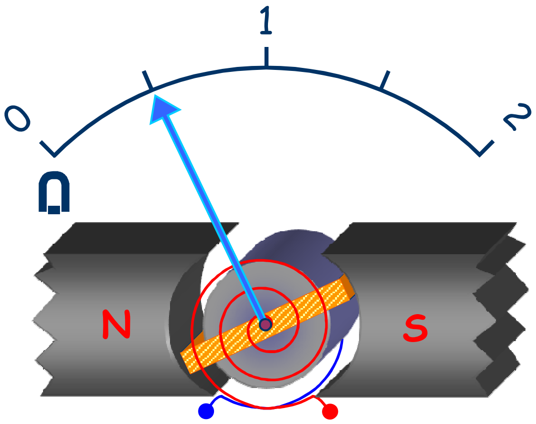
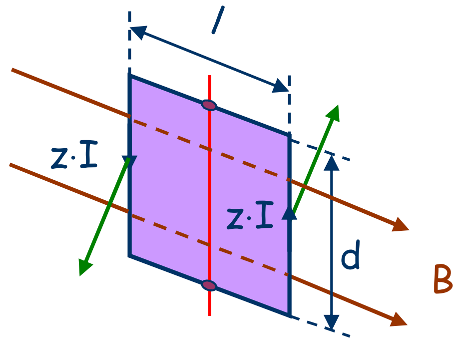
Digital measurements and digital instruments

Analog instruments – some history

Moving-coil ammeter

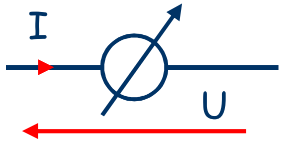




$$M_I = B \cdot I \cdot z \cdot d \cdot l$$

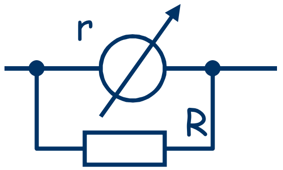
$$M_S = k \cdot \alpha$$

$$M_I = M_S \Rightarrow \alpha = \frac{B \cdot z \cdot d \cdot l}{k} I = c \cdot I$$



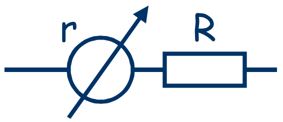
$$U = r \cdot I$$

range expansion coefficient



ammeter

$$1 + \frac{r}{R}$$



voltmeter

$$1 + \frac{R}{r}$$

Types of digital instruments

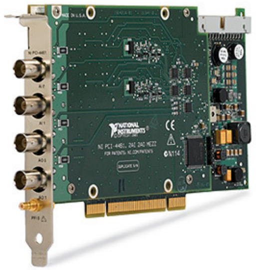
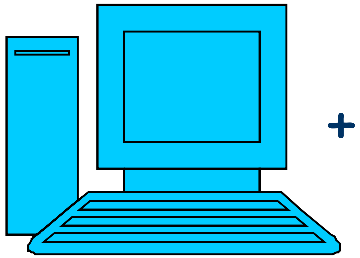
1. „Traditiona" instruments

    bench-top and hand-held multimeters
    R, L, C, Z, Q – meters
    frequency meters
    counters



2. Computer cards

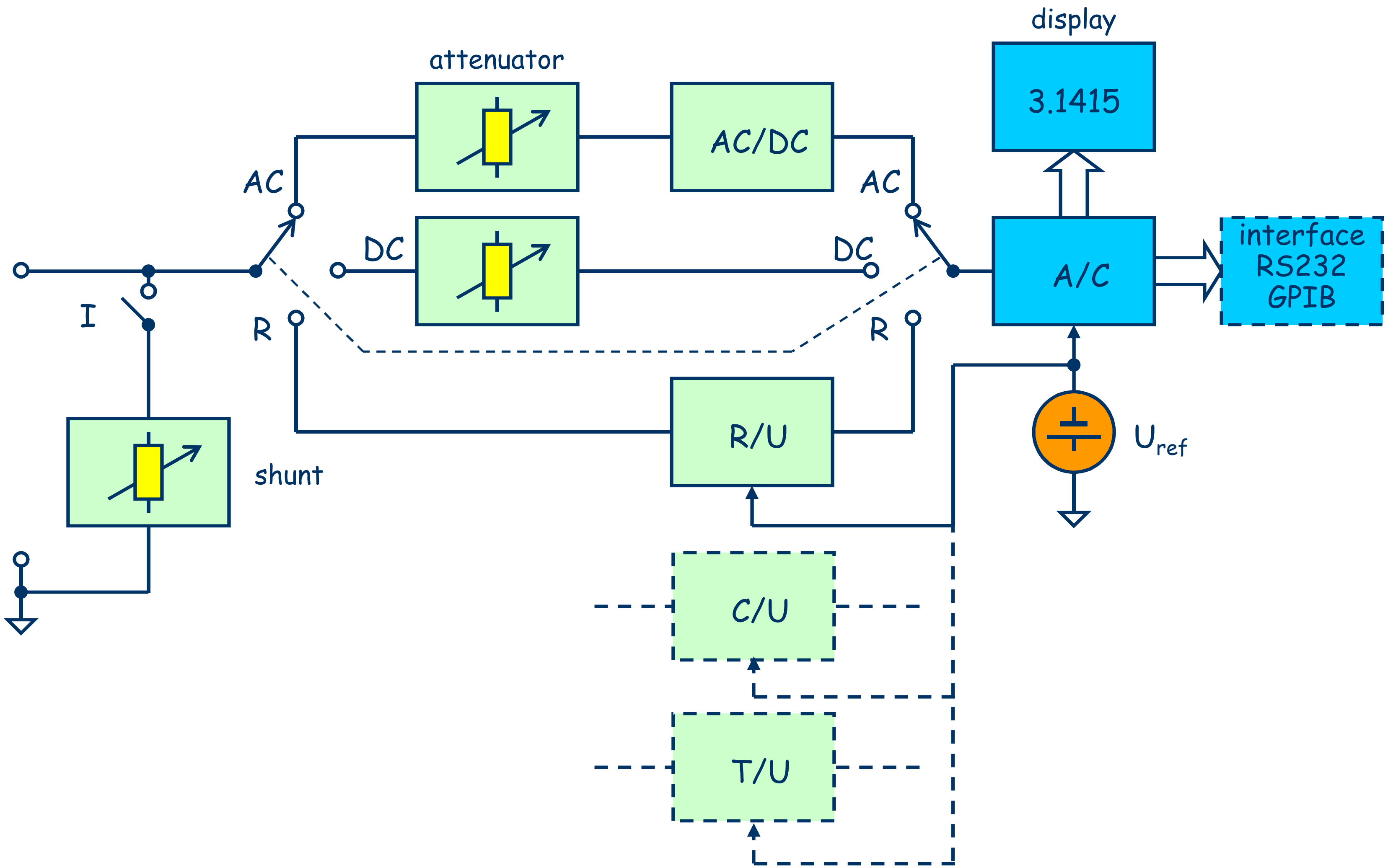
    electrical signals
    time, frequency
    non-electrical quantities

 +  + software

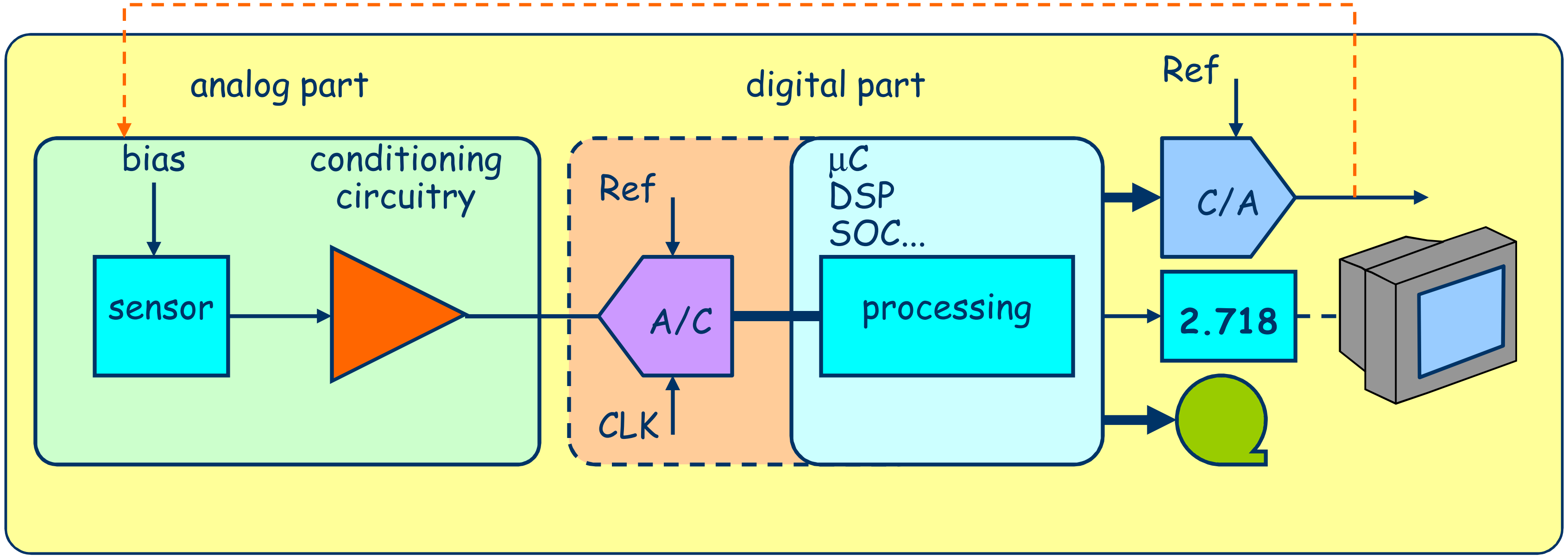
3. Virtual Instruments

    computer processeing and visualization of data

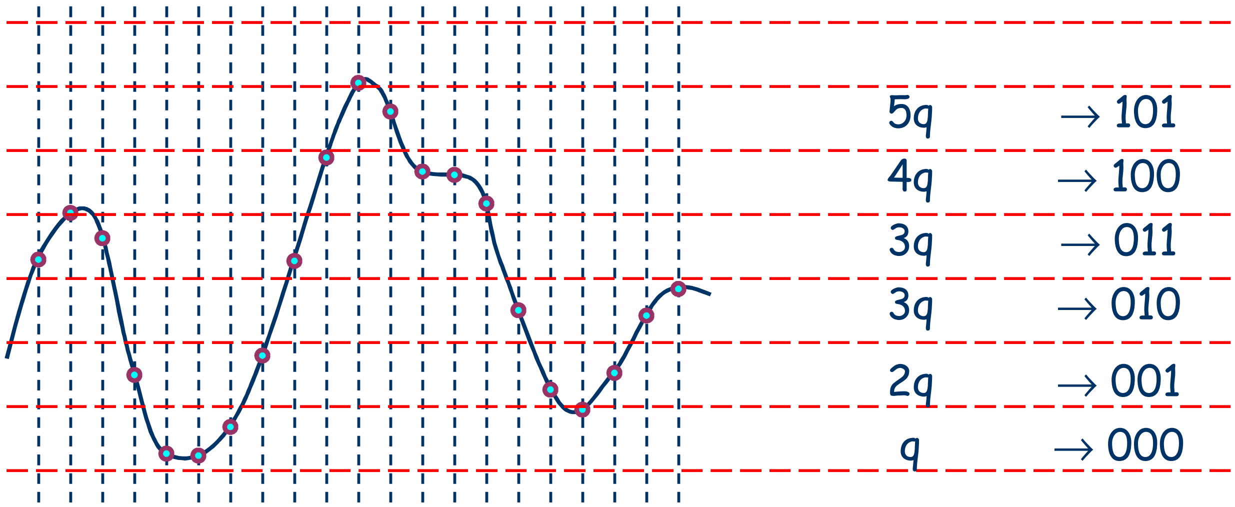
Digital multimeter (DMM)



Digital measurement chain



Ideal Analog-to-Digital Converter (ADC)

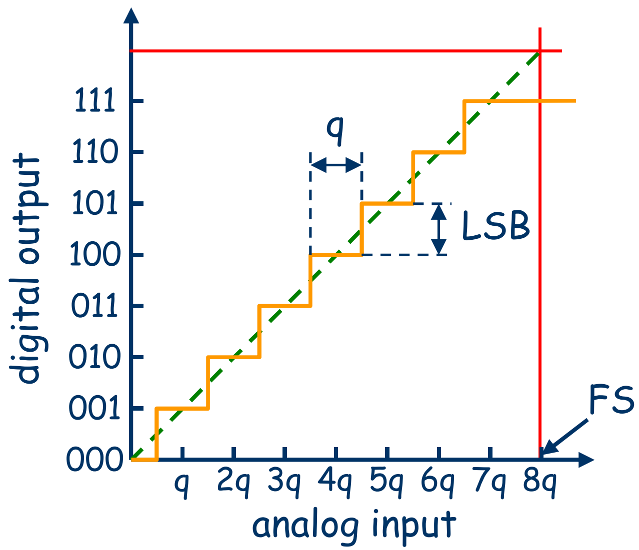


natural binary code
codes with sign
BCD
Gray code

sampling
(constant rate)

quantization

coding



$FS = q \cdot 2^N$

N	2^N	q (FS=1.024V)	q [dB FS]
8	256	4 mV	-48
10	1024	1 mV	-60
12	4096	250 μV	-72
16	65536	15.625 μV	-96
24	16777216	61 nV	-144

Basic codes

Natural binary code

$$a_{N-1}a_{N-2}\ldots a_1 a_0 \in \{0,1\}: L = \sum_{i=0}^{N-1} a_i \cdot 2^i$$

000000 → 0 (e.g. 0)
11111111 → 2^{N}-1 (e.g. 255)

Two's complement code

$$a_{N-1}a_{N-2}\ldots a_1 a_0 \in \{0,1\}: L = -a_{N-1} \cdot 2^{N-1} + \sum_{i=0}^{N-2} a_i \cdot 2^i$$

100000 → -2^{N-1} (e.g. -128)
01111111 → 2^{N-1}-1 (e.g. 127)

Binary Coded Decimal (BCD) code

D_3 | D_2 | D_1 | D_0

$b_3b_2b_1b_0$ | $b_3b_2b_1b_0$ | $b_3b_2b_1b_0$ | $b_3b_2b_1b_0$

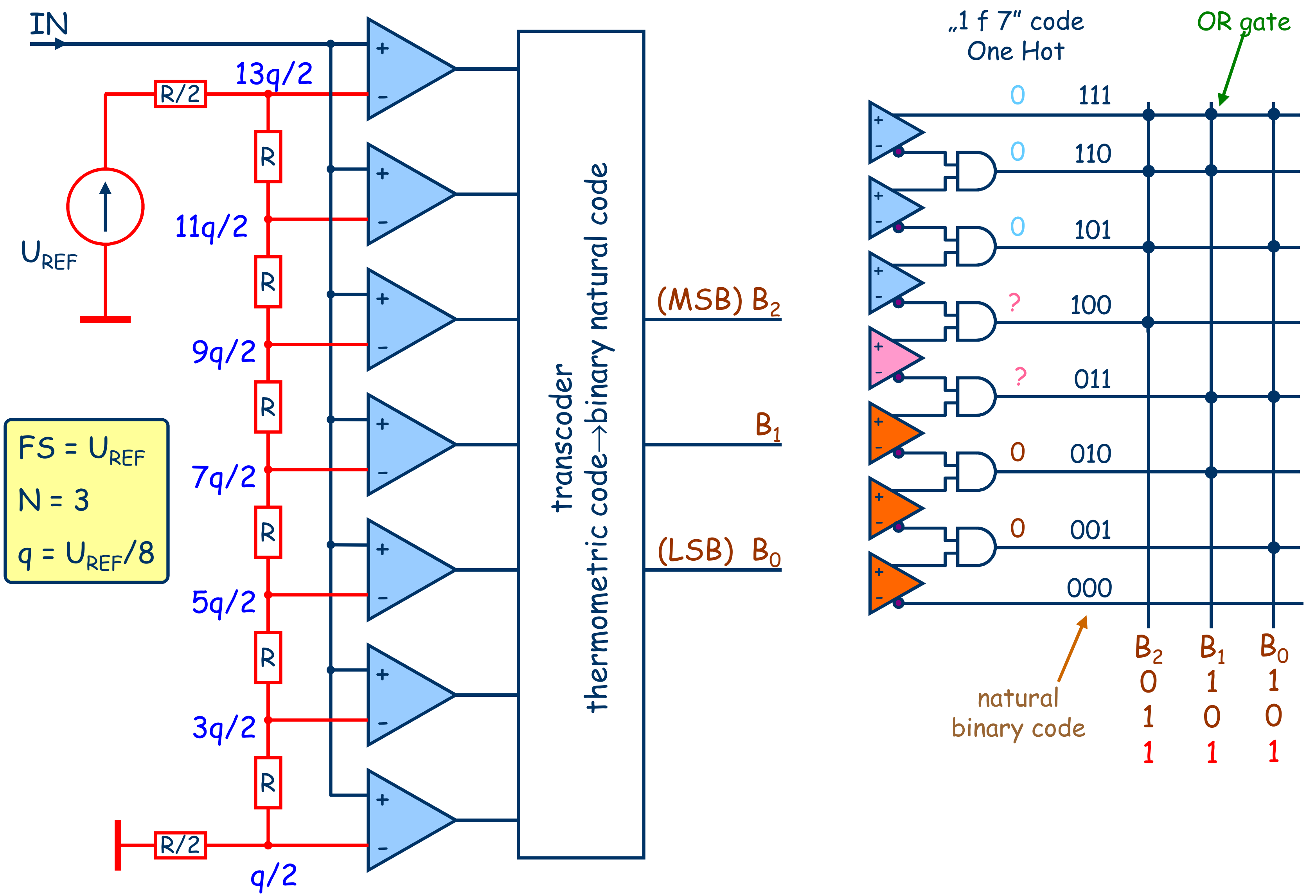
$$L = \sum_{j=-k}^{l} 10^j \sum_{i=0}^{3} a_{ij} \cdot 2^i$$

3.5 digits: ±1999 ; ±3999
$3\frac{3}{4}$ digits: ±8999
4.5 digits: ±51000
5.5 digits: ±119999

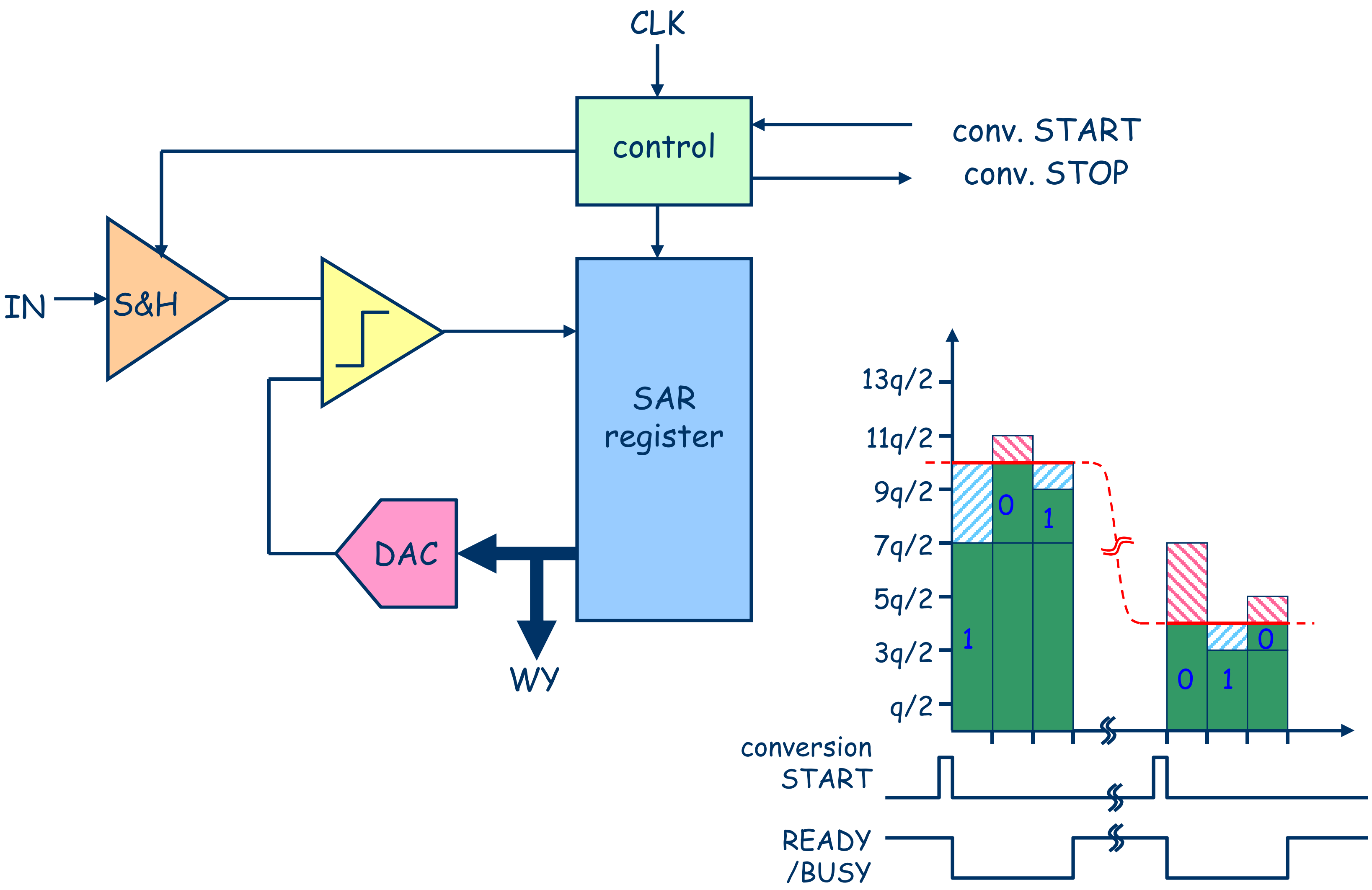
Gray code

0	00000
1	00001
2	00011
3	00010
4	00110
5	00111
6	00101
7	00100
8	01100
9	01101
10	01111
11	01110
12	01010
13	01011
14	01001
15	01000
16	11000

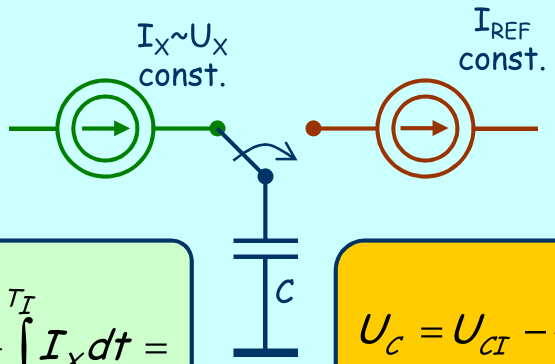
An example ADC: Flash Converter



Succesive Approximation (SAR)

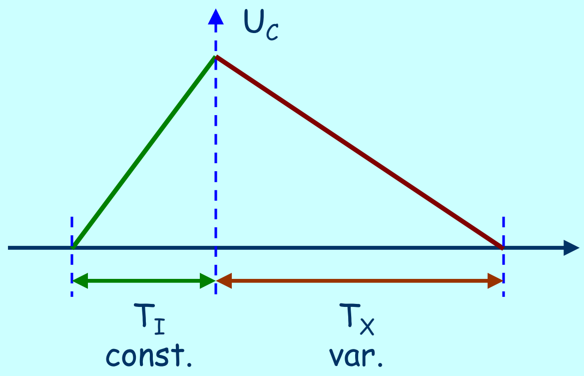


Dual Slope Converter



$$U_{CI} = \frac{1}{C}\int_0^{T_I} I_X dt = \frac{I_X \cdot T_I}{C}$$

$$U_c = U_{CI} - \frac{1}{C}\int_0^{T_X} I_{REF} dt = U_{CI} - \frac{I_{REF} \cdot T_X}{C}$$

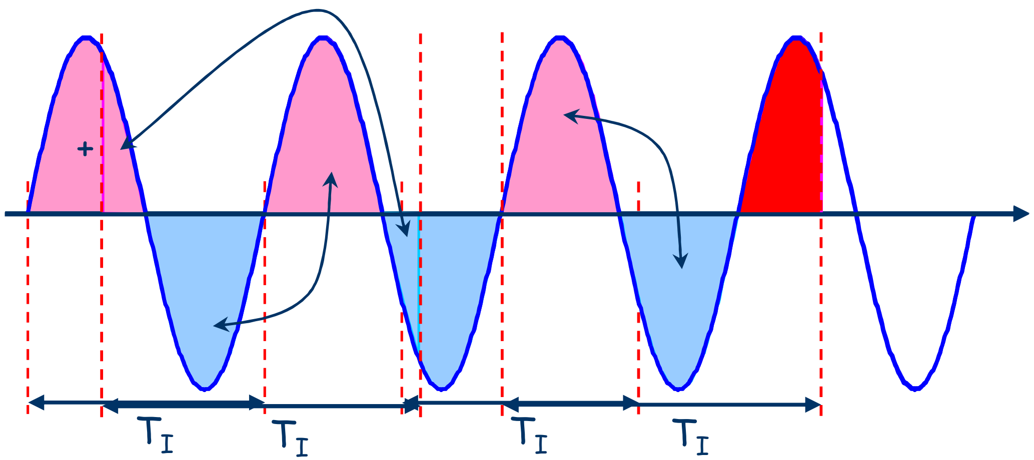


$$U_c = 0 \Rightarrow I_X = I_{REF}\frac{T_X}{T_I}$$

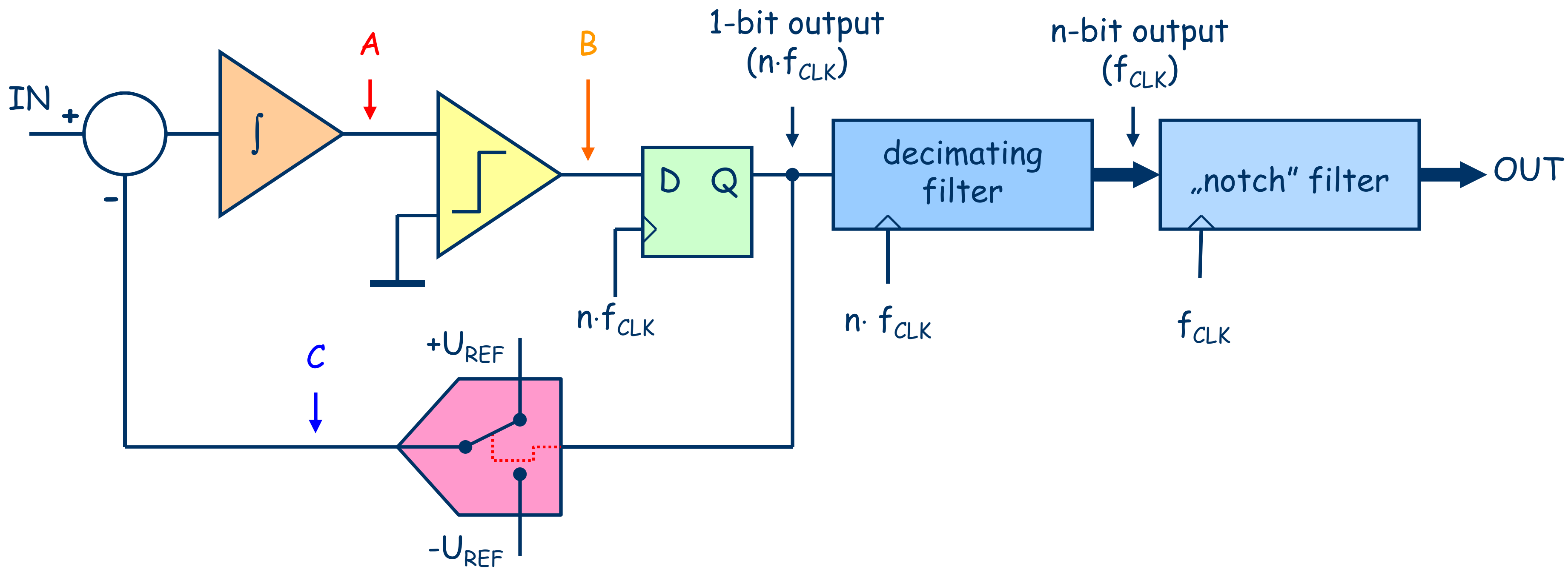
Noise immunity (suppression):

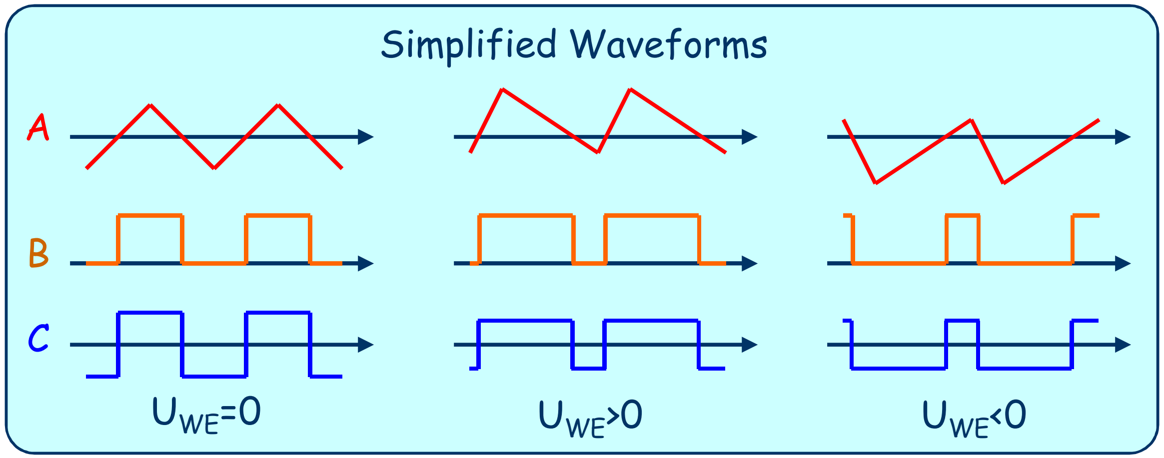
$$\int_0^{T_I} \sin(\omega t) dt = \frac{\cos(\omega T_I) - 1}{\omega}$$

$$\omega = \frac{\pi/2 + n \cdot 2\pi}{T_I} \Rightarrow \int_0^{T_I} \ldots = 0$$

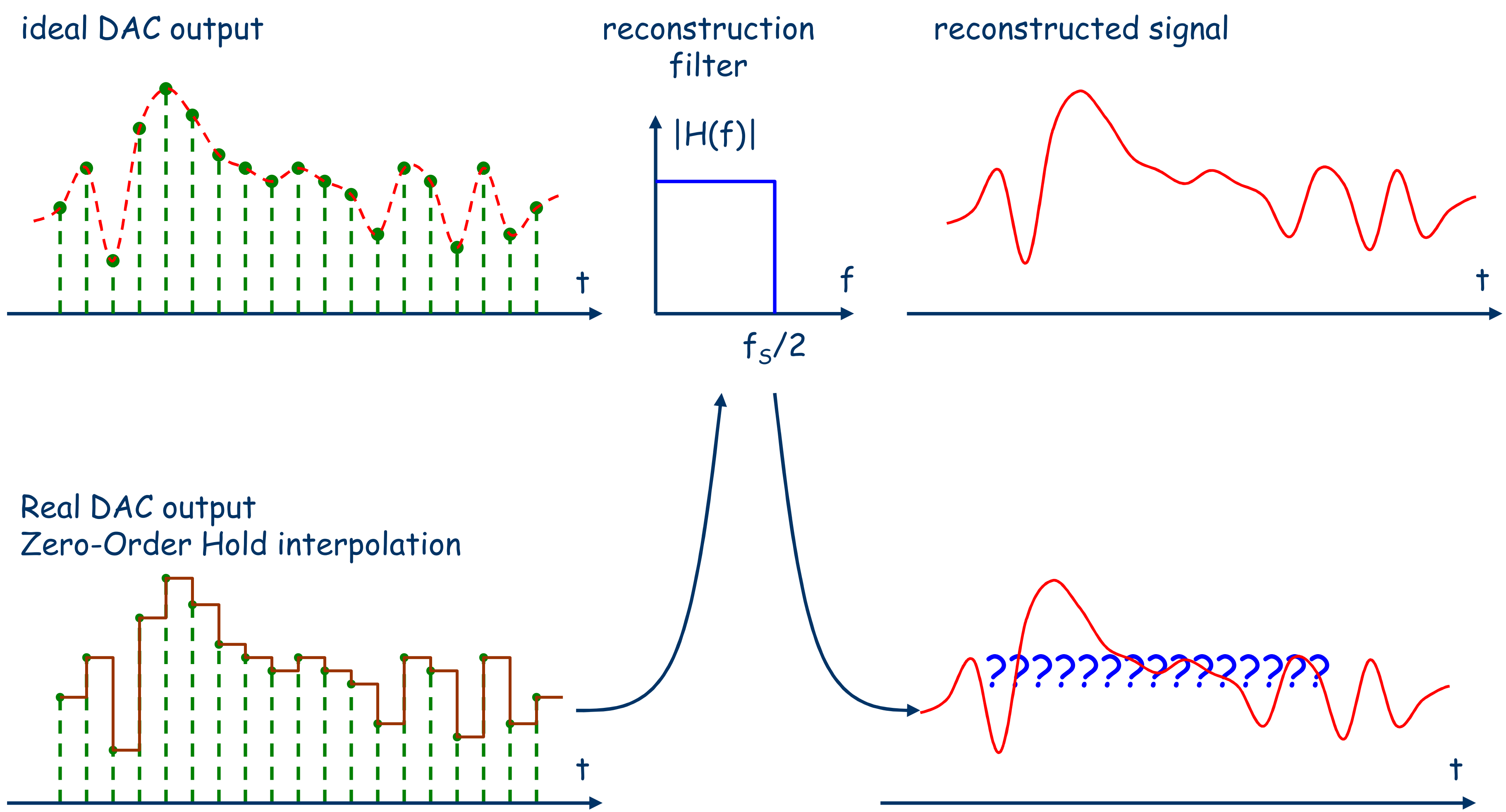


Σ-δ (sigma-delta) Converter



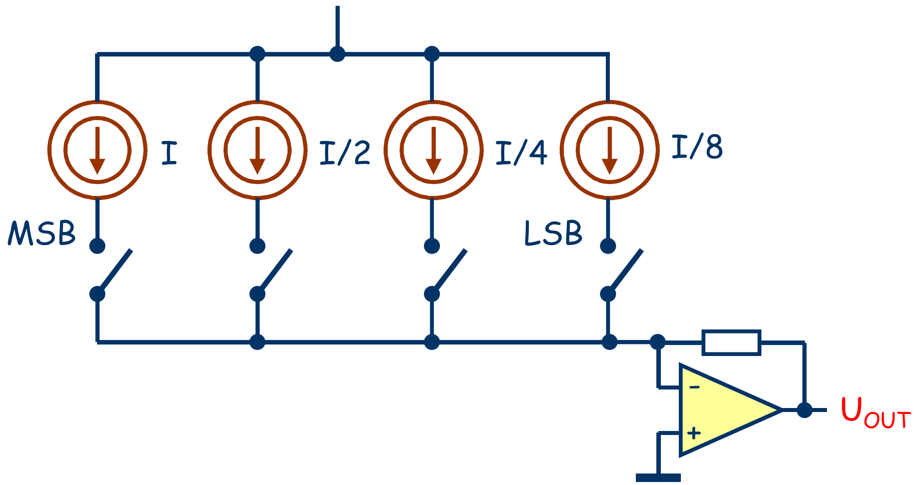


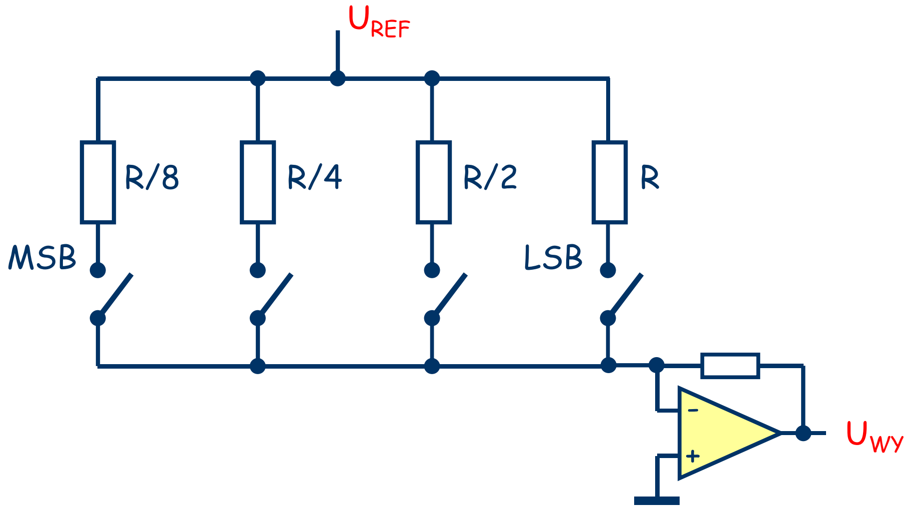
Digital-to-Analog Conversion (DAC)



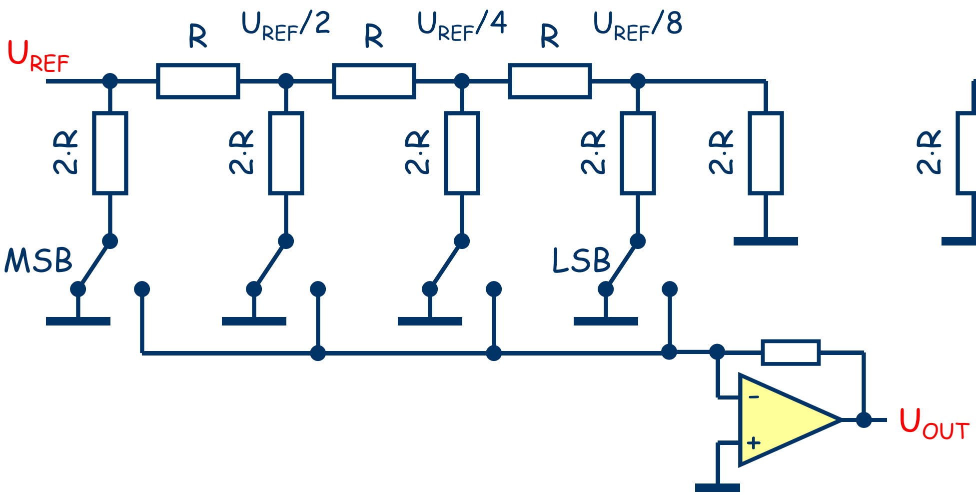
Basic DACs

Binary-weighted DAC

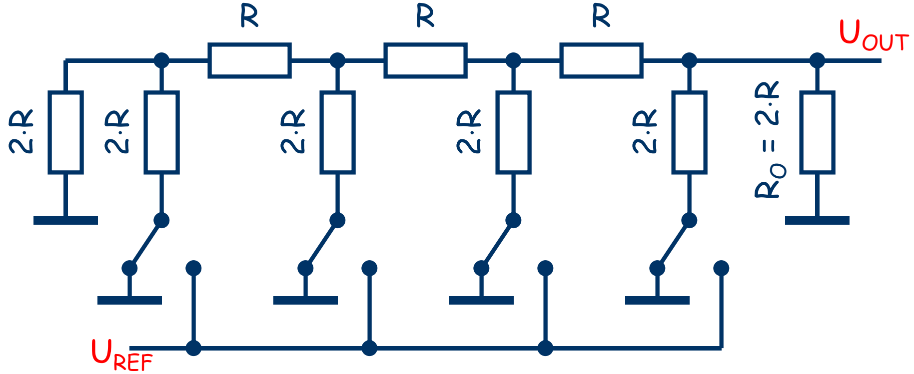




R-2R ladder DAC

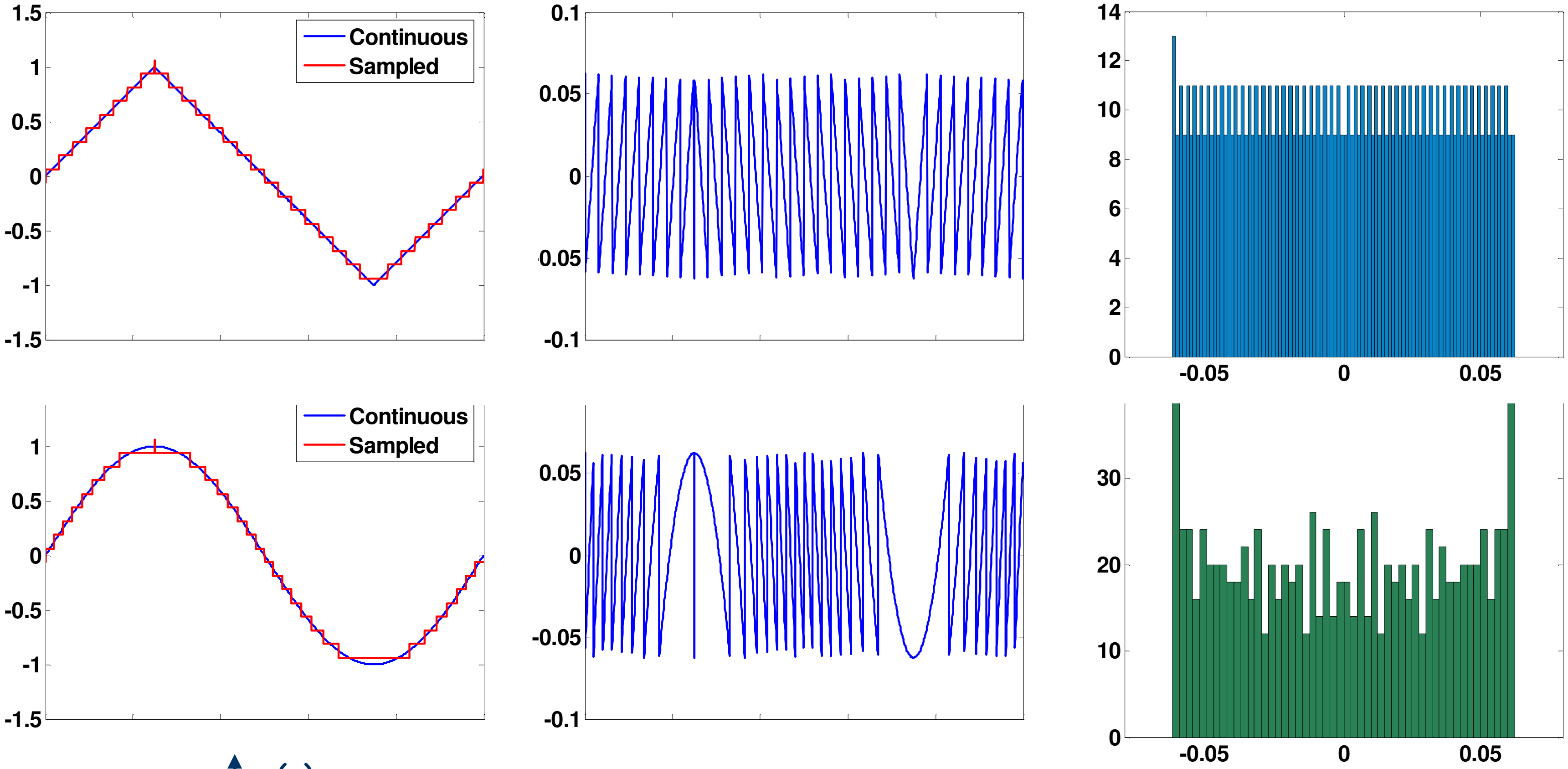


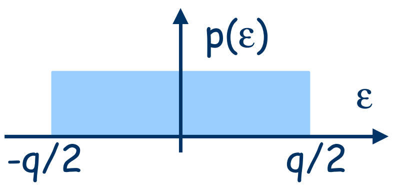
Inverted R-2R ladder DAC



Conversion errors - quantization noise

4-bit quantizer | quantization error ε | error histogram





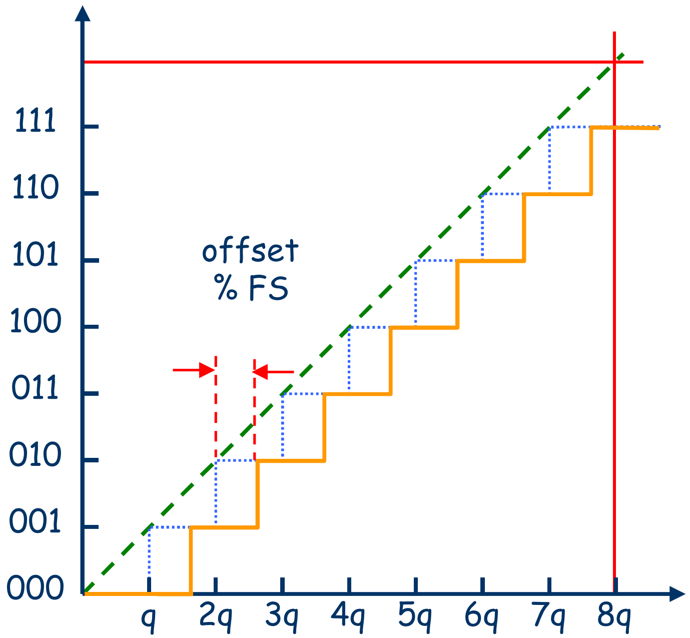
$$\varepsilon_{RMS} = \frac{q}{\sqrt{12}} \approx 0.289 \cdot q$$

$$SNR = 20\log\frac{S_{RMS}}{\varepsilon_{RMS}} = 6.02 \cdot N + 1.76 \text{ [dB]}$$

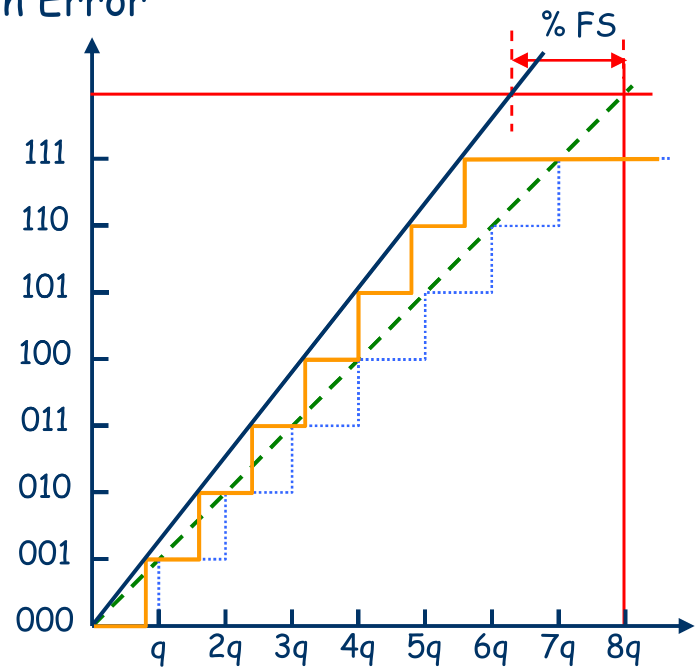
$$DR = 20\log\frac{FS}{q} = 20\log\frac{2^N q}{q} = 6.02 \cdot N \text{ [dB]}$$

ADC Errors

Offset Error

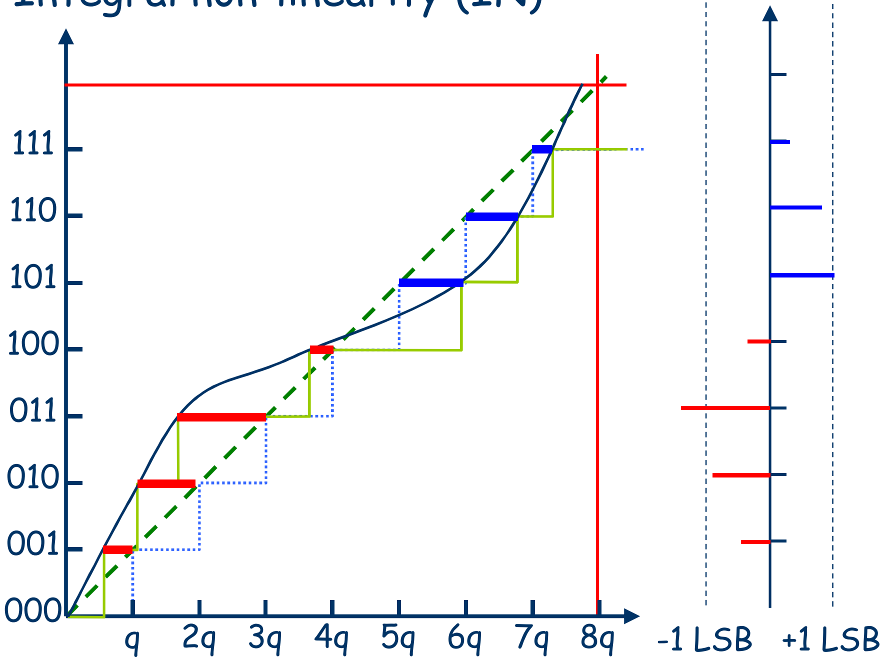


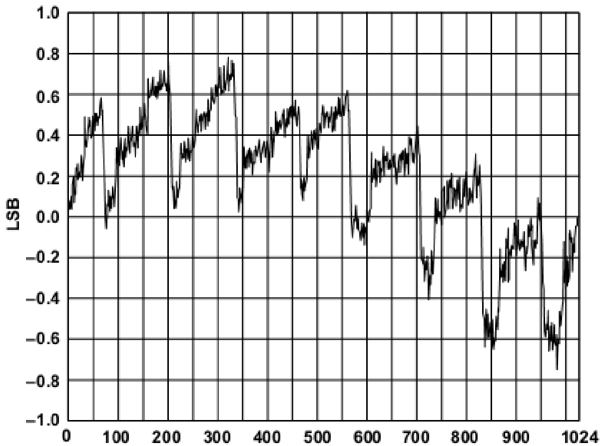
Gain Error



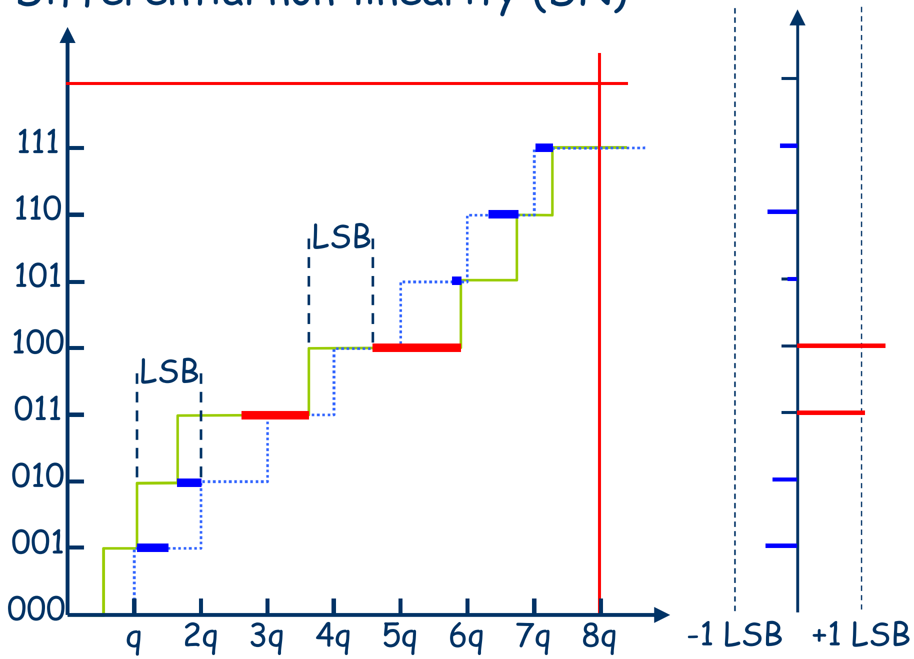
Conversion non-linearity

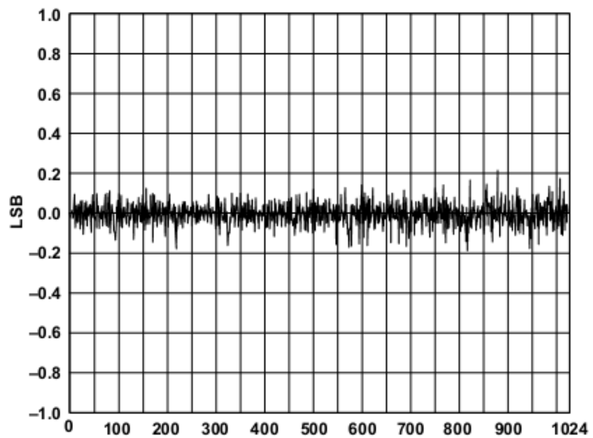
Integral non-linearity (IN)





Differential non-linearity (DN)

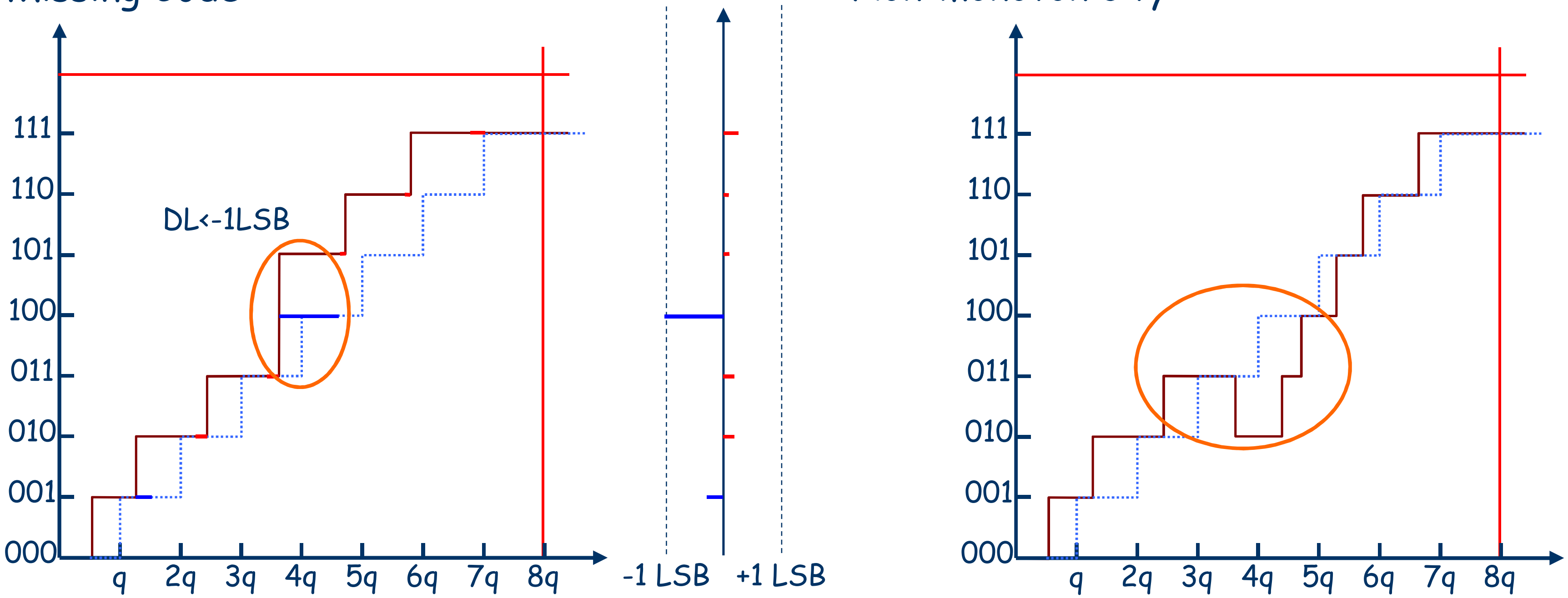




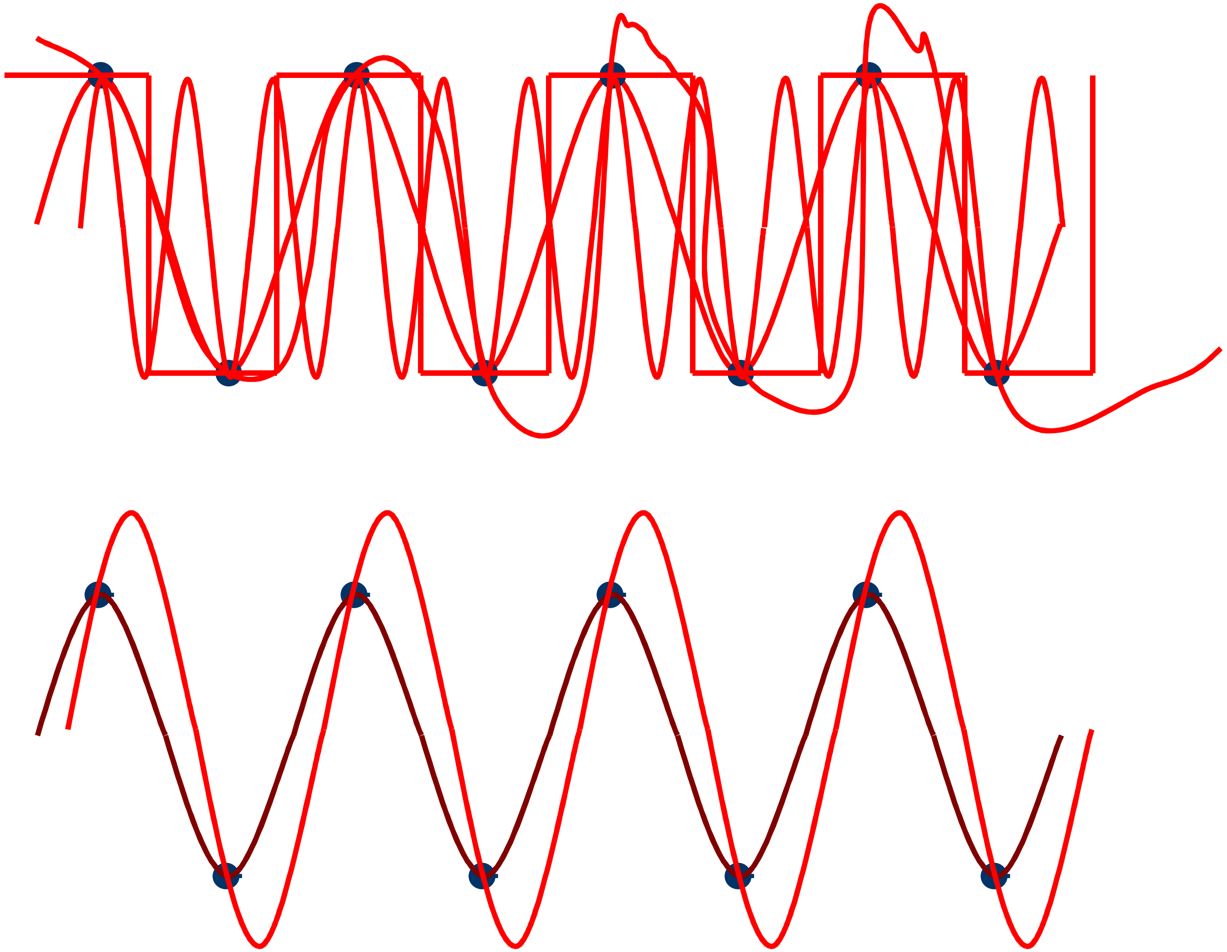
Some drastic examples

Missing Code

Non-monotonicity



Does sampling give us the full information about the signal?



the answer is „YES"
provided that we respect...

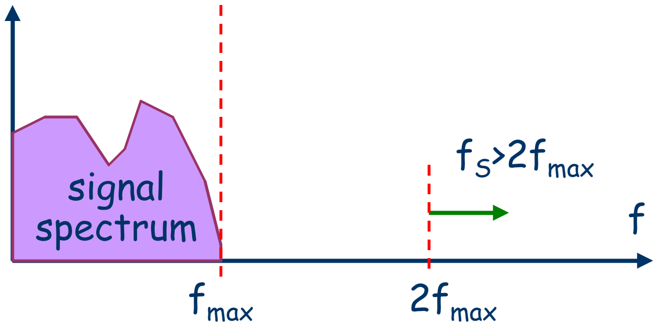
Sampling theorem

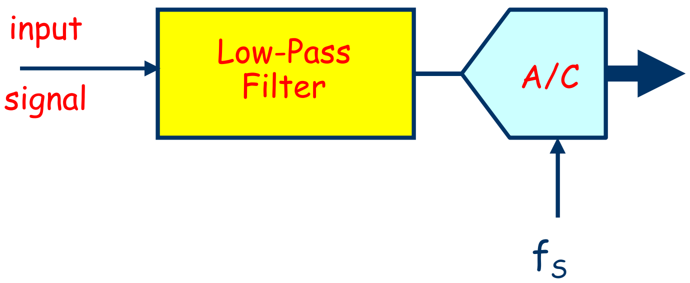
Shannon-Kotielnikov, Nyquist-Shannon, ...

It is possible to recover the signal exactly from its samples taken at the constant rate f_S if the the signal is band-limited and there are no spectral components above frequency $f_S/2$.

f_S – sampling frequency
$f_S/2$ –Nyquist frequency

or stating it more directly:
Sampling rate must be at least two times higher than the bandwidth occupied by the signal.



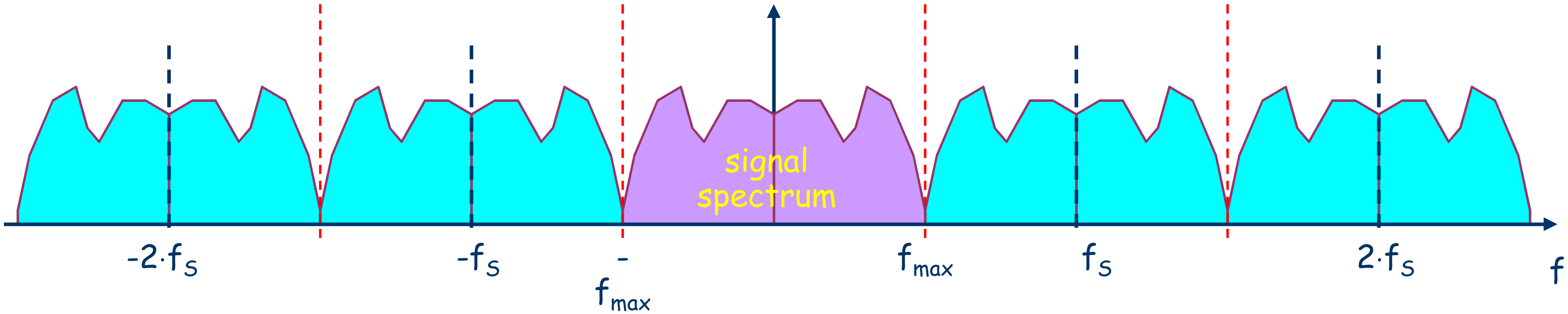


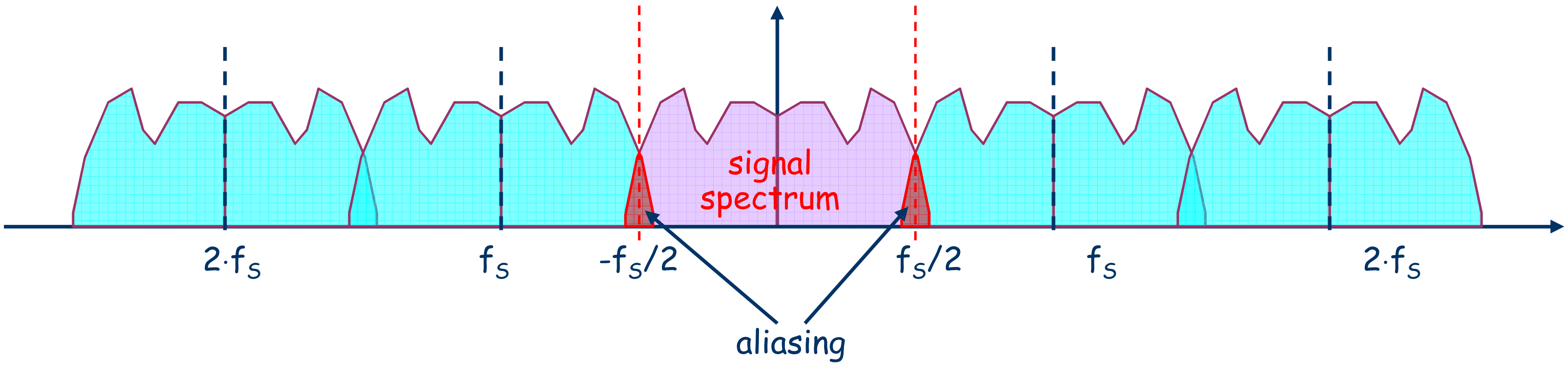
Aliasing

X(f) – signal spectrum (two-sided)

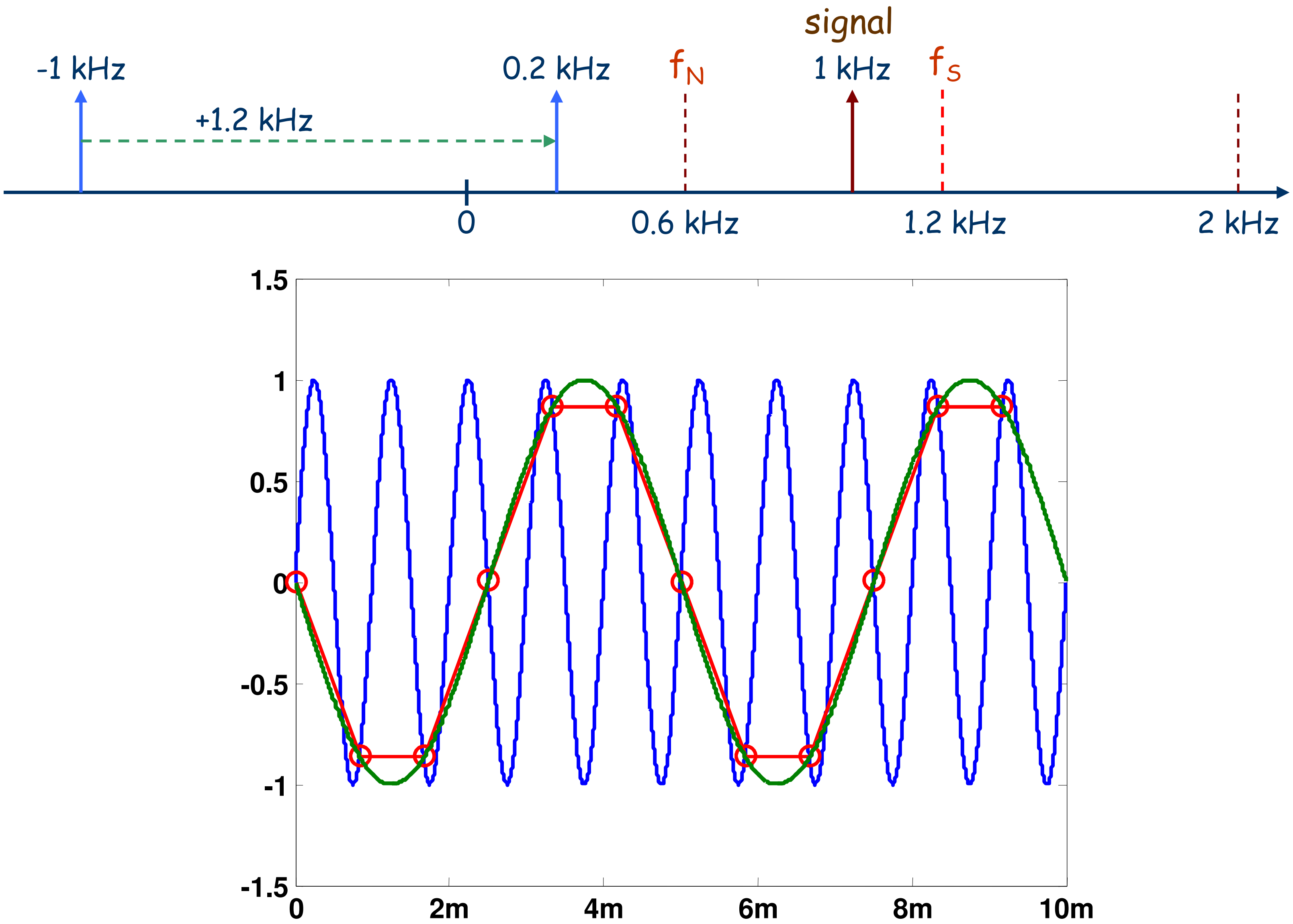
spectrum of sampled signal:

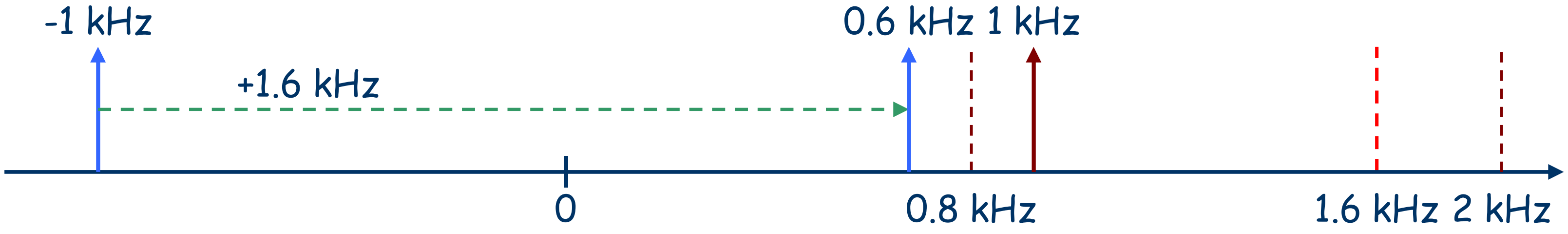
$$X_s(f) \sim \sum_{n=-\infty}^{+\infty} X(f - n \cdot f_s)$$



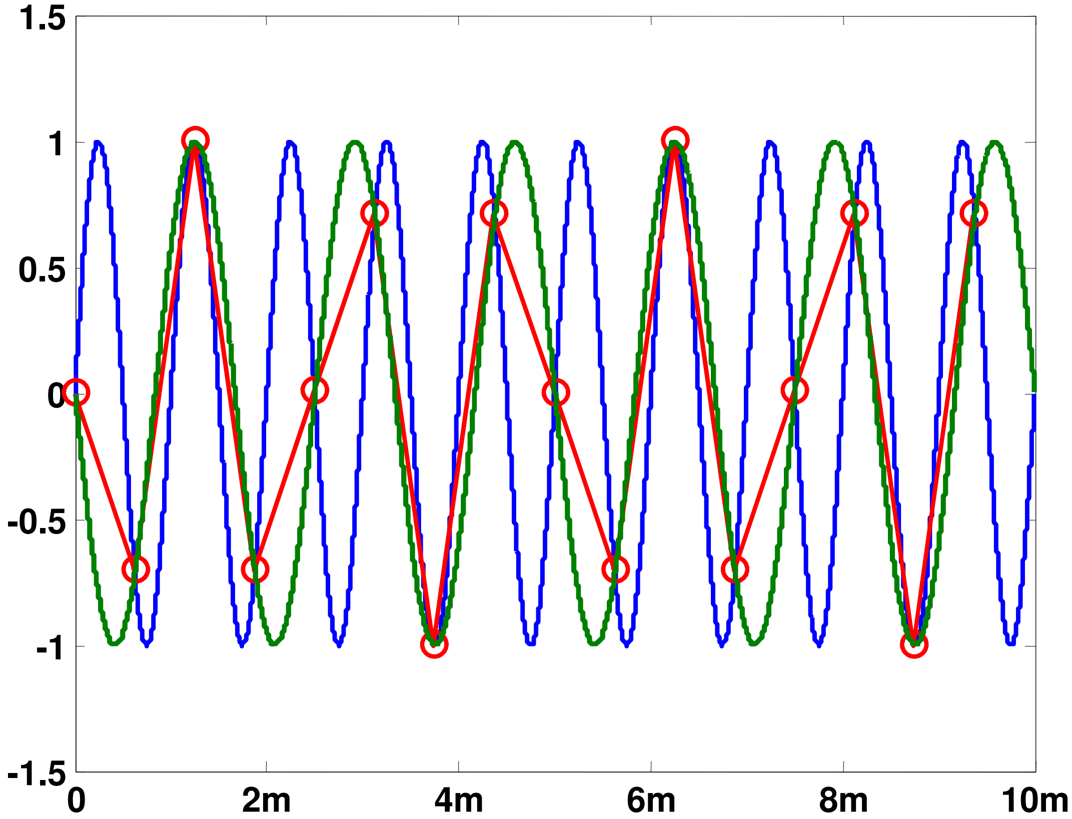


Aliasing more accesible...





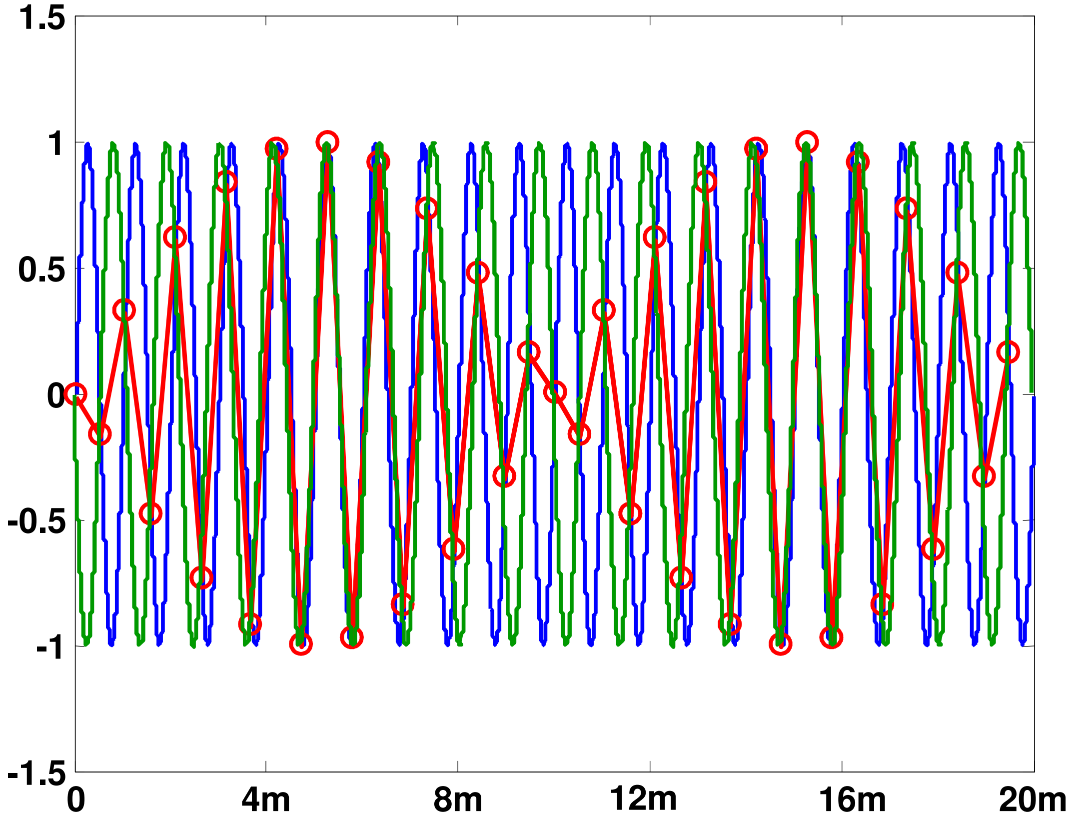
-1 kHz
+1.6 kHz
0.6 kHz 1 kHz
0
0.8 kHz
1.6 kHz 2 kHz



1.5
1
0.5
0
-0.5
-1
-1.5
0
2m
4m
6m
8m
10m

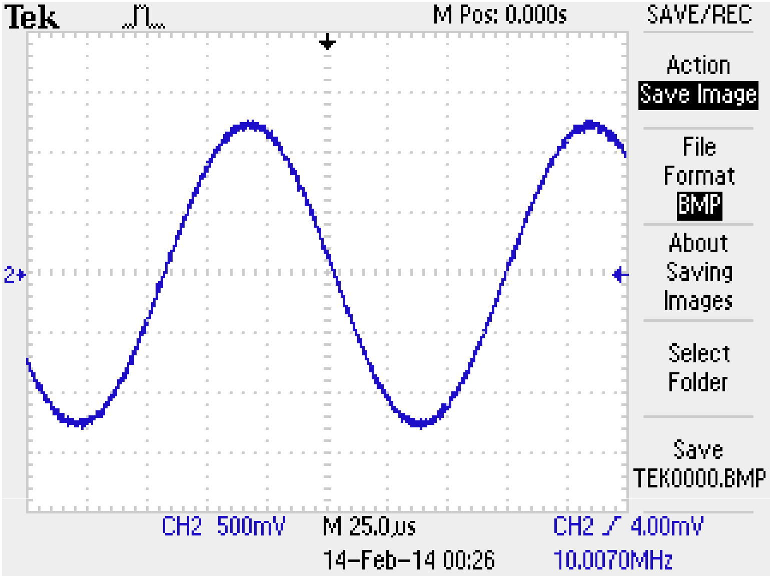
…

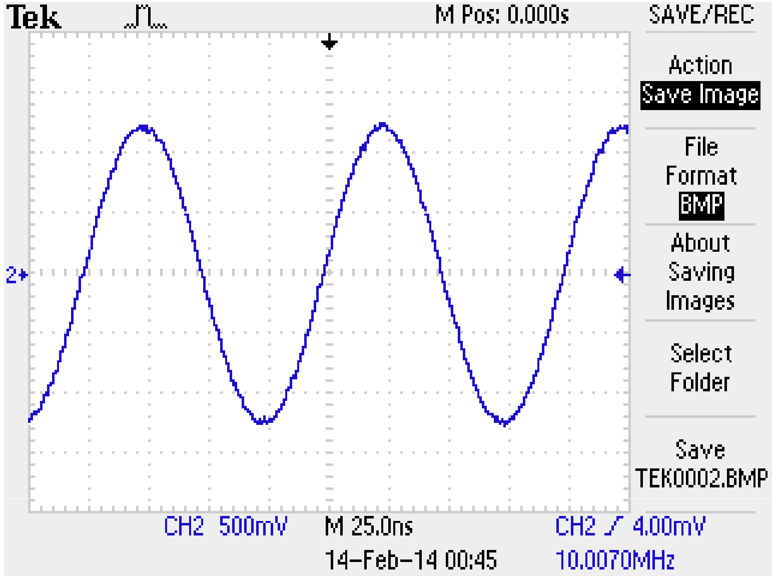
-1 kHz
+1.9 kHz
0.9 kHz
1 kHz
1.9 kHz
0
0.95 kHz
2 kHz

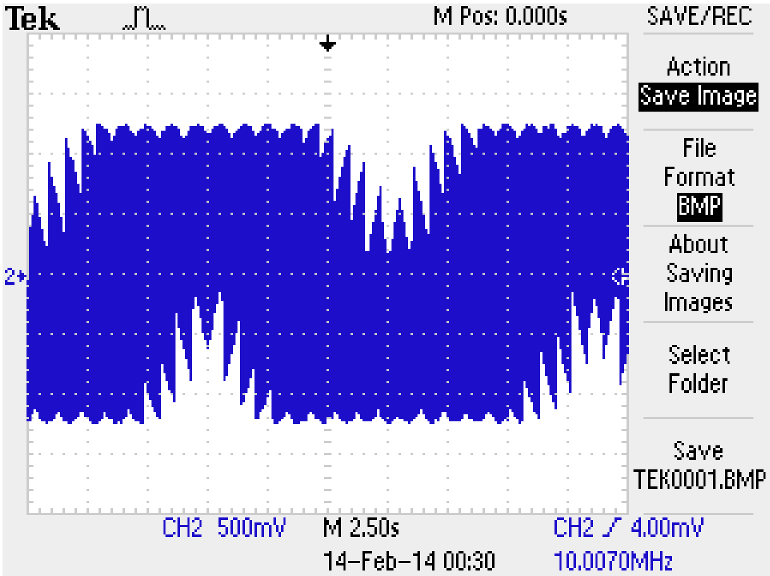


1.5
1
0.5
0
-0.5
-1
-1.5
0
4m
8m
12m
16m
20m

Aliasing in practice it can appear with a DSO!



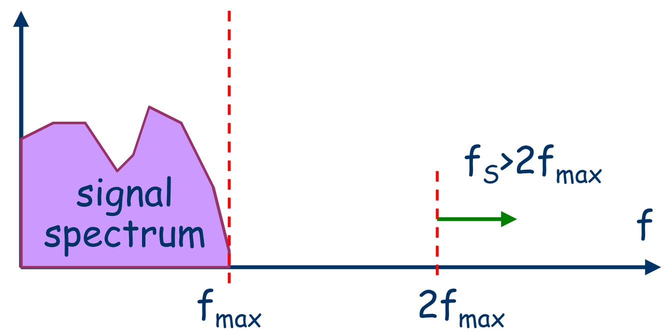


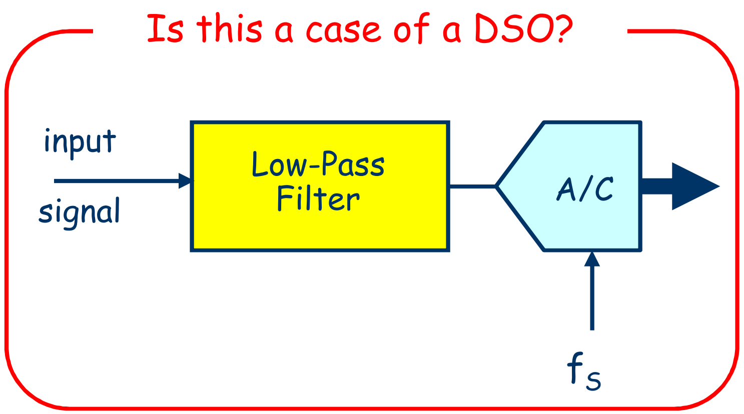


Why?

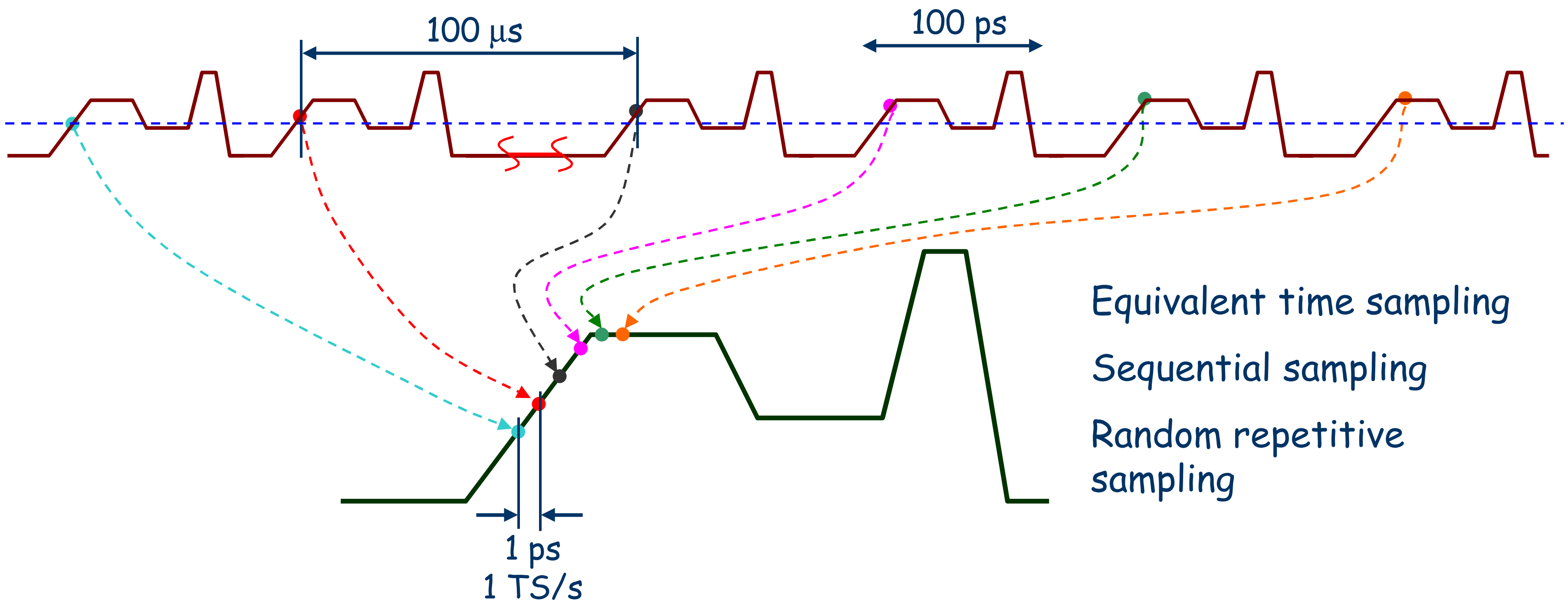
Sampling theorem (in short)

Sampling rate must be at least two times higher than the bandwidth occupied by the signal.





Is it possible to have $f_s < 2 \times f_{max}$???



Equivalent time sampling

Sequential sampling

Random repetitive sampling