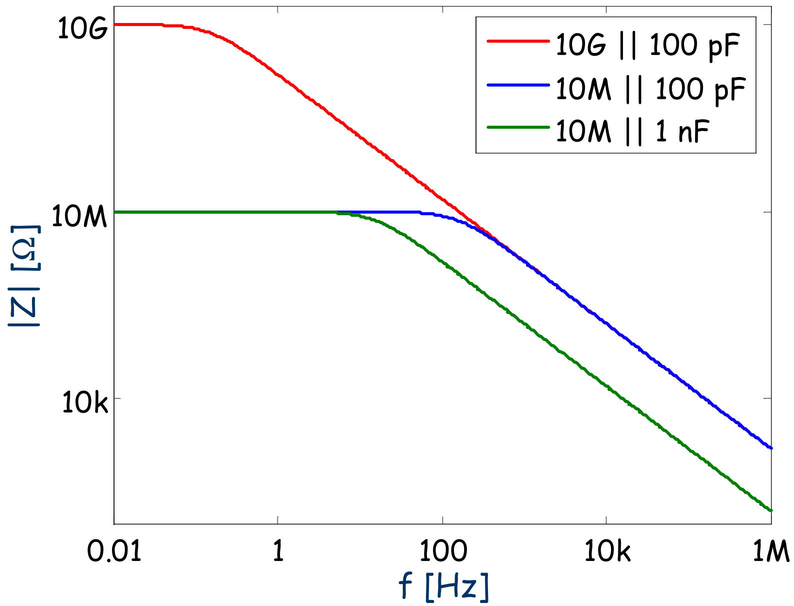
$$U_V = U_X \frac{R_V}{R_Z + R_V}$$

$$\frac{\Delta U_X}{U_X} = \frac{R_Z}{R_Z + R_V}$$

$$\frac{\Delta U_X}{U_X} < 1\% \Rightarrow$$

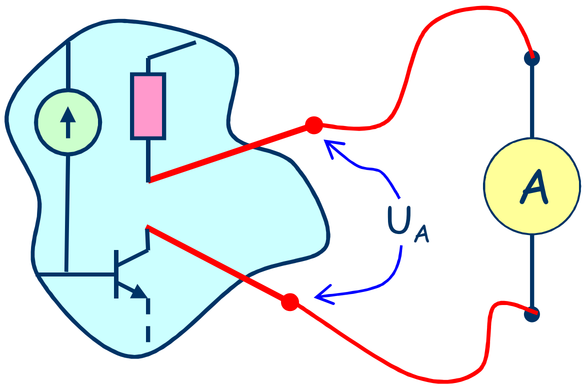
$$R_V > 100 \cdot R_Z$$

impedance of a voltmeter



Measurement primer – current

ideal ammeter

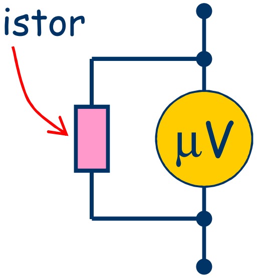


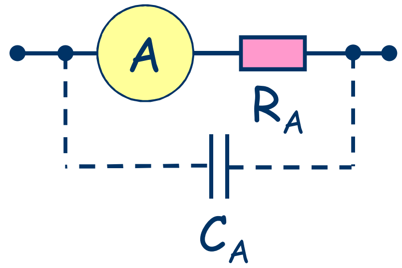
$U_A = 0 \Rightarrow R_A = 0$

$Z_V = 0$

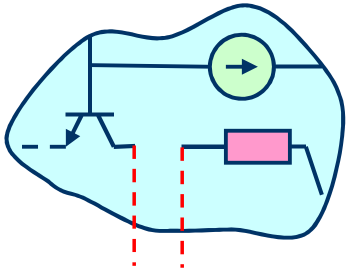
real ammeter

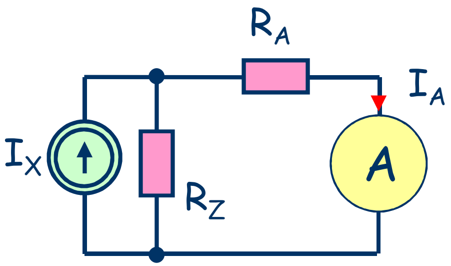
shunt resistor





current divider effect





$$I_A = I_X \frac{R_Z}{R_Z + R_A}$$

$$\frac{\Delta I_X}{I_X} = \frac{R_A}{R_Z + R_V}$$

$$\frac{\Delta I_X}{I_X} < 1\% \Rightarrow$$

$$R_A < 0.01 \cdot R_Z$$

R_A

0-400 μA:	1 mV/μA
0-400 mA:	1 mV/mA
0-20 A:	10 mV/A

0-5 mA:	100 Ω
0-500 mA:	1 Ω
0-10 A:	0.01Ω

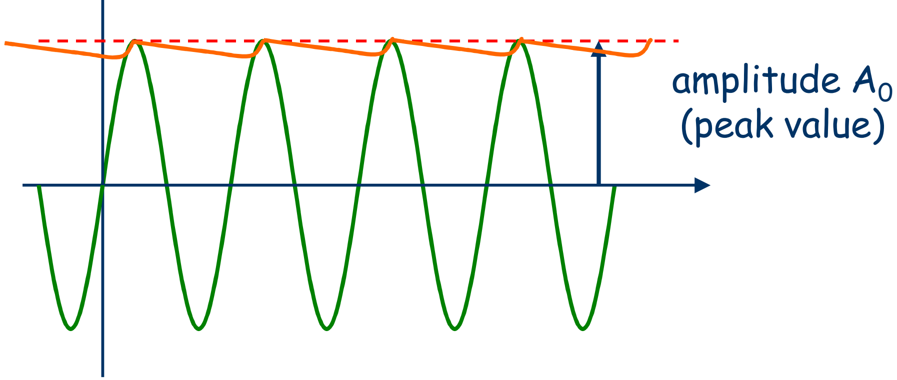
C_A

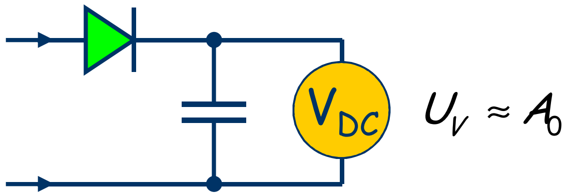
~ ???

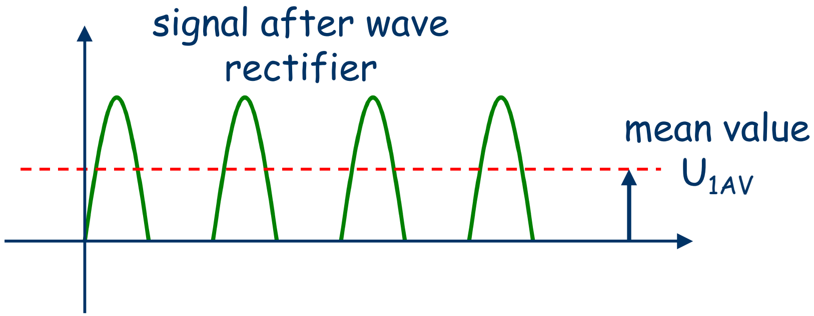
Measurement primer – AC voltages/currents

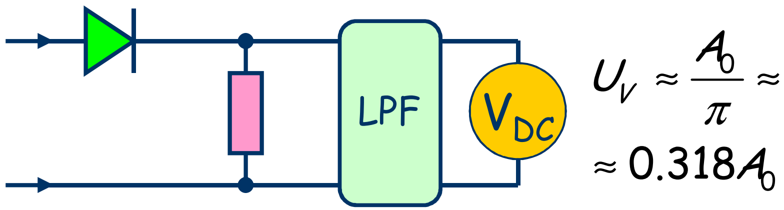
acuuracy af DC measurements is much greater than accuracy of AC measurements

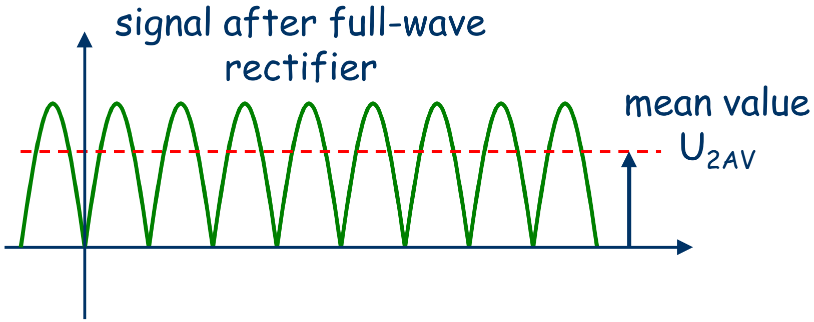
$\Rightarrow$ AC $\rightarrow$ DC conversion

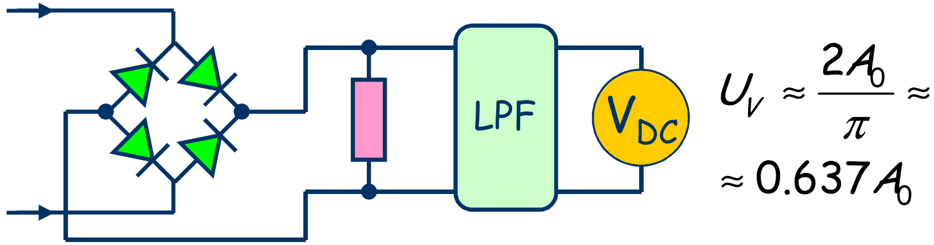












Measurement primer – AC voltages/currents

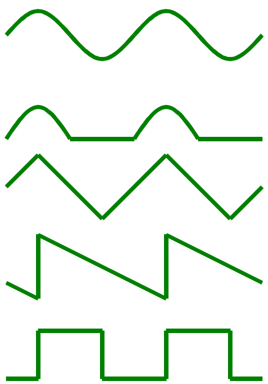
I would like to measure the RMS value ???

waveform factor

$$k_W = \frac{y_{RMS}}{y_{1AV}}$$

crest factor

$$k_C = \frac{y_{max}}{y_{RMS}}$$



	k_W	k_C
sinusoid	1.111	1.414
rectified	1.571	2
triangle	1.155	1.732
sawtooth	1.155	1.732
square wave	1	1

sinusoid

$$U_{RMS}^{sin} = k_W \cdot U_{1AV} \approx 1.11 \cdot U_{1AV}$$

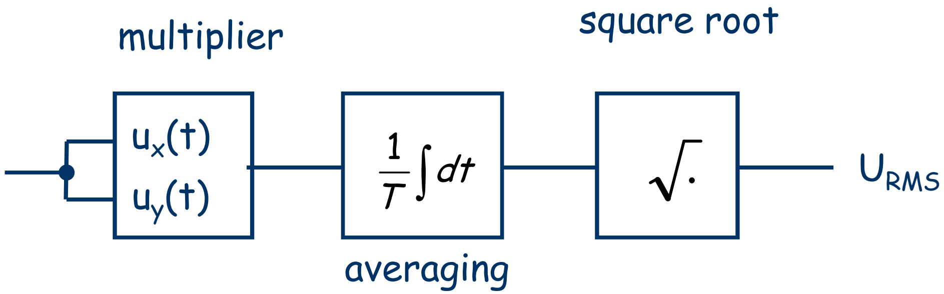
distorted signal

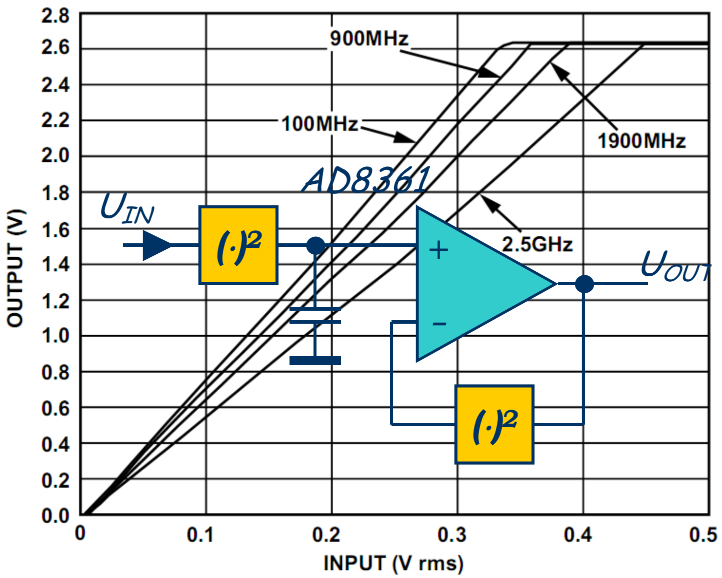
$$U_{RMS}^{d} = \left(U_{RMS}^{sin} / k_W^{sin} \right) \cdot k_W^{d}$$

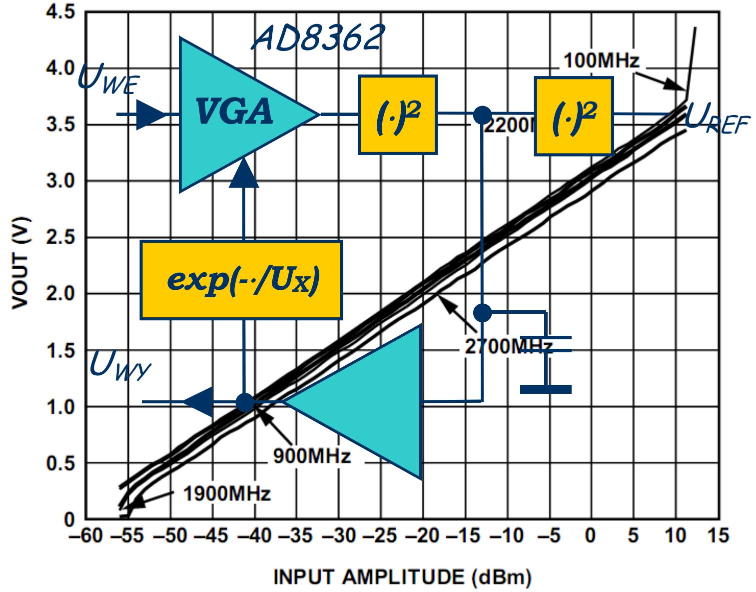
RMS value measurement

„exact" approach –True RMS Detector

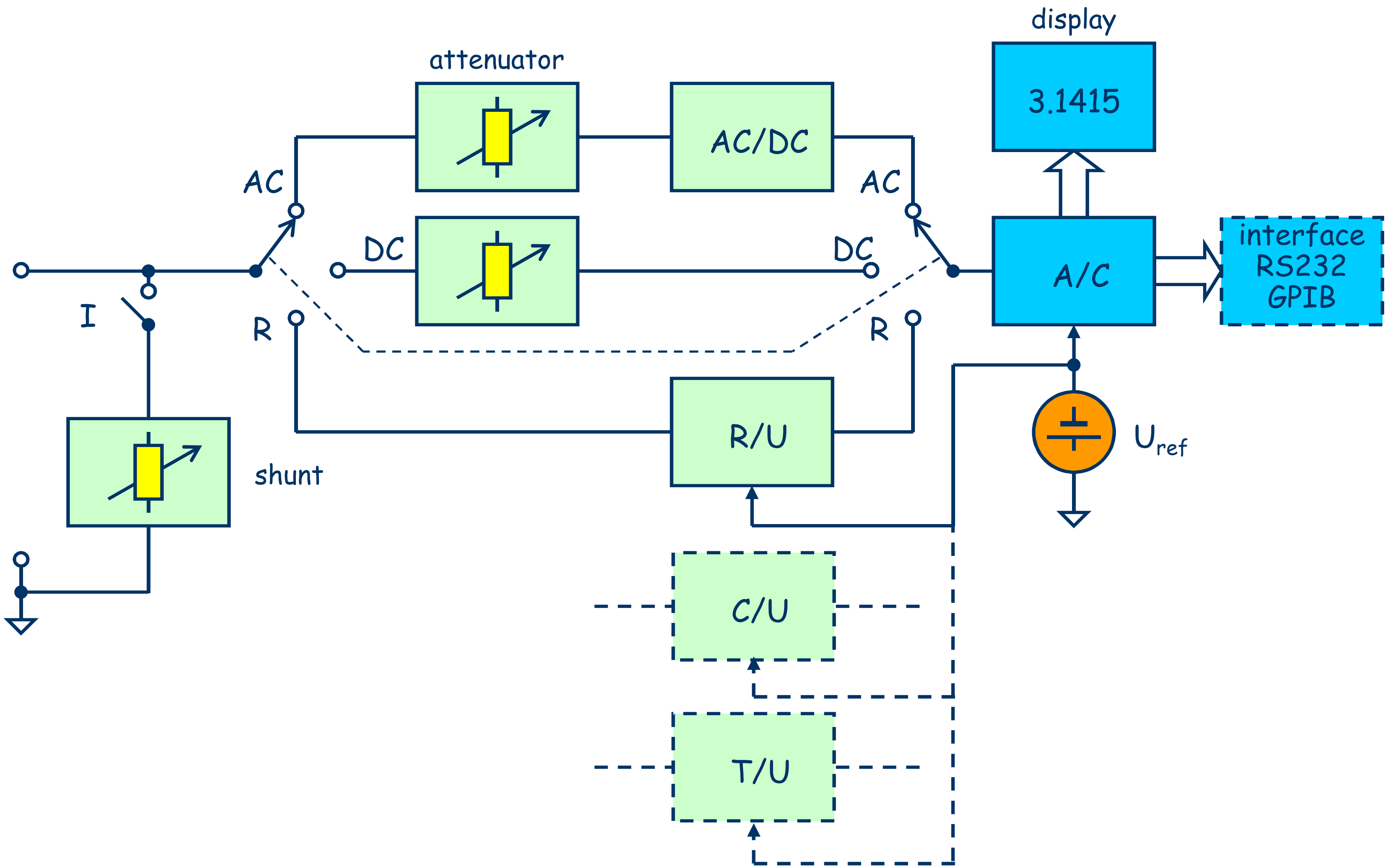
$$y_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} y^2(t)\,dt}$$





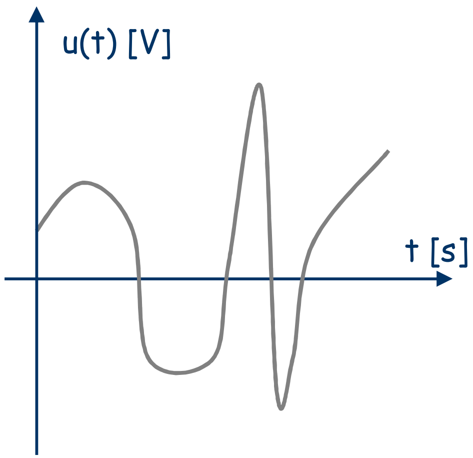


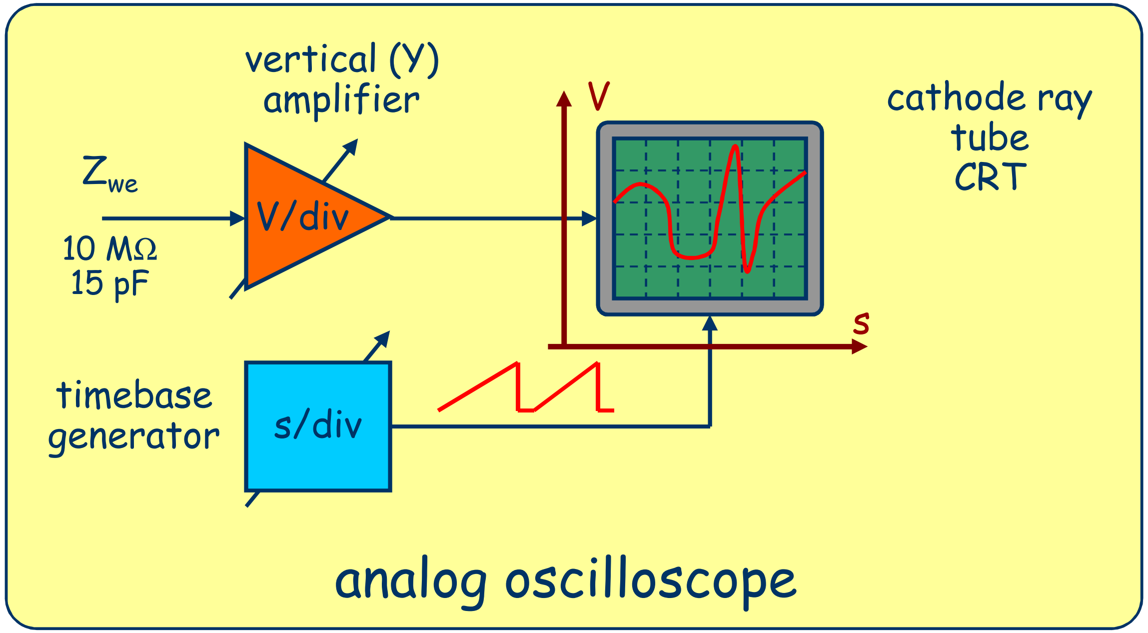
Primer – multimeter (digital – DMM)

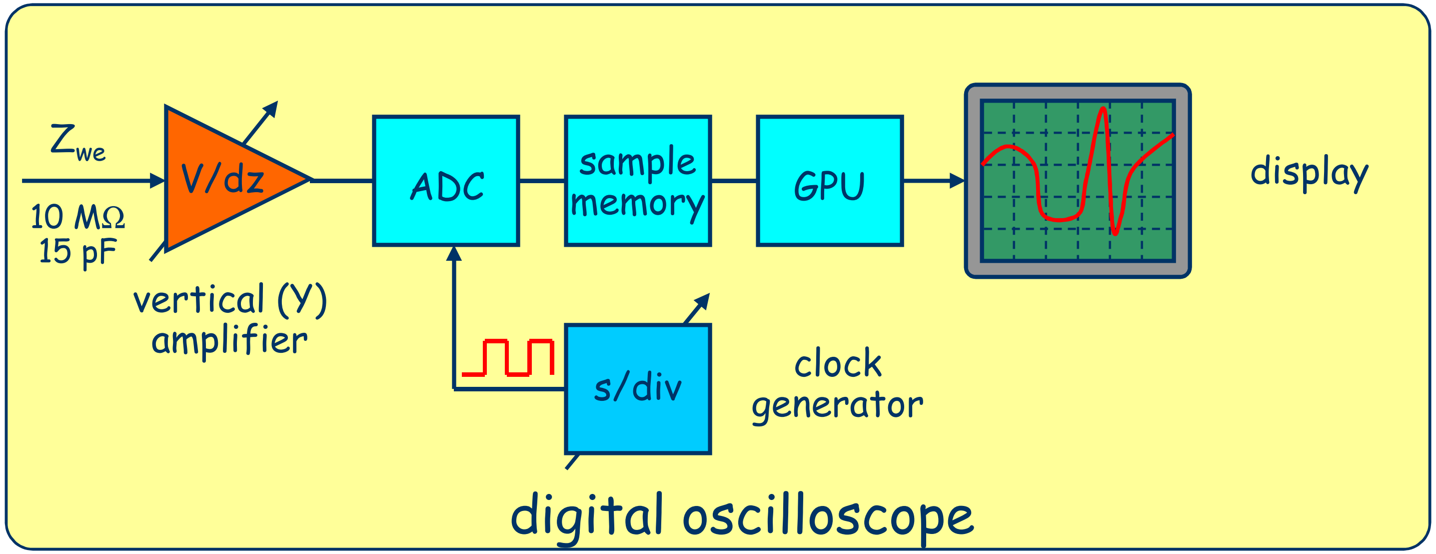


Primer – time parameters

oscilloscope (scope)







Primer – spectrum measurements