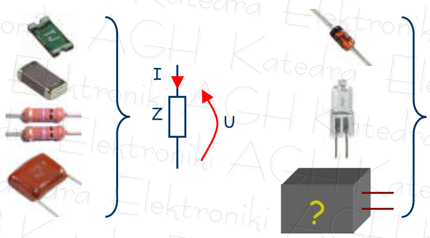
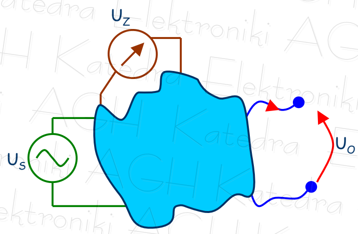
Impedance measurements

Impedance of two-terminals

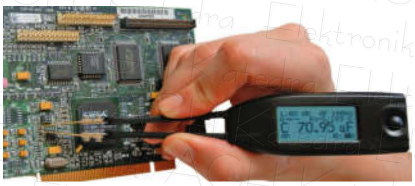
How one may understand the „impedance" concept...





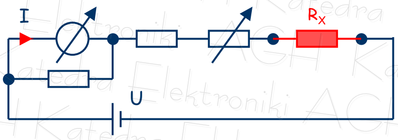
$U = Z(\omega)\cdot I$ $\quad U = f(I,\omega,T,...)$ $\quad U_O = f(U_S,U_Z,I_O,\omega,...T,...)$

Good or wrong?



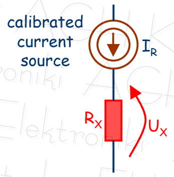
Resistance measurement: the ohmmeter

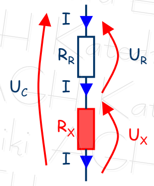
„Analog" ohmmeter



ohmmeter scale

„Digital" ohmmeter





$$R_X = \frac{U_X}{I_R}$$

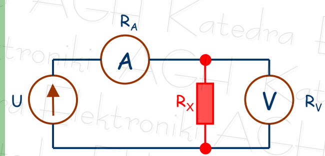
$$R_X[k\Omega] = U @ I_R = 1mA$$

$$R_X[M\Omega] = U @ I_R = 1\mu A$$

1. $R_X = \frac{U_X}{U_R} R_R$
2. $R_X = \frac{U_X}{U_C - U_X} R_R$

Resistance measurement: V-A method

Accurate voltmeter method



searched resistance

$$R_m = \frac{U_V}{I_A} = \frac{R_X R_V}{R_X + R_V} < R_X$$

measurement error

$$\Delta R_X = R_m - R_X$$

$$\Delta R_X = -\frac{R_X^2}{R_X + R_V} = -\frac{R_m^2}{R_V - R_m}$$

relative error

$$\delta R_X = \frac{\Delta R_X}{R_X}$$

nominal relative error

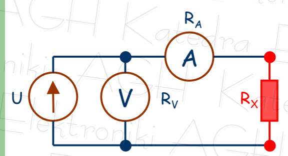
$$\delta_N R_X = \frac{\Delta R_X}{R_m}$$

correction

$$P = -\Delta R_X$$

$$P = \frac{R_m^2}{R_V - R_m}$$

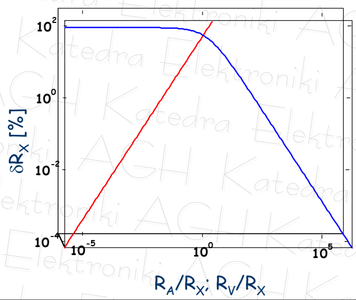
Accurate ammeter method



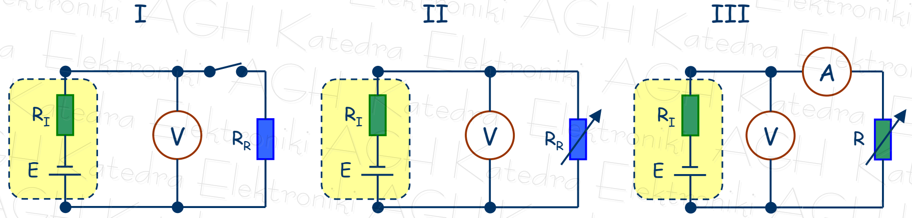
searched resistance

$$R_m = \frac{U_V}{I_A} = R_X + R_A > R_X$$

$$\Delta R_X = R_A$$



Source resistance



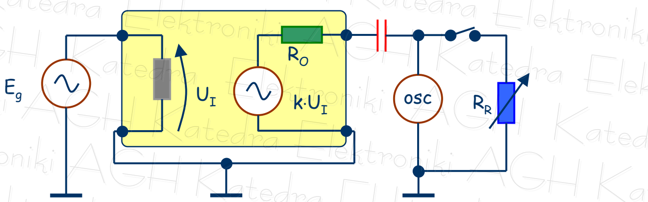
I	II	III
1. $U_{V1} = E$	1. $U_{V1} = E\dfrac{R_{R1}}{R_{R1}+R_I}$	1. $U_{V1} = E$
2. $U_{V2} = E\dfrac{R_R}{R_R+R_I}$	2. $U_{V2} = E\dfrac{R_{R2}}{R_{R2}+R_I}$	2. $E - I_{A2}R_I = U_{V2}$

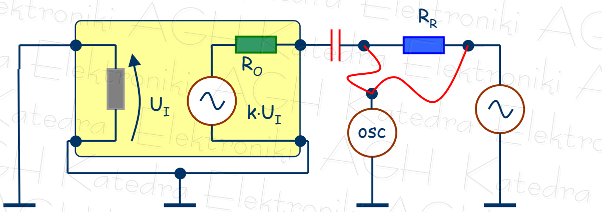
Estimation of limiting error R_I

$$R_I = \frac{U_{V1}-U_{V2}}{I_{A2}} \Rightarrow \Delta_{gr}R_I = \left(\left|\frac{\partial R_I}{\partial U_{V1}}\right| + \left|\frac{\partial R_I}{\partial U_{V2}}\right|\right)\Delta_{gr}U + \left|\frac{\partial R_I}{\partial I_{A2}}\right|\Delta_{gr}I$$ limiting error

$$\delta_{gr}R_I = \frac{2\Delta_{gr}U}{|U_1-U_2|} + \frac{\Delta_{gr}I}{I_{A2}}$$ relative limiting error

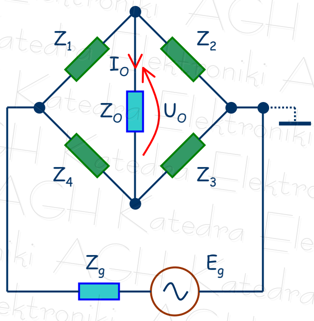
Output resistance of a two-port





Bridges

General structure



Balanced bridge:

Z_1, Z_2, Z_3, Z_4: $U_O, I_O = 0$

Balance condition:

$$\frac{Z_2}{Z_1+Z_2} = \frac{Z_3}{Z_3+Z_4} \quad \Leftrightarrow \quad Z_1Z_3 = Z_2Z_4$$

Unbalanced bridge:

Z_1, Z_2, Z_3, Z_4: $U_O, I_O \neq 0$

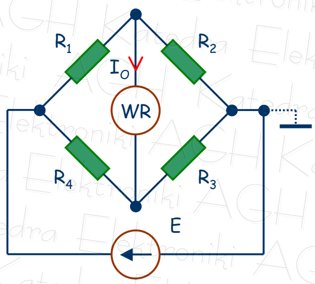
$$U_O = f_U(Z_1, Z_2, Z_3, Z_4, Z_g, Z_O, E_g)$$
$$I_O = f_I(Z_1, Z_2, Z_3, Z_4, Z_g, Z_O, E_g)$$

In general it is rather complicated circuit, so the designer should be a bit clever...

1). $Z_g \approx 0,\ Z_O \approx \infty$ 2). $Z_g \approx 0$

$$U_O = E_g \frac{Z_2Z_4 - Z_1Z_3}{(Z_1+Z_2)(Z_3+Z_4)} \qquad I_O = E_g \frac{Z_2Z_4 - Z_1Z_3}{R_g(Z_1+Z_2)(Z_3+Z_4) + Z_3Z_4(Z_1+Z_2) + Z_1Z_2(Z_3+Z_4)}$$

DC Wheaston bridge



Balance condition:

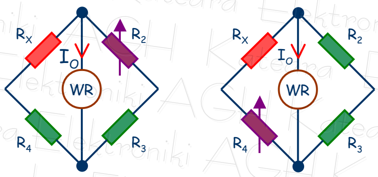
$$\frac{R_2}{R_1+R_2} = \frac{R_3}{R_3+R_4}$$

$$R_1R_3 = R_2R_4 \qquad \frac{R_1}{R_2} = \frac{R_4}{R_3}$$

Limiting error estimation

$$R_X = R_2\frac{R_4}{R_3} \Rightarrow \delta_{gr}R_X = |\delta_{gr}R_2| + |\delta_{gr}R_3| + |\delta_{gr}R_4|$$

How to balance the bridge?

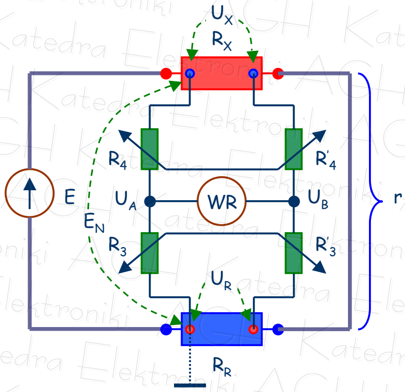


Transpose method

$$\left.\begin{aligned} &1).\ R_X = R_2'\frac{R_4}{R_3} \\ &2).\ R_X = R_2''\frac{R_3}{R_4} \end{aligned}\right\} \quad R_X = \sqrt{R_2'R_2''}$$

Limiting error???

Small resistances: Thomson bridge



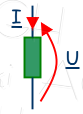
$$U_A = E_N \frac{R_3}{R_3 + R_4}$$

$$U_B = (E_N - U_X - U_R)\frac{R'_3}{R'_3 + R'_4} + U_R$$

Balance condition: $U_A = U_B$

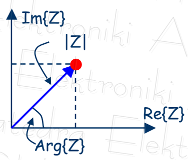
$$R_X = R_R \frac{R'_4}{R'_3}$$

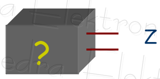
Impedance: component modelling

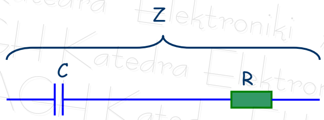


$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \mathrm{Re}\{\underline{Z}\} + j\,\mathrm{Im}\{\underline{Z}\} = R + jX = |\underline{Z}|e^{jArg\{\underline{Z}\}}$$

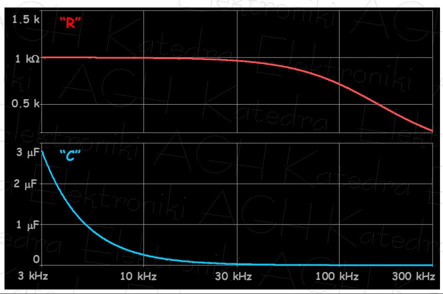
$$\underline{Y} = \frac{\underline{I}}{\underline{U}} = \mathrm{Re}\{\underline{Y}\} + j\,\mathrm{Im}\{\underline{Y}\} = G + jB = |\underline{Y}|e^{jArg\{\underline{Y}\}}$$

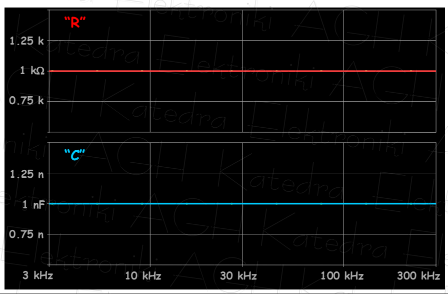




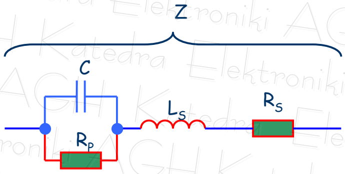


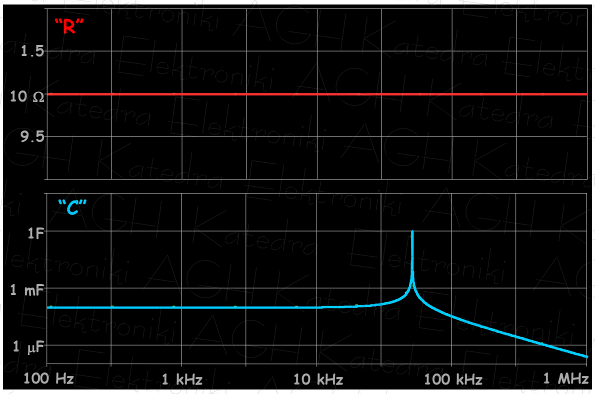
$$C = \frac{-1}{\omega\,\mathrm{Im}\{\underline{Z}\}} \qquad R = \mathrm{Re}\{\underline{Z}\}$$





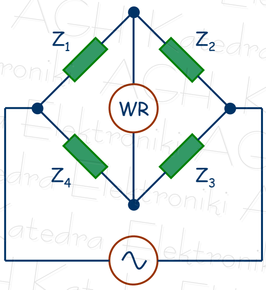
„Real" capacitor





R_S value enlarged for better illustration

AC bridges



Balance condition:

$$Z_1 Z_3 = Z_2 Z_4$$

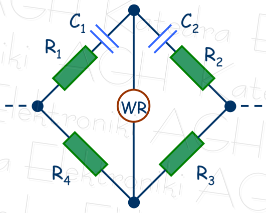
$$Z_i = R_i + jX_i :$$

1). $R_1 R_3 - X_1 X_3 = R_2 R_4 - X_2 X_4$

2). $R_1 X_3 - X_1 R_3 = R_2 X_4 - X_2 R_4$

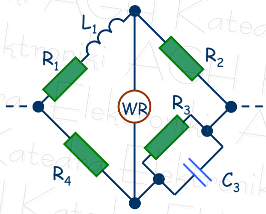
ballance = f(R_1, R_2, R_3, R_4, X_1, X_2, X_3, X_4, ω)

e.g.: Z_3, $Z_4 \rightarrow R$



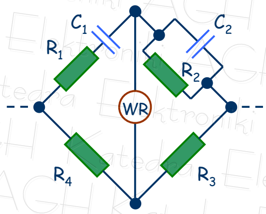
Wien bridge

e.g.: Z_2, $Z_4 \rightarrow R$



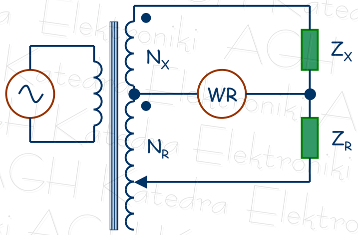
Maxwell – Wien bridge

e.g.: Z_3, $Z_4 \rightarrow R$

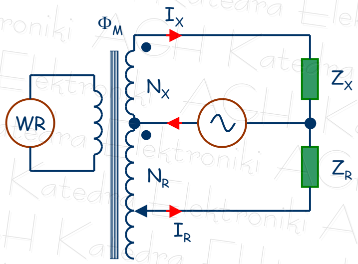


Wien – Robinson bridge

Transformer bridges



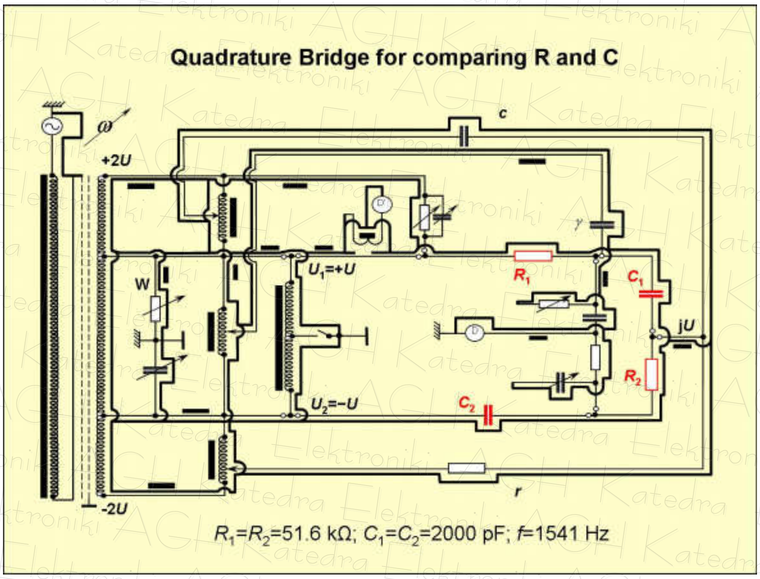
$$Z_X = Z_R \frac{N_X}{N_R}$$



$$I_X N_X = I_R N_R \Rightarrow \Phi_M = 0$$

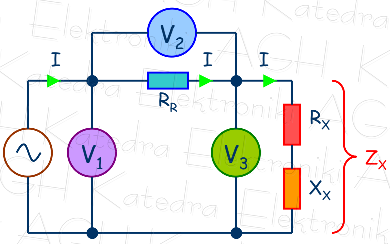
$$Z_X = Z_R \frac{N_X}{N_R}$$

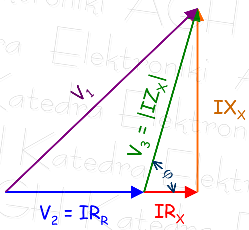
Special purpose bridge...



Source: http://www.bipm.org/utils/common/img/elec/quadrature_bridge.jpg

Method of „three voltmeters”





$$\cos\varphi = \frac{V_1^2 - V_2^2 - V_3^2}{2V_2V_3}$$

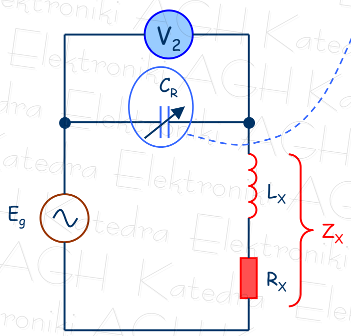
$$|Z| = \frac{V_3}{I} = \frac{V_3}{V_2}R_R$$

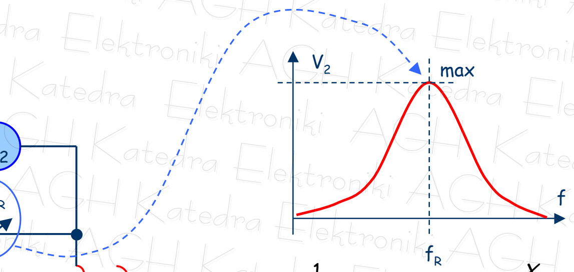
$$R = |Z|\cos\varphi$$

$$X = |Z|\sin\varphi$$

e.g.: Agilent E4982
1 MHz - 3 GHz
140 mΩ - 4.8 k Ω

Resonant method



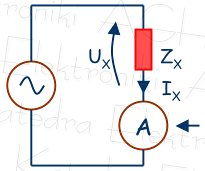


$$f_R = \frac{1}{2\pi\sqrt{L_X C_R}}$$

$$Q = \frac{X_L}{R_X} = \frac{X_C}{R_X}$$

$$U_C = ?$$

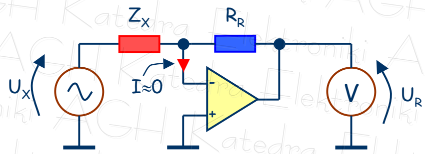
Voltage-current method



$$Z_X = \frac{U_X}{I_X} = \frac{|U_X|}{|I_X|} e^{j(Arg(U_X)-Arg(I_X))}$$

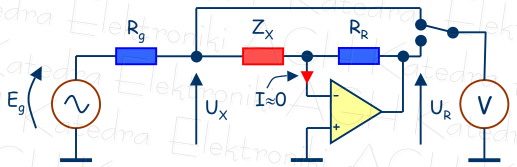
ideal ammeter (vector)

Basic circuit using operational amplifier



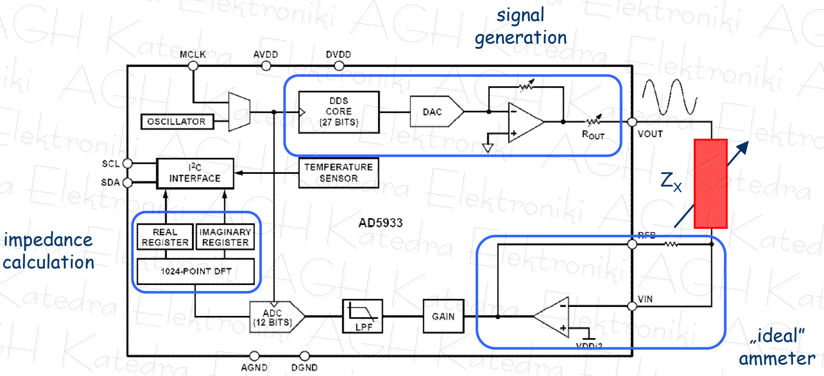
$$Z_X = R_R \frac{U_X}{U_R}$$

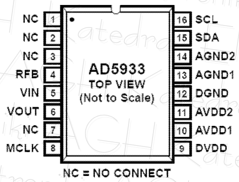
Practical realization



vector voltmeter

Integrated solution: impedance converter



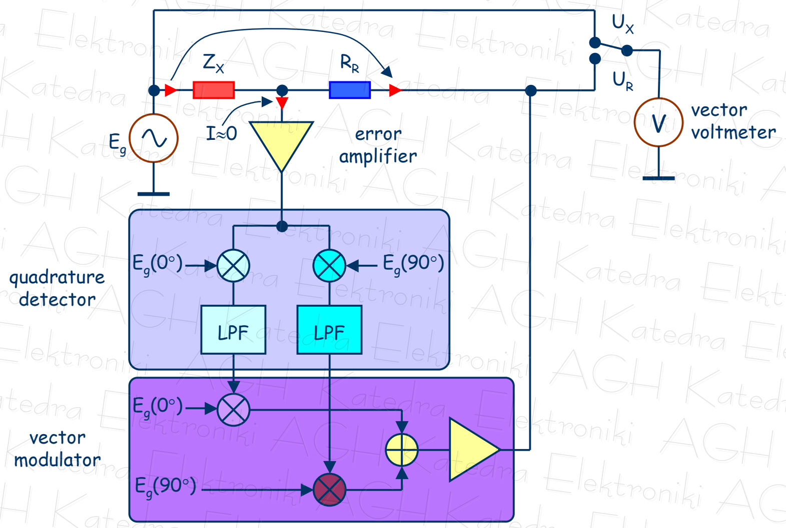


Z_X: 1 kΩ - 1 M Ω
1 - 100 kHz

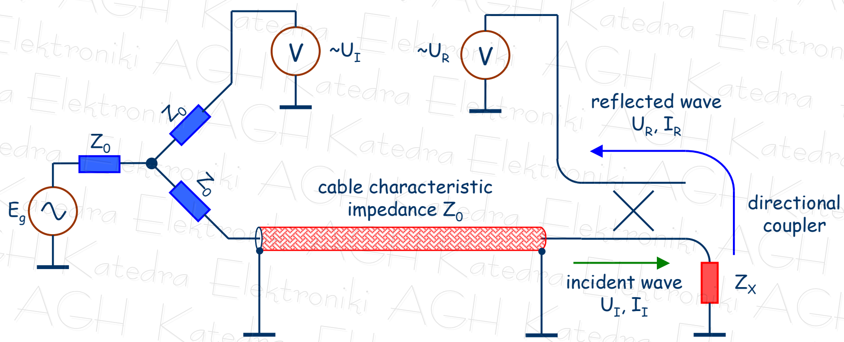
Source: Analog Devices

Broadband measurement: automatic bridge

Auto-Balancing Bridge



Even broader bandwith: network analyser



$$Z_X = \frac{U_X}{I_X} = \frac{U_I + U_R}{I_I - I_R}$$