AGH University of Science
and Technology

On Negation, Abduction, Deduction and Inconsistency Elimination. A Note on Diagnosis from Logical Perspective

Antoni Ligęza

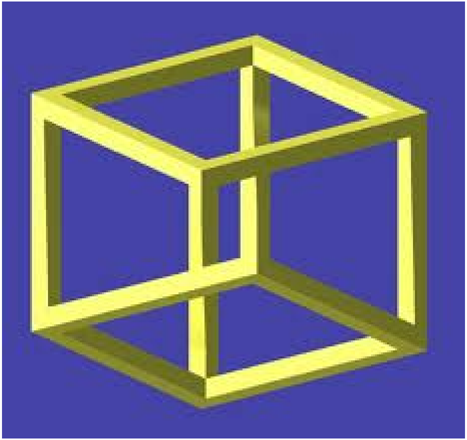
Faculty of EEACSE
Department of Automatics

DPS'2011
September 21, 2011 — Zamość, Poland

Outline

1. Introduction
2. An Example: Multiplier-Adder
3. A Theory of Consistency-Based Diagnosis
4. Search for Conflicts: Causal Graphs
5. Negation, Abduction, Deduction
6. Qualitative Diagnoses. Multi-Component Multi-Valued Diagnoses
7. Qualitative Diagnoses: Example
8. Concluding Remarks

Inconsistency — Fascinating and Inspiring



About the Title

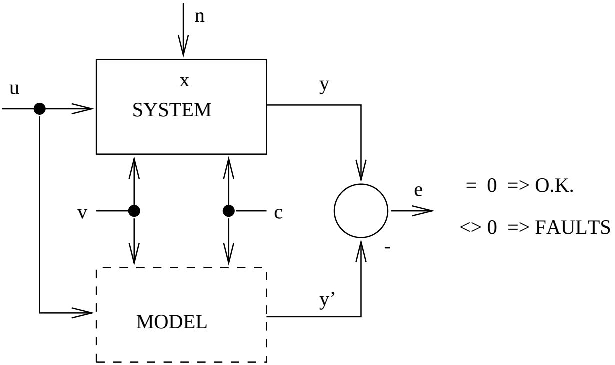
Logical components

- **negation** — its understanding and role
- **abduction** — hypotheses generation
- **deduction** — inferring consequences
- **inconsistency elimination** — elimination of hypotheses
- **logic** — tool for reasoning

The challenge of diagnosis

- diagnosis = speculative reasoning
- incomplete knowledge available
- positive models may be sufficient! (no experience, no records)
- hypothetical reasoning (guess)
- deductive inference (what-if): causal reasoning

Diagnosis — A Basic Scheme



u - control

v - external signals

z - noise

x - state

c - components

y - output

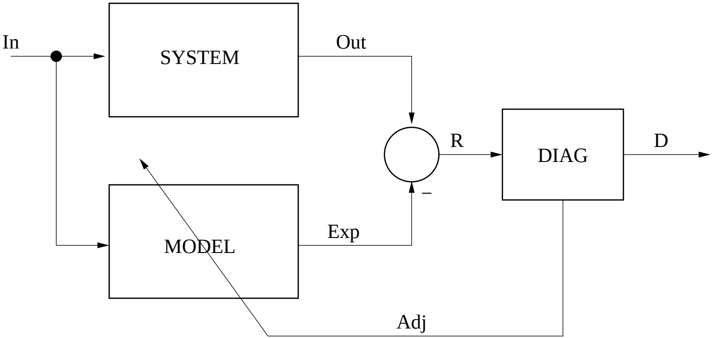
Diagnosis — How Does it Go?

Typical stages in a diagnostic process

- System observation — monitoring
- Detection of faulty behavior of the system (negation)
    - manifestations of faults
    - auxiliary observations
- Classification of this behavior — mode(e.g. $+$ or $-$)
- Search for and determination of causes of the observed misbehavior:
    - generation of potential diagnoses (abduction)
    - elimination of inconsistent ones (deduction: inconsistency elimination)
    - verification of consistent diagnoses
    - selection of the correct one
- Repair plan
- Repair action

FDI — Fault Detection and Isolation

Diagnosis — How Do We Do It?



Approaches to Diagnosis: Where Are We?

Learning: Pattern Recognition Type

- pattern recognition (classifiers)
- artificial neural networks
- decision trees, decision tables
- rule-based systems, expert systems (induction)
- case-based reasoning
- nearest neighbor

Characteristics

- experimental data necessary — faults must have happened
- training/learning necessary — time consuming, error rate
- distance-based methods — mostly numerical data
- shallow expert knowledge — no in-depth analysis

Approaches to Diagnosis: Where Are We?

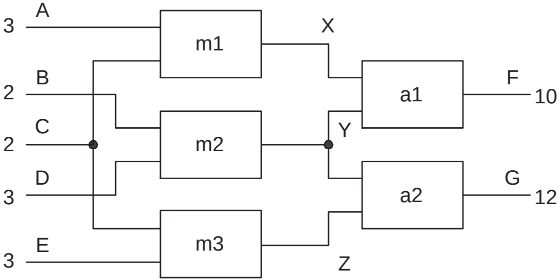
Model-Based Diagnosis

- causal graphs, causal relations
- abductive reasoning
- causal logical graphs (AND/OR/NOT causal graphs)
- analytical models (e.g. differential equations) (FDI)
- ◀consistency-based reasoning▶ AI/DX

Characteristics

- no experimental data necessary
- no training/learning necessary
- no distance-based methods
- deep expert knowledge — models are necessary (OK behavior)

Example: Multiplier-Adder



Diagnosis — How Does it Go?

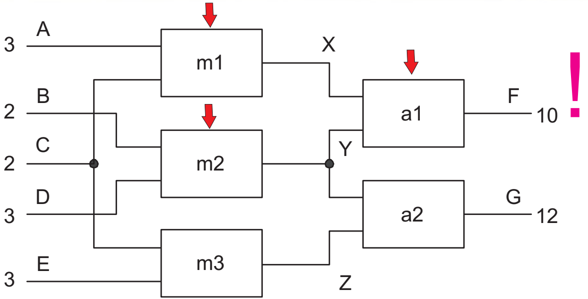
Multiplier-Adder Model

Components: $COMP = \{m1, m2, m3, a1, a2\}$

SD - *System Description*:

- $\text{ADD}(x) \wedge \neg \text{AB}(x) \Rightarrow Output(x) = Input1(x) + Input2(x)$
- $\text{MULT}(x) \wedge \neg \text{AB}(x) \Rightarrow Output(x) = Input1(x) * Input2(x)$
- $\text{ADD}(a1),\ \text{ADD}(a2),\ \text{MULT}(m1),\ \text{MULT}(m2),\ \text{MULT}(m3)$
- $\text{Output}(m1) = \text{Input1}(a1)$
- $\text{Output}(m2) = \text{Input2}(a1)$
- $\text{Output}(m2) = \text{Input1}(a2)$
- $\text{Output}(m3) = \text{Input2}(a2)$
- $\text{Input2}(m1) = \text{Input1}(m3)$
- $X = A * C,\ Y = B * D,\ Z = C * E$
- $F = X + Y,\ G = Y + Z$

Example: Multiplier-Adder — First Conflict



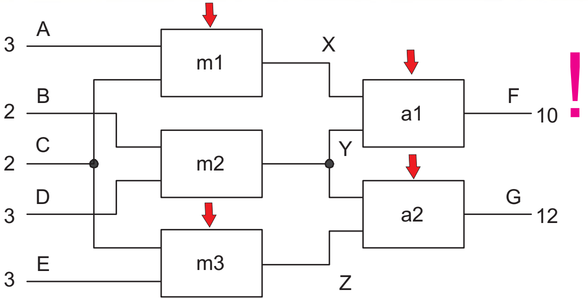
OBS - Observations:

$$\text{OBS} = \{A = 3, B = 2, C = 2, D = 3, E = 3, F = 10, G = 12\}$$

SD becomes inconsistent with OBS! Conflict = disjunctive diagnosis:

$$DCF_1 = \{a1, m1, m2\}$$

Example: Multiplier-Adder — Second Conflict



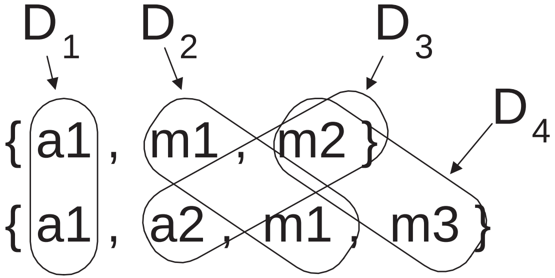
OBS - Observations:

$$\text{OBS} = \{A = 3, B = 2, C = 2, D = 3, E = 3, F = 10, G = 12\}$$

SD becomes inconsistent with OBS! Conflict = disjunctive diagnosis:

$$DCF_2 = \{a1, a2, m1, m3\}$$

Example: Multiplier-Adder — Diagnoses



- $DCF_1 = \{a1, m1, m2\}$
- $DCF_2 = \{a1, a2, m1, m3\}$
- $D_1 = \{a1\}$
- $D_2 = \{m1\}$
- $D_3 = \{a2, m3\}$
- $D_4 = \{m2, m3\}$

Consistency-Based Diagnosis: Reiter's Theory

System

$$System = (SD, COMPONENTS)$$

- SD — system description (model)
- $COMPONENTS$ — system elements

Diagnosis

A diagnosis for the system $(SD, COMPONENTS)$ with observations specified by OBS, is any minimal set $\Delta \subseteq$ *COMPONENTS*, such that the set

$$SD \cup OBS \cup \{AB(c) \mid c \in \Delta\} \cup$$

$$\{\neg AB(c) \mid c \in COMPONENTS - \Delta\}$$

is consistent.

Consistency-Based Diagnosis: Reiter's Theory

Conflict Set

A conflict set (*SD*, *COMPONENTS*, *OBS*) is any set $\{c_1, \ldots, c_k\} \subseteq$ *COMPONENTS*, such that the theory below is inconsistent.

$$SD \cup OBS \cup \{\neg AB(c_1), \ldots, \neg AB(c_k)\}$$

A conflict set is *minimal* if any of its proper subsets is not a conflict set.

Hitting Set

Let C be any family of sets. A hitting set for C is any set

$$H \subseteq \bigcup_{S \in C} S$$

such that $H \cap S \neq \emptyset$ for any set $S \in C$. A hitting set is *minimal* if and only if any of its proper subsets is not a hitting set for C.

Consistency-Based Diagnosis: Reiter's Theory

Theorem 1

$\Delta \subseteq$ COMPONENTS is a diagnosis for (SD, COMPONENTS, OBS) if and only if Δ is a minimal hitting set for the family of conflict sets for (SD, COMPONENTS, OBS).

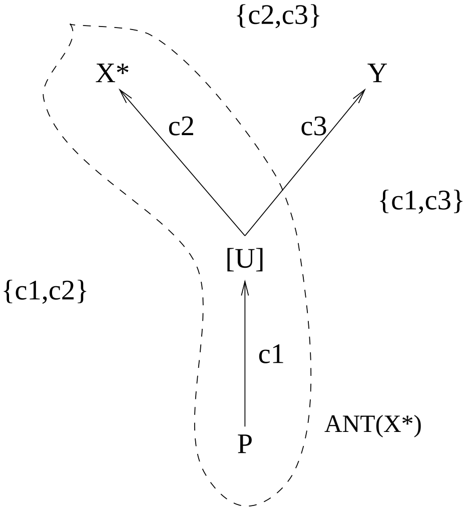
Theorem 2

H is a minimal hitting set for the collection of all conflict sets for (SD, COMPONENTS, OBS) iff H is a minimal hitting set for the collection of all *minimal conflict sets* for (SD, COMPONENTS, OBS).

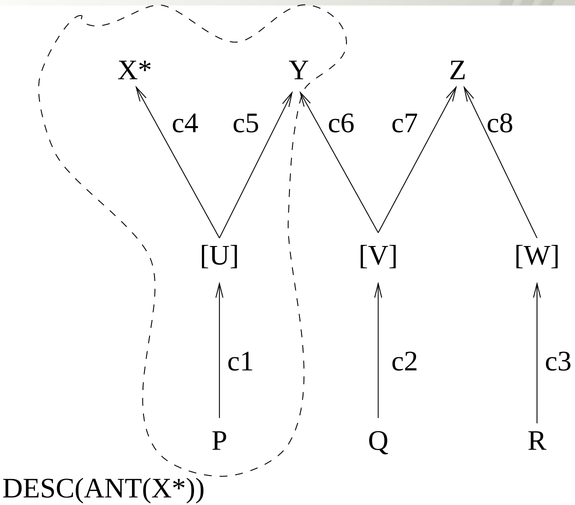
Corrolary

$\Delta \subseteq$ COMPONENTS is a diagnosis for (SD, COMPONENTS, OBS) if and only if Δ is a minimal hitting set for the family of *minimal* conflict sets for (SD, COMPONENTS, OBS).

Where to Search: Simple Conflicts

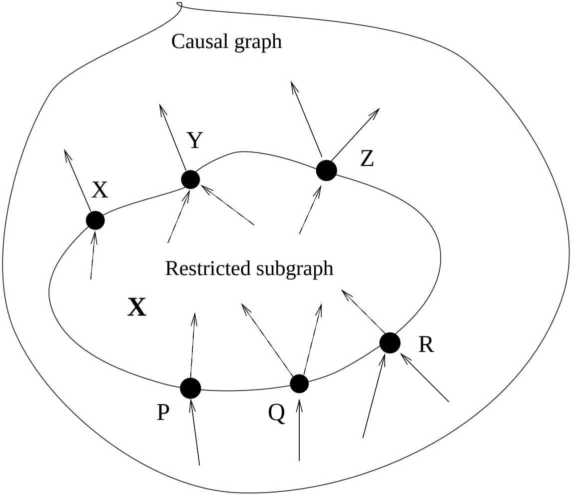


Where to Search: Complex Conflicts



C = {c4,c5,c6,c7,c8,c3}

Where to Search: Information Closure

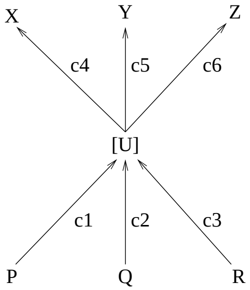


Potential Conflict Structure

Definition

A PCS structure defined for variable X on m variables is any subgraph of the causal graph, such that:

- it contains exactly m variables (including X),
- the values of all the variables are measured or calculated (they are well-defined),
- the value of variable X is double-defined,
- in the considered PCS all the values of the m variables are necessary for X in order to be double-defined.



Potential conflicts:

{c1,c2,c3,c4}

{c1,c2,c3,c5}

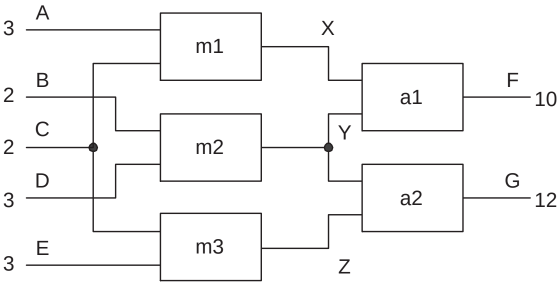
{c1,c2,c3,c6}

{c4,c5}

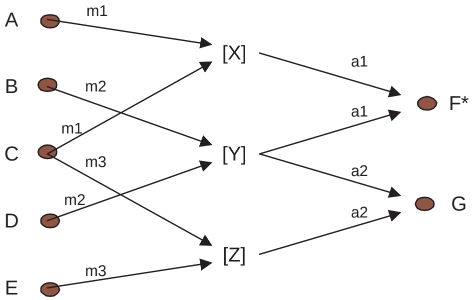
{c5,c6}

{c4,c6}

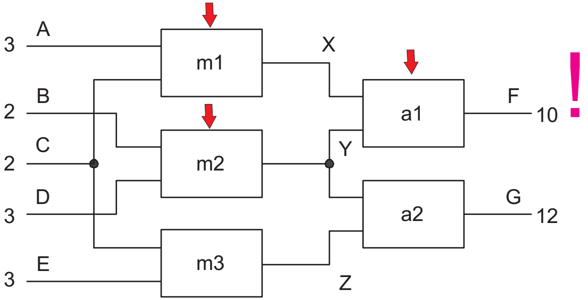
Example: Back to Multiplier-Adder



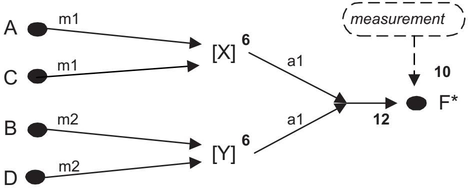
Example: Back to Multiplier-Adder



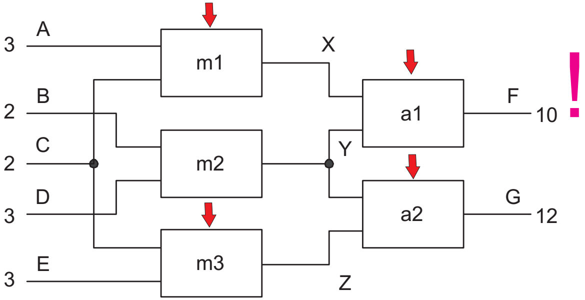
Example: Back to Multiplier-Adder



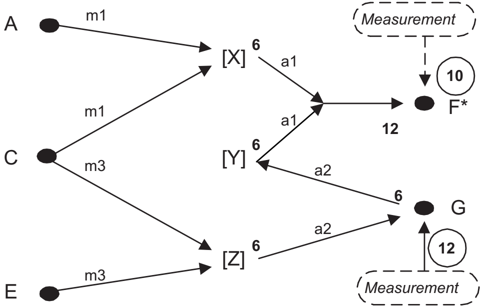
Example: Back to Multiplier-Adder



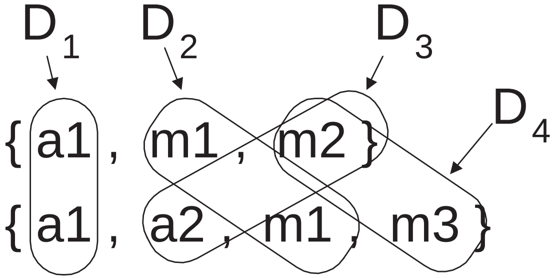
Example: Back to Multiplier-Adder



Example: Back to Multiplier-Adder



Example: Multiplier-Adder — Diagnoses



- $DCF_1 = \{a1, m1, m2\}$
- $DCF_2 = \{a1, a2, m1, m3\}$
- $D_1 = \{a1\}$
- $D_2 = \{m1\}$
- $D_3 = \{a2, m3\}$
- $D_4 = \{m2, m3\}$

The Two-Layer Approach: Causal Graph

Multiplier-adder: causal graph for multiple-fault diagnoses

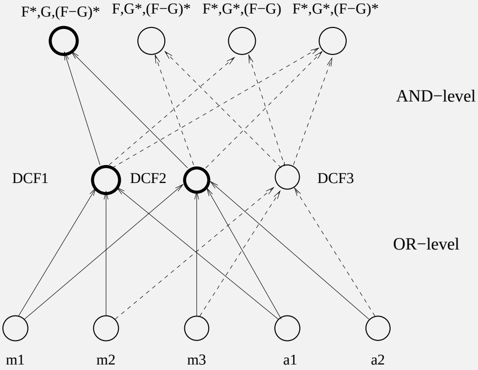


Figure: An AND/OR causal graph for the example multiplier-adder system

Multiplier-adder: final multiple-fault diagnoses

Table: Final possible diagnoses

Manifestations	Diagnoses
F^*,G, (F-G)*	$\{a1\}$, $\{m1\}$, $\{a2, m2\}$, $\{m2, m3\}$
F, G^*, (F-G)*	$\{a2\}$, $\{m3\}$, $\{a3, m2\}$, $\{m1, m2\}$,
F^*, G^*, (F-G)	$\{m2\}$, $\{a1, a2\}$, $\{a1, m3\}$, $\{a2, m1\}$, $\{m1, m3\}$
F^*, G^*, (F-G)*	$\{a1, a2\}$, $\{a1, m2\}$, $\{a1, m3\}$, $\{a2, m1\}$, $\{a2, m2\}$,$\{m1, m2\}$, $\{m2, m3\}$, $\{m1, m3\}$

Can we find more precise diagnoses?

Negation

Basic facts about negation

- $I: p \longrightarrow \{true, false\}$
- $I(p) = true \quad \Rightarrow \quad I(\neg p) = false$
- $I(p) = false \quad \Rightarrow \quad I(\neg p) = true$
- Principle of Contradiction: $\not\models p \land \neg p$
- Principle of Excluded Middle: $\models p \lor \neg p$

Some consequences

- Logical inconsistency may occur in systems with negation
- Problem: everything can be proved and disproved
- Let $U = \{black, white\}$; then
- $\neg[color = black] \equiv [color = white]$ and $\neg[color = white] \equiv [color = black]$
- $ok(c) \equiv \neg faulty(c)$ and $faulty(c) \equiv \neg ok(c)$

Negation in Extended Universe

Three-valued case

- Basic idea: $\neg[signal = ok] \equiv [signal = low] \vee [signal = high]$
- $I\colon c \longrightarrow \{low, ok, high\}$ $(\{-, 0, +\})$
- Notation: $ok(c)$ = $c(0)$, $faulty(c, +)$ = $c(+)$, $faulty(c, -)$ = $c(-)$
- Principle of Contradiction: $\not\models c(0) \wedge c(+)$, $\not\models c(0) \wedge c(-)$, $\not\models c(-) \wedge c(+)$
- Principle of Excluded Middle: $\models c(0) \vee c(-) \vee c(+)$

Some consequences

- Logical inconsistency still may occur
- Negation gives no unique result:

$$\neg c(0) \equiv c(+) \vee c(-)$$

- Notation: $c(+) \vee c(-) \;\equiv\; c(\{-, +\}) \;\equiv\; c(-, +)$

Negation

Negation — the 3 values case

proposition	negated proposition
$c(0)$	$c(+,-)$
$c(+)$	$c(0,-)$
$c(-)$	$c(+,0)$

Negation — consequences

proposition	negated proposition
$c(+,-)$	$c(0)$
$c(0,-)$	$c(+)$
$c(+,0)$	$c(-)$

Observation: Negation as complement can extend and refine logical value.

Deduction

Basic schemes

- The *Modus Ponens* or *Law of Detachment* rule:

$$\frac{\alpha, \alpha \Longrightarrow \beta}{\beta}$$

- The *Modus Tollens* or *Disjunctive Syllogism* rule:

$$\frac{\alpha \Longrightarrow \beta, \neg\beta}{\neg\alpha}$$

- The *Resolution* rule:

$$\frac{\alpha \vee q, \beta \vee \neg q}{\alpha \vee \beta}$$

- Deduction is a kind of *forward chaining*
- Deduction preserves *truth* (logical consequence)
- Deduction leads to inconsistency $\Rightarrow$ initial knowledge inconsistent!

Abduction

Basic scheme

$$\frac{\alpha \Longrightarrow \beta, \beta}{\alpha}$$

- $SD \cup EXP \models OBS \Rightarrow$ the hypotheses fully explain current observations taking into account knowledge about the system SD,
- $SD \cup EXP$ must be consistent.

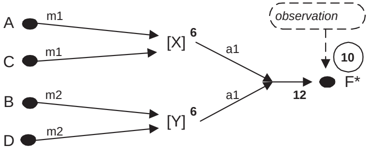
Observations

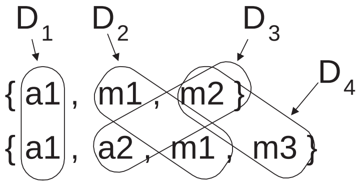
- Abduction is a kind of *backward chaining*
- Abduction does not preserve *truth* (it is not legal inference rule)
- Abduction leads to alternative hypotheses explaining observations
- Sherlock Holmes used to use abduction!

Inconsistency Elimination

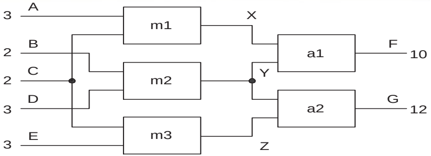
Role of Abduction, Deduction and Inconsistency Elimination

- Abduction — generation of potential diagnoses D such that $SD \cup D \models OBS$
- Abduction — performed with backtrack search
- Deduction — detection of inconsistency $(SD(ok) \cup OBS)$
- Inconsistency elimination:
  - regaining consistency through hitting sets use
  - elimination of inconsistent D with deduction and qualitative rules





Extended Formal Setting



Example: Multiplier-Adder once more

- $O = \{A, B, C, D, E, F, G\}$ — observable variables,
- $H = \{X, Y, Z\}$ — hidden variables,
- $D = \{m1, m2, m3, a1, a2\}$ — components,
- $\{-, 0, +\}$ — truth values,
- SM — model (the set of equations),
- OBS — current observations,
- Qualitative inference rules.
- Qualitative diagnoses — diagnostic hypotheses refinement

Extended Diagnoses

Qualitative diagnosis

A qualitative diagnosis is s set of the form:

$$D = \{d_1(\#), d_2(\#), \ldots, d_k(\#)\}$$

- $\# \in \{-, 0, +\}$
- minimal, fully explaining OBS (complete, consistent, minimal)

Transformation of diagnoses into qualitative diagnoses

$$\{d\} \quad \Rightarrow \quad \{d(-), d(+)\}$$

$$\{d_1, d_2\} \quad \Rightarrow \quad \{(d_1(-), d_2(-)), (d_1(-), d_2(+)), (d_1(+), d_2(-)), (d_1(+), d_2(+))\}$$

Example qualitative diagnoses: $m1(-)$, $m1+$, $(a2(-), m2(-))$, $(a2(-), m2(+))$, $(a2(+), m2(-))$, $(a2(+), m2(+))$, ...

Qualitative Rules: I

Type I rules: normal inputs, faulty component rules

Assumption: $input1(Comp, 0)$ and $input2(Comp, 0)$

$$c(< value >) \longrightarrow output(< value >)$$

Example rules

$$m1(-) \longrightarrow X(-)$$

$$m1(+) \longrightarrow X(+)$$

$$a1(-) \longrightarrow F(-)$$

$$a1(+) \longrightarrow F(+)$$

There are 10 such rules (2 for each component)

Qualitative Rules: II

Type 2 rules: deviated inputs, normal component ($c(0)$)

$$c_1(< value >) \wedge c_2(< value >) \longrightarrow output(< value >)$$

Example rules

$$a_1(0) \wedge a_2(0) \longrightarrow output(0)$$

$$a_1(-) \wedge a_2(0) \longrightarrow output(-)$$

$$a_1(-) \wedge a_2(+) \longrightarrow output(?)$$

inputs	-	0	+
-	-	-	?
0	-	0	+
+	?	+	+

Qualitative Rules: III

Type 3 rules: deviated inputs, faulty component rules

$$c_1(< value >) \wedge c_2(< value >) \wedge c(< value >) \longrightarrow output(< value >)$$

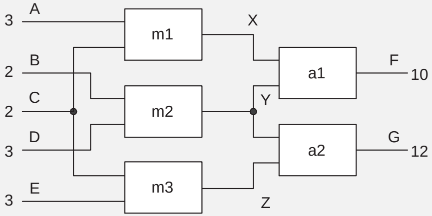
Example rules

input1	input2	Component Mode	Output
-	-	-	-
-	0	-	-
0	-	-	-
0	0	-	-
+	+	+	+
+	0	+	+
0	+	+	+
0	0	+	+

$$Y(+) \wedge Z(0) \wedge a2(+) \longrightarrow G(+)$$

Qualitative Diagnoses: Example

The multiplier-adder system to be diagnosed



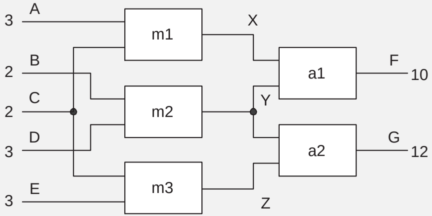
Case: $D = \{m1\}$

$OBS = \{F(-), G(0)\}$

- $\{m1(-)\}$: OK $[X(-), F(-)]$
- $\{m1(+)\}$: inconsistent $[X(+), F(+)]$.

Qualitative Diagnoses: Example

The multiplier-adder system to be diagnosed



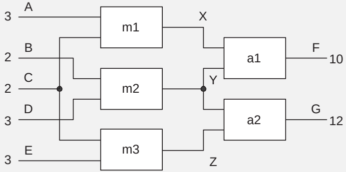
Case: $D = \{a1\}$

$OBS = \{F(-), G(0)\}$

- $\{a1(-)\}$: OK $[F(-)]$
- $\{a(+)\}$: inconsistent $[F(+)]$.

Qualitative Diagnoses: Example

The multiplier-adder system to be diagnosed



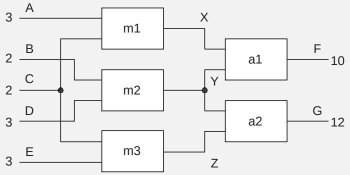
Case: $D = \{a2, m2\}$

$OBS = \{F(-), G(0)\}$

- $\{a2(-), m2(-)\}$: inconsistent $[Y(-), F(-), G(-)]$
- $\{a2(-), m2(+)\}$: inconsistent $[Y(+), F(+)]$
- $\{a2(+), m2(-)\}$: OK $[Y(-), F(-), G(?)]$
- $\{a2(+), m2(+)\}$: inconsistent $[Y(+), F(+), G(+)]$

Qualitative Diagnoses: Example

The multiplier-adder system to be diagnosed



Case: $D = \{m2, m3\}$

$OBS = \{F(-), G(0)\}$

- $\{m2(-), m3(-)\}$: inconsistent $[Y(-), F(-), Z(-), G(-)]$
- $\{m2(-), m3(+)\}$: OK $[Y(-), F(-), Z(+), G(?)]$
- $\{m2(+), m3(-)\}$: inconsistent $[Y(+), F(+)]$
- $\{m2(+), m3(+)\}$: inconsistent $[Y(+), F(+), Z(+), G(+)]$

Concluding Remarks

Conclusions

- Negation, Abduction, Deduction and Inconsistency Elimination are useful logical concepts for diagnosis
- Qualitative diagnoses are more informative,
- Qualitative analysis based on simple constraint rules allows for elimination of spurious (inconsistent) diagnostic hypotheses:
  - $\{a1\}$, $\{m1\}$, $\{a2, m2\}$, $\{m2, m3\}$ classic diagnoses,
  - 12 potential qualitative diagnoses,
  - $\{a1(-)\}$, $\{m1(-)\}$,$\{a2(+), m2(-)\}$, $\{m2(-), m3(+)\}$ four qualitative diagnoses,
  - further elimination possible with a priori knowledge about potential faults,
- Extensions: more qualitative values,
- Extensions: more specific rules,
- Extensions: additional test/measurements of system variables can reduce the number of diagnoses.

Acknowledgegments

My sincere thanks go to people who worked with me in the area of diagnosis:

- dr Pilar Fuster Parra (University of Balearic Islands, Palma de Mallorca)
- dr Bartłomiej Górny (most of the pictures!) (AGH,Comarch)
- dr Jakub Oleksiak (AGH, somwhere in Australia)
- Prof. Josep Aguilar Martin (LAAS, Toulouse; UdG, Girona)
- Prof. Jan Maciej Kościelny (UW, Warsaw)

and to my wife Ewa, being a source of continuous inspiration in the field of inventing and managing inconsistency.

[Waterfall. M.C. Escher, 1961]