Introduction to Mathcad 15 – part 4

Differential equations

Regardless of chosen method to solve differential equation in Mathcad you have to specify three things:

- starting conditions (or in other words Cauchy problem),
- range of points, where Mathcad will search for solutions,
- the equation – written in specific form depending on the type of function use to solve a problem.

To solve differential equations you can either use command

Given ……. Odesolve(x,n), where

- *x - variable*
- *n – ending point*

this command works exactly the same way as block solve discussed previously,

or

rkfixed(yp,x1,x2,m,D), where:

- yp – vector of initial condition (dim (yp) =n =rank of equation)
- x1,x2 – range of finding solution
- m – parameter stating how many points will be calculated to create matrix of solutions
- D – function D(x,yp)-contains first differentials of searched functions. (for equations of higher order(lets say =n) D contains differentials up to n-1 order)

Example:

Solving equation: **y'+3y=0**

1. **Odesolve**

Given

d/dx y(x) + 3y(x) = 0, y(0)=3 (initial condition)

Sol:=Odesolve(x,10)

2. **Rkfixed**

$y_{0:}$=3 -> initial value

D(x,y) :=-3y_0

Sol:=rkfixed(y,0,10,100,D)

The above commands also allow to solve more complicated equations of higher ranks. The difference lays in defining larger vector yp and function must contain as many derivatives as the rank of equation.

System of differential equations

Are solved in exactly the same way as differential equations. Both ***Given ……. Odesolve(x,n) & rkfixed(yp,x1,x2,m,D)*** command can be applied. The only difference is that you have to be careful about appropriate number of initial conditions.

Exercise:

1. Solve the 4^{th} rank equation as follows:

$$y^{(4)} - 2k^2y^{(2)} + k^4y = 0, \quad for\ k = 3$$

Initial condition are given by the vector y=(0,1,2,3). Use the method you find more suitable in this task.

2. Solve the following system of differential equations:
   u''(t)=2v(t);
   v''(t)=4v(t)-2u(t)
   with following initial conditions: u(0)=1,5; u'(0)=1; v(0)=1; v'(0)=1

3. Consider an installation of three cascading tanks with given areas S_i and outlet streams coefficients k_i. Liquid is delivered to the first tank in the stream given by equation (1):

S_1=1 m^2, S_2=1,3m^2, S_3=1,5m^2

k_1=0.08, k_2=0.06, k_3=0.04 [m^3/s*$m^{1/2}$]

$$Q1(t) = 50 * 10^{-3} * \exp(-0.02 * t)$$

a) Assuming that all tanks are empty at the beginning, calculate the change of heights of liquid in time. Create appropriate plot.
b) Calculate the maximum heights of liquid level for each tank.

TIP

General mass balance for each tank is given by the equation:

$$\boldsymbol{\rho S \frac{dh}{dt} = (Q_{in} - k\sqrt{h})\rho}$$