AGH University of Science and Technology in Cracow

Department of Electronics

Laboratory Manual
Physics_1

Title:

Hall effect

Experiment No.

13

2009 r.

1. Goal

To determine the sample conductivity, Hall coefficient, mobility and concentration of carriers.

2. What to learn?

Magnetic field $\vec{B}$. Magnetic force $\vec{F}_B$. Hall effect. Current density. Drift speed. Resistivity. Conductivity. Microscopic view of Ohm's law. The relationship between the electric field and potential difference. Electromagnet histeresis.

3. Background

Consider a semiconductor sample placed in the magnetic field that is in the *z*-direction (Fig. 1).

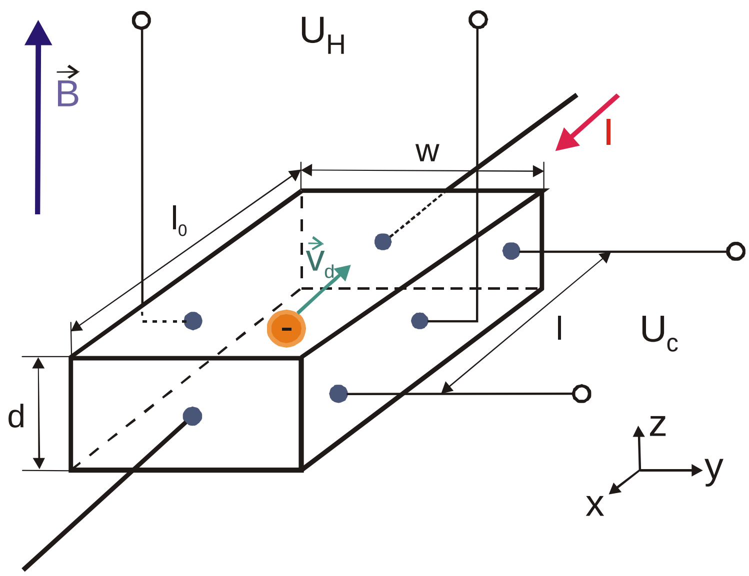

Fig. 1. Sample configuration in a Hall measurement set up. A “rectangular” block is used as the sample.

Suppose we pass a current through that sample perpendicular to the magnetic field, say in the *x*-direction.

The charge carriers are electrons and they drift with drift speed $\vec{v}_d$ in the opposite direction –x..

At the instant shown in Fig. 2a, an external magnetic field $\vec{B}$ pointing out of the plane of the figure has just been turned out. Magnetic force $\vec{F}_B$ acts on each drifting electron, pushing it toward the left edge of the sample.

As time goes on, electrons move to the left piling up on the left edge of the strip, leaving uncompensated positive charges in fixed positions at the right edge. The separation of positive and negative charges produces an electric field $\vec{E}$ within the sample pointing from right to left (Fig. 2b). This field acts an electric force $\vec{F}_E$ on each electron tending to push it to the left.

In an equilibrium, the electric force $\vec{F}_E$ balances the magnetic force $\vec{F}_B$ so as to keep current flowing along the x-direction.

$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

$$F_B = ev_x B_z$$

$$\vec{v} = (v_x, 0, 0) \ , \ \vec{B} = (0, 0, B_z)$$

$$\vec{F}_E = q\vec{E}$$

$$F_E = eE_y$$

$$\vec{E} = (0, E_y, 0)$$

Upsurge of the electrical field in the y-direction is called *Hall Effect.* A Hall potential difference U_H is associated with the electric field across the sample of width w as follows:

$$U_H = E_y w$$

Considering the balance of forces on the charge carriers we see that :

$$eE_y = ev_x B_z$$

The total current density is according to the microscopic Ohm's Law $j_x = nev_x$ which yields:

$$eE_y = \frac{eB_z j_x}{ne}$$

$$E_y = \frac{1}{ne} j_x B_z = R_H j_x B_z$$

The Hall coefficient R_H is defined as:

$$R_H = \frac{E_y}{j_x B_z} = \frac{1}{ne}$$

Note that the sign of R_H indicates if the sample is *n-type* or *p-type*. When the carriers are electrons, the Hall coefficient is negative. When the carriers are holes, the Hall coefficient is positive. The Hall coefficient enables also establishing the concentration of the carriers in the sample *n*. Carrier concentration is the number or carriers per unit volume.

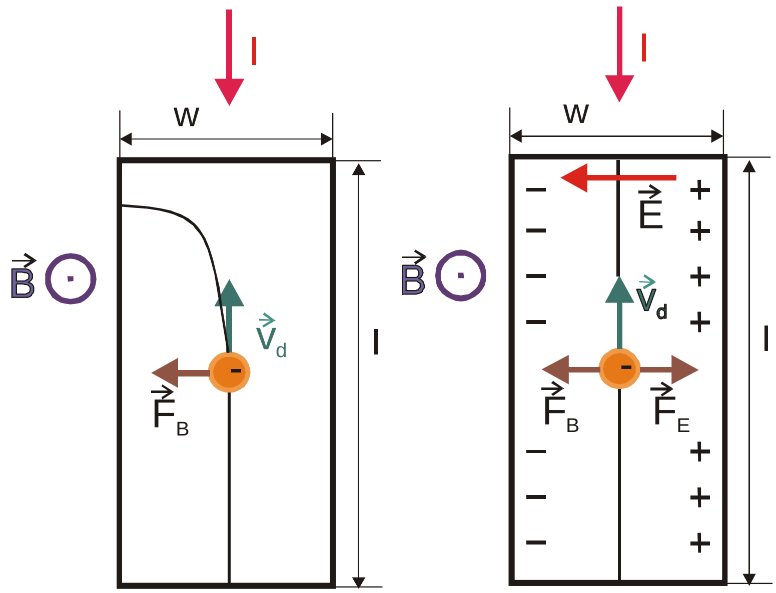

Fig. 2. The idea of Hall effect

Electron velocity v_d is taken to vary linearly with the field E_x :

$$v_d = \mu E_x$$

which is only true under ohmic condition. The proportionality constant between v and E is μ – mobility - and it can be expressed as :

$$\mu = \frac{v_d}{E_x} = \frac{j_x}{neE_x} = \frac{j_x}{E_x}\frac{1}{ne} = \frac{j_x}{E_x}\frac{E_y}{j_x B_z} = \frac{1}{B_z}\frac{E_y}{E_z} = \frac{1}{B_z}\frac{U_H}{w}\frac{l}{U_c} = \frac{1}{B_z}\frac{U_H}{U_c}\frac{l}{w}$$

Voltage $U_c = E_x l$ is called the conductivity voltage i.e. voltage between the contacts in the direction of the current and $U_H = E_y w$ is called the Hall voltage and it is the resultant voltage between the contacts that are perpendicular to the direction of the current.

The mobility μ is used to express conductivity $\sigma = ne\mu$.

Taking into account the dimensions of the sample, the total current is related to the density of charge carriers n and their drift velocity v_d as:

$$I_x = j_x S = j_x wd = (env_d)(wd)$$

i.e. as the product of the current density j_x and the cross-sectional area of the sample S.

Finally, we obtain the following expressions for the :

$$\mu = \frac{1}{B_z}\frac{U_H}{U_c}\frac{l}{w} = R_H\sigma$$

$$R_H = \frac{E_y}{j_x B_z} = \frac{U_H}{w}\frac{wd}{I_x}\frac{1}{B_z} = \frac{U_H}{I_x}\frac{d}{B_z}$$

$$\sigma = ne\mu = \frac{1}{R_H}\mu = \frac{I_x}{U_H}\frac{B_z}{d}\frac{1}{B_z}\frac{U_H}{U_c}\frac{l}{w} = \frac{I_x}{U_c}\frac{1}{dw}$$

4. Equipment

Electromagnet. Hall probe. Measurement panel containing DC power supply to furnish current to Hall probe, voltmeter of high input impedance to measure Hall voltage U_H, voltmeters to measure conductivity voltage U_c and voltmeter to measure voltage U_R across the resistor $R=1000\ \Omega \pm 1\Omega$ connected in series with the Hall probe.

5. Measurements

Fig. 3 shows the measurement set-up.

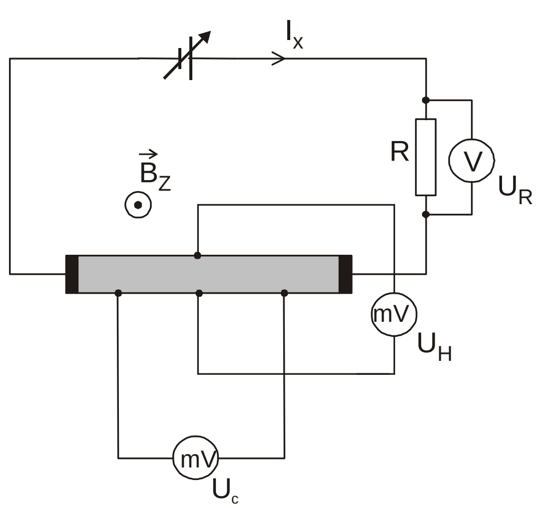

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Fig. 3. The measurement set-up.

Fig. 4 shows the view of the front of the measurement panel. Fig. 5 shows the view of electromagnet.

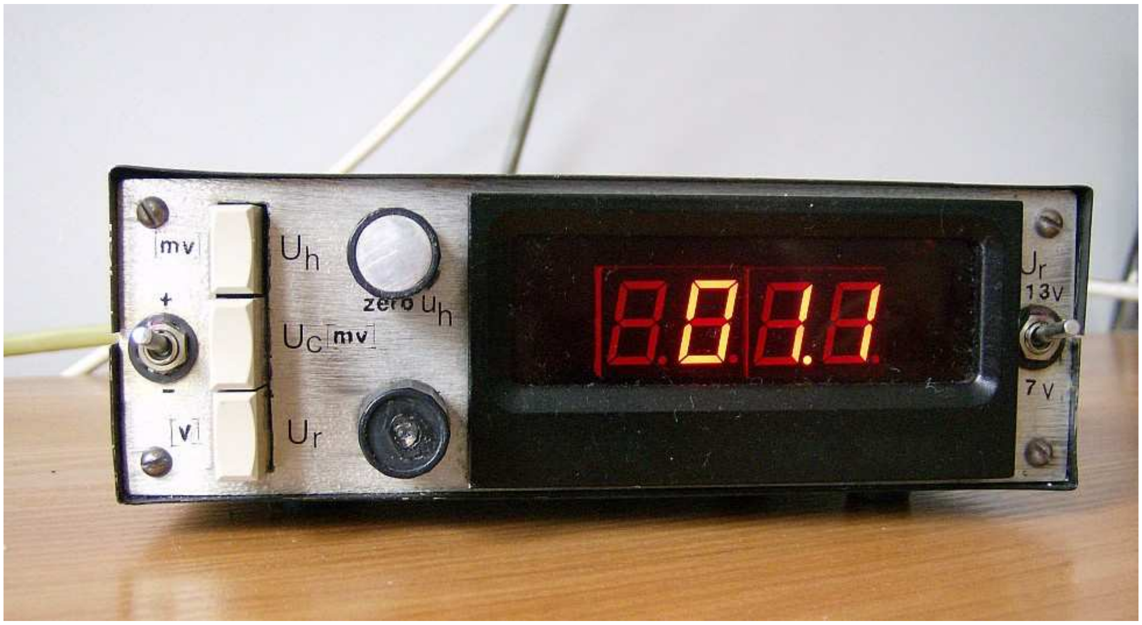

Fig. 4. Measurement panel

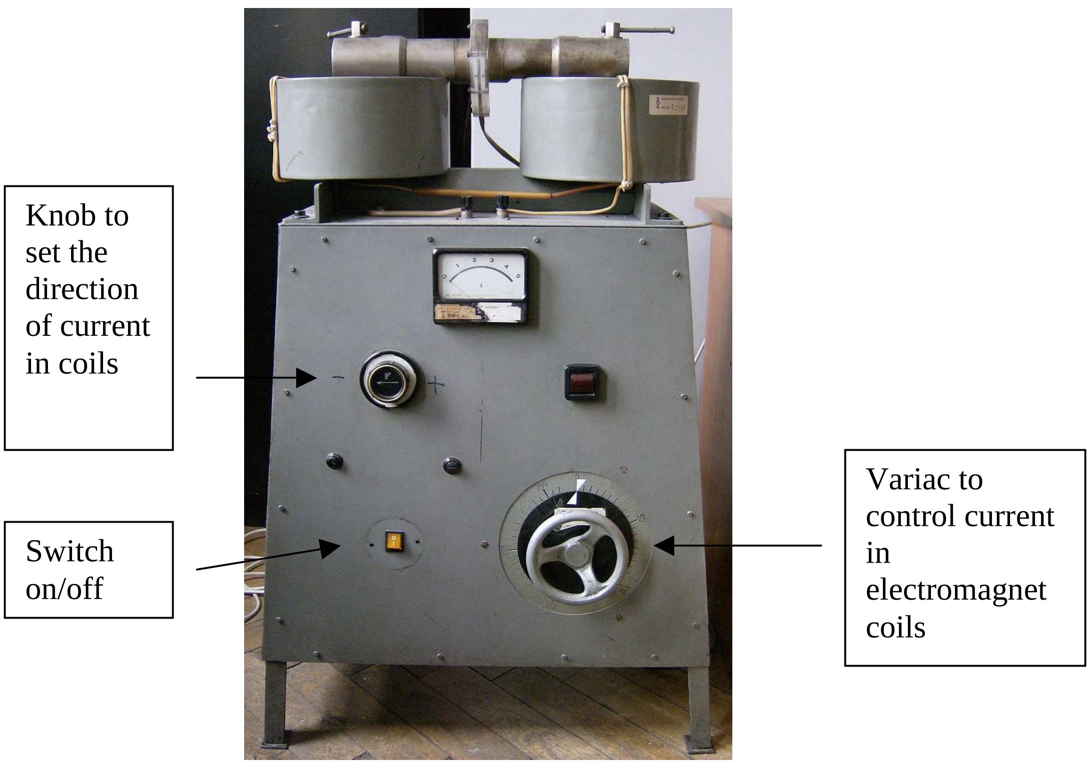

Fig. 5. Electromagnet

1. Turn the measurement panel on (Fig. 4).

2. Push the knob U_R and then set U_R with the lower potentiometer on the front of the measurement panel. The instructor will choose the certain values of U_R for your experiment. U_R

is a voltage across the resistor $R = 1000\ \Omega \pm 1\%$, which is connected in series with the Hall probe and it is used to calculate current I_x flowing through the Hall probe:

$$I_x = \frac{U_R}{R}\ .$$

3. Turn the electromagnet on (Fig. 5). Variac to control current in electromagnet coils should be set for zero current.

4. Set the direction of current through the electromagnet coils " to the right" using the knob with a white arrow on the front of electromagnet.

5. Push the knob U_H and using the upper potentiometer on the front of the measurement panel set U_H for zero.

6. Set the current through the electromagnet coils for a certain values given by the instructor. Do not exceed electromagnet current of 6A. Always gradually increase and decrease current in electromagnet coils. Large inductive voltage surges may damage the insulation of coils.

7. For every setting of the current measure U_{H1} and U_{c1} voltage using voltmeters included in the measurement panel. Push the knob U_H and measure the Hall voltage U_{H1} . Push the knob U_c and measure the voltage conductivity U_{c1}.

8. Decrease the current in electromagnetic coils to zero and change the direction of current in coils "to the left" using the knob with white arrows on the front of electromagnet. After this change adjust Hall voltage U_H to zero using the upper potentiometer on the front of the measurement panel.

9. Set the current through the electromagnet coils for the same values as in the point 7. For every setting of the current measure U_{H2} and U_{c2} voltage.

10. Use the electromagnet characteristic $B_z = f(I_{em})$ to establish the magnetic field between the electromagnet poles on the basis of current I_{em} in electromagnet coils (Fig. 6).

11. Write down the results in the Table 1.

6. Data handling

1. Write down the results in the Table 1

2. Draw the relationships: $U_H=f(U_c)$, $U_H=f(U_R)$, $U_R=f(U_c)$ for each B_z that corresponds to I_{em} = 1, 1.5, 2, 2.5, 3.0, 3.5, 4.0 A, respectively.

3. For each relationship find the "best fit" straight line through the data using the linear regression method, find the regression coefficients of the line *a* and *b* and the uncertainties of coefficients Δa and Δb.

4. Find the carrier mobility μ, Hall coefficient R_H and conductivity σ of the Hall probe using the relationships:

$$U_H = \mu B_z \frac{w}{l} U_c$$

$$U_H = R_H \frac{B_z}{Rd} U_R$$

$$U_R = \sigma \frac{Rwd}{l} U_c$$

and the proper linear regression equations. Be careful: use the one parameter linear regression (b=0)

5. Find the concentration of carriers n on the basis of the relationship $R_H = \frac{1}{ne}$.

6. Calculate the uncertainties $\Delta\mu$, ΔR_H, $\Delta\sigma$, Δn .

Literature:

1. Halliday, Resnick "Fundamentals of Physics - 8th edition", John Wiley 2007,

Table 1 . Results

			←		→			
U_R [V]	I_{em} [A]	B_z [Wb/m2]	U_{H1} [mV]	U_c [mV]	U_{H2} [mV]	U_{c2} [mV]	U_H [mV]	U_c [mV]
4	1							
	1,5							
	2							
	2,5							
	3							
	3,5							
	4							
5	1							
	1,5							
	2							
	2,5							
	3							
	3,5							
	4							
6	1							
	1,5							
	2							
	2,5							
	3							
	3,5							
	4							

$$U_H = \frac{|U_{H1}| + |U_{H2}|}{2} \;, \; U_c = \frac{U_{c1} + U_{c2}}{2}$$

Dimensions of the Hall probe:
d = $(3{,}7 \pm 0{,}1) \times 10^{-3}$ m,

w = $(1{,}7 \pm 0{,}1) \times 10^{-3}$ m,

l = $(11{,}5 \pm 0{,}2) \times 10^{-3}$ m,

$l_0 = (22{,}4 \pm 0{,}2) \times 10^{-3}$ m.

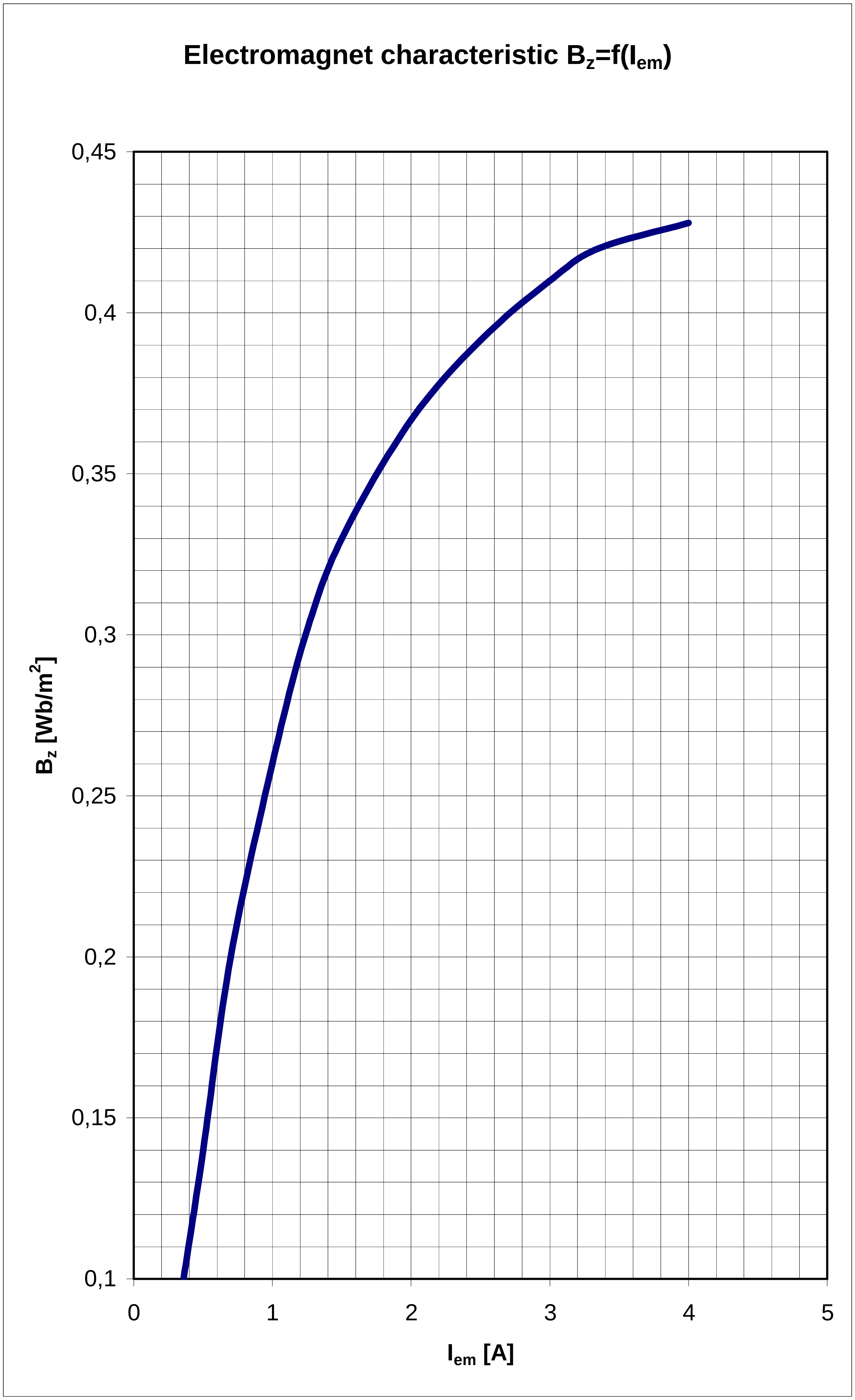

Fig. 6. Electromagnet characteristic

6. Hints to the report.

1. $U_H=f(U_c)$ *for Bz=const.*

On the basis of Table 1 create the sub-table:

I_{em} [A] =1A	U_c [mV]	U_H [mV]	μ [......]	$\frac{\Delta\mu}{\mu}$ [......]
B_z = ….. Wb/m^2				

Draw the plot $U_H=f(U_c)$.

Find the regression coefficient *a* and the uncertainty σ_a for these data.

Use the one parameter linear regression (b=0)

Compare the slopes:

$$U_H = a \cdot U_c$$

$$U_H = \mu B_z \frac{d}{l} \cdot U_c$$

$$a = \mu B_z \frac{d}{l}$$

$$\mu = \frac{al}{dB_z}$$

Find the unit of μ: $[\mu] = \frac{[m]}{\left[m\frac{Wb}{m^2}\right]} = \left[\frac{m^2}{Wb}\right] = \left[\frac{m^2}{Vs}\right]$

Calculate the uncertainty Δμ/μ using the method of logarithmic derivative:

$$\mu = \frac{al}{dB_z}$$

Calculate logarithm of the both sides of the equation:

$$\ln\mu = \ln a + \ln l - \ln B_z - \ln d$$

Derivate both sides of the equation:

$$\left|\frac{\Delta\mu}{\mu}\right| = \left|\frac{\Delta a}{a}\right| + \left|\frac{\Delta l}{l}\right| + \left|\frac{\Delta B_z}{B_z}\right| + \left|\frac{\Delta d}{d}\right|$$

where:

$\Delta a = \sigma_a$ is the uncertainty of the regression coefficient a

$\Delta l = 0{,}2 \cdot 10^{-3} m$, $\Delta b = 0{,}1 \cdot 10^{-3} m$ are the geometrical uncertainties of the probe,

$\Delta B_z =$ is the uncertainty of the B_z established on the basis of the electromagnet characteristic.

Build-up such sub-tables for each values of I_{em} = 1, 1.5, 2, 2.5, 3.0, 3.5, 4.0 A and respective B_z. Calculate μ and Δμ/μ in each case and collect the results in one final table

B_z [Wb/m2]	μ [........]	$\frac{\Delta\mu}{\mu}$ [.......]

.

2 . *$U_H=f(U_R)$ for Bz=const.*

On the basis of Table 1 create the sub-table:

I_{em} [A] =1A	U_R [mV]	U_H [mV]	R_H [......]	$\frac{\Delta R_H}{R_H}$ [......]
B_z = ….. Wb/m^2				

Draw the plot $U_H=f(U_R)$.

Find the regression coefficient *a* and the uncertainty σ_a for these data.

Use the one parameter linear regression (b=0).

Compare the slopes:

$$U_H = a \cdot U_R$$

$$U_H = R_H \frac{B_z}{Rd} U_R$$

$$a = R_H \frac{B_z}{Rd}$$

$$R_H = \frac{aRd}{B_z}$$

Find the unit of R_H .

Calculate the uncertainty of $\Delta R_H/R_H$ using the method of logarithmic derivative:

Build-up such sub-tables for each values of I_{em} = 1, 1.5, 2, 2.5, 3.0, 3.5, 4.0 A and respective B_z. Calculate R_H and $\Delta R_H/R_H$ in each case and collect the results in one final table

B_z [Wb/m^2]	R_H [........]	$\Delta R_H/R_H$ [.......]

3 . $U_R=f(U_{c\_average})$

On the basis of Table 1 create the subtable:

I_{em} [A] =1A	U_c [mV]	U_R [mV]	σ [......]	$\frac{\Delta\sigma}{\sigma}$ [......]
B_z = ….. Wb/m^2		4 000		
		5 000		
		6 000		

Draw the plot $U_R=f(U_c)$.

Find the regression coefficient a and the uncertainty σ_a for these data.

Use the one parameter linear regression (b=0)

Compare the slopes:

$$U_R = a \cdot U_c$$

$$U_R = \sigma \frac{Rwd}{l} U_c$$

$$a = \sigma \frac{Rwd}{l}$$

$$\sigma = \frac{al}{Rwd}$$

Find the unit of σ .

Calculate the uncertainty of $\Delta\sigma/\sigma$ using the method of logarithmic derivative:

Build-up such sub-tables for each values of I_{em} = 1, 1.5, 2, 2.5, 3.0, 3.5, 4.0 A and respective B_z. Calculate R_H and $\Delta R_H/R_H$ in each case and collect the results in one final table

B_z [Wb/m^2]	σ [........]	$\Delta\sigma/\sigma$[.......]

4. $n = \frac{1}{R_H e}$

Remember: e= 1.6 x 10^{-19} C.

Determine the unit of *n*. Calculate the uncertainty Δn using the total differential method:

$$\Delta n = \left|\frac{\delta n}{\delta R_H}\right| \cdot \left|\Delta R_H\right|$$

Collect the results in the final table:

B_z [Wb/m^2]	*n* [........]	Δn [.......]

Updated: 15.02.2009, 11.03.2010 by Barbara Dziurdzia