Mathematical Model of Warm Drawing Process of Magnesium Alloys in Heated Dies

Andrzej Milenin, Piotr Kustra

Faculty of Metal Engineering and Industrial Computer Science, AGH University of Science and Technology, Krakow / Poland, milenin@agh.edu.pl

Due to high compatibility and solubility in human organism, magnesium alloys are often applied in bioengineering. Production of surgical threads to integration of tissue can be application of these types of alloys. This sort of application calls for fine wires with diameters from 0.1 to 0.9 mm. Low workability of the Mg alloys makes a drawing process very complex. The warm drawing process in heated dies is proposed to increase the workability. The purpose of this paper is development of a mathematical model of a warm drawing process of wires made of MgCa0.8 and ZEK100 alloys and determination of optimal parameters with the objective function defined as maximum of workability. The first part of investigation is focused on development of a numerical model, which is based on FE solution of a thermal problem in the die and coupled thermal-mechanical problem in the alloy. A boundary problem is solved accounting for such phenomena as plastic deformation, heat transfer, wire heating due to deformation and friction. Solution of the boundary problem is obtained using the variation principle of rigid-plastic theory. The second part of paper is focused on experimental upsetting and tensile tests. Basing on these tests the flow stress and ductility models were obtained. Tests were performed on the Zwick Z250 machine. The inverse method was used to interpret experimental results. The FE modelling of upsetting and tension tests was used to determine conditions of material fracture. The flow stress and workability models were obtained for temperatures 20 - 300 ^{0}C, strain rates 0.1 - 5 s^{-1} and triaxility factors -0.6 - 0.6. The proposed model is implemented into the Authors' FE code, which is dedicated to modelling of drawing processes. The technical problem defined as determination of optimal drawing velocity, which is helpful to obtain the temperature in deformation zone taking into account fracture criterion, is solved. The optimum drawing schedule for MgCa0.8 and ZEK100 alloys is proposed in the paper. The new models of fraction and flow curves for MgCa0.8 and ZEK100 alloys are the main output of the paper. These material models can be used for other processes of warm deformation of these Mg alloys.

Keywords: Magnesium alloys, Warm drawing process, Formability, Finite element method

Introduction

Due to their high compatibility and solubility in human organism, magnesium alloys (eg. MgCa0.8, ZEK100) are often applied in medicine. These alloys are alternatives for bio-inert materials such as titanium, tantalum and 316L steel. In the Institute of Materials Science of the Leibniz Universität Hannover, new degradable Mg-Ca alloys with an increased biocompatibility have been developed [1-4]. Corrosion research performed in human environment for Mg alloys showed possibility of implants solubility, what can eliminate necessity of surgery operation for implant elimination [2-3]. Production of surgical threads to integration of tissue is an application of these types of alloys which requires fine wires with diameters from 0.1 mm to 0.9 mm. The low plasticity of these alloys causes difficulties in the cold drawing process. In work [5] a new manufacture technology of tubes made of Mg alloys is proposed. In this technology the metal is heated by a hot die and the process of warm deformation is performed. The description of this process is represented in the papers [6,7].

The model of ductility is very important element of FE program for simulation of drawing. It enables the optimization of the process of wire drawing on the basis of simulations. The problem of the prediction of ductility for the magnesium alloys is described in the literature [7-9]. However, these works account only for few parameters of drawing, such as the die angle and the reduction ratio. Aluminium and zinc containing magnesium alloys (e.g. AZ31) are the investigated materials, which have a bigger plasticity than MgCa0.8 and ZEK100 alloys. The yield stress and ductility models of the latter alloys for warm deformation are not available in the literature.

The purpose of this paper is the development of mathematical models of yield stress and ductility for MgCa0.8 and ZEK100 alloys, implementation of these models into FE code and simulations of wire drawing processes in heated die. The practical conditions of drawing processes for thin wire made of Mg alloys are proposed in the paper.

FEM Model of Wire Drawing

The FE code Drawing2d developed by A. Milenin [10] is used. The FE model solves a boundary problem considering such phenomena as metal deformation, heat transfer in die and wire, metal heating due to deformation and friction.

Model of Metal Deformation. Solution of boundary problem is obtained using variation principle of rigid-plastic theory:

$$J = \int_V \int_0^{\xi_i} \sigma_s(\varepsilon_i, \xi_i, t) d\xi_i dV + \int_V \sigma_0 \xi_0 dV - \int_S \sigma_\tau v_\tau dS \tag{1}$$

where ξ_i is the effective strain rate, σ_s is the yield stress, ε_i is the effective strain, t is the temperature, V is the volume, σ_0 is the mean stress, ξ_0 is the volumetric strain rate, S is the contact area between the alloy and the die, σ_τ is friction stress and v_τ is the alloy slip velocity along area of die.

The friction stress is determined according to law:

$$\sigma_\tau = f_{tr} \frac{\sigma_s}{\sqrt{3}} \left[1 - \exp\left(-\frac{1.25\sigma_n}{\sigma_s} \right) \right] \tag{2}$$

where f_{tr} is the friction coefficient and σ_n is the normal stress on contact between the alloy and the die.

The stress tensor σ_{ij} is calculated on the basis of strain rate tensor ξ_{ij} according to following equation:

$$\sigma_{ij} = \delta_{ij}\sigma_0 + \frac{2\sigma_s}{3\xi_i}\xi_{ij} \tag{3}$$

The stationary formulation of the task is used. The tensor ε_{ij} is calculated by integration along the flow lines:

$$\varepsilon_{ij} = \int_0^{\tau}\xi_{ij}(\tau)d\tau = \sum_{p=1}^{p=p_\tau}\xi_{ij}^{(p)}\Delta\tau^{(p)} \tag{4}$$

where $\Delta\tau^{(p)}$ ia the time increment and $\xi_{ij}^{(p)}$ is the strain rate tensor determined according to equation:

$$\xi_{ij}^{(p)} = \sum_{n=1}^{n_{nd}} N_n \xi_{ijn} \tag{5}$$

where N is the finite element shape functions, ξ_{ijn} is the nodal strain rate tensor for current finite element, n_{nd} is the number of nodes in element.

The points of flow lines are determined on the basis of the values of the velocity at point p, which are calculated according to the formula:

$$v_i^{(p)} = \sum_{n=1}^{n_{nd}} N_n v_{in} \tag{6}$$

The calculation of the position of the next point (p+1) of flow line is carried out according to the equation:

$$x_i^{(p+1)} = x_i^{(p)} + v_i^{(p)}\Delta\tau \tag{7}$$

FEM Solution of Thermal Problem in Alloy. This problem is solved by applying the following method. The passage of the section through the zone of deformation is simulated. For this section at each time step the non-stationary temperature problem is examined:

$$\lambda\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right) + Q_d = c\rho\frac{dt}{d\tau} \tag{8}$$

where $Q_d = 0.9\sigma_s\xi_i$ is the deformation power, c is the specific heat, ρ is the alloy density, τ is the time and λ is the thermal conductivity coefficient (the following values are used for MgCa0.8 and ZEK100 alloys: c = 624 J /kgK, ρ = 1738 kg /m^3, λ = 126 J /mK). Heat exchange between the alloy and the die is defined as:

$$q_{conv} = \alpha(t - t_{die}) \tag{9}$$

where t_{die} is the die temperature and α is the heat exchange coefficient.

The generation of heat from the friction is calculated according to the formula:

$$q_{fr} = 0.9\sigma_\tau v_\tau \tag{10}$$

FEM Solution of Thermal Problem in Die. The model of temperature distribution in the die is based on the solution of Fourier equation in the cylindrical coordinate system:

$$\lambda\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial y^2}\right) + Q_h = 0 \tag{11}$$

where Q_h is the power of the heating element.

The heat Q_h is generated in the finite elements, which correspond to the position of heating device. The boundary problem is solved on the basis of the variation formulation of Eq. (11). For the areas, which are in contact with the metal, the temperature of the alloy is obtained from the solution of the thermal problem for the metal.

Materials Tests

In the present paper, the flow stress and ductility models for MgCa0.8 (Mg 99.2 %, Ca 0.8 %) and ZEK100 (Mg 98%, Zn 1 %, rare earths 0.5 %, Zr 0.5 %) alloys were obtain. Upsetting and tensile tests were performed on the Zwick Z250 machine at the AGH University of Science and Technology. Results of the upsetting tests were used to determine the flow stress model and results of both tests were used for identification of workability model.

Conditions and Results of Experiment. Cylindrical samples ø8 mm, h = 10 mm were used for upsetting tests. The sample for the tensile tests is presented in the **Figure 1**. Conditions and selected results of experiment are presented in **Table 1** (upsetting tests) and **Table 2** (tensile tests). For the upsetting samples 1 and 2 (Table 1) the destruction of the sample was not initiated.

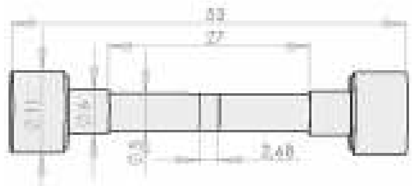


Figure 1. Drawing of sample for tensile tests.

Table 1. Conditions of upsetting tests.

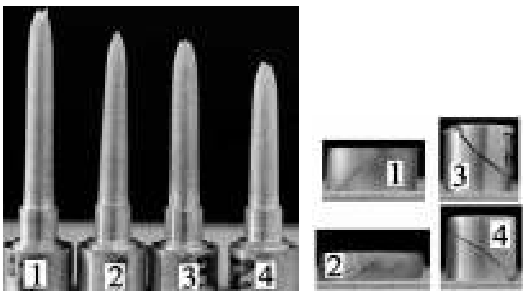
Sample	Initial temperature, ^{0}C	Tool velocity, mm/min	The deformation, which corresponds to destruction of sample, mm (MgCa0.8 /ZEK100)
1	300	60	5.80* / 5.04*
2	300	600	5.60* / 5.56*
3	250	60	6.10 / 4.15
4	250	600	4.70 / 2.96
5	200	60	3.00 / 2.76
6	200	600	2.30 / 2.26
7	100	60	1.80 / 1.55
8	20	10	1.50 / 1.57

*The destruction of the sample did not occur

The results of experiment show that at temperatures above 250 ^{o}C the MgCa0.8 alloy is more ductile than the ZEK100. Opposite tendency is observed at low temperature (tests 3-8 in Table 2 and test 8 in Table 1). The essential influence of the deformation rate on the workability is also observed. In the process of upsetting tests the initiation of cracks was the criterion of the interrupting the test. Examples of samples after destruction are shown in **Figure 2**. Figure 2(a) shows the samples after tensile tests, which correspond to the following tests in Table 2: samples 1, 2 – test 1, samples 3, 4 – test 2. Figure 2(b) shows the samples after upsetting tests, which correspond to the following tests from Table 1: samples 3, 4 – test 8, samples 1, 2 – test 3.

Table 2. Conditions of tension tests.

Sample	Initial temperature, ^{0}C	Tool velocity, mm/min	The deformation, which corresponds to destruction of sample, mm (MgCa0.8 / ZEK100)
1	300	60	22.5 / 16.9
2	300	600	16.0 / 13.4
3	250	60	14.0 / 14.1
4	250	600	8.50 / 10.7
5	200	60	7.50 / 11.3
6	200	600	- / 9.34
8	20	10	1.55 / 3.94



(a) (b)

Figure. 2. Samples of MgCa0.8 (2, 4) and ZEK100 (1 ,3) after deformation in tensile tests (a) and upsetting tests (b) for conditions from Tables 1 and 2.

Yield Stress Model. For obtaining the model of flow stress the load-displacement curves from upsetting tests were used. Model of yield stress was proposed as a modified Henzel-Spittel equation:

$$\sigma_s = A e^{-m_1 t} \varepsilon_i^{m_2} \xi_i^{m_3 \left(\frac{t-20}{280}\right)^{m_6}} e^{\frac{m_4}{\varepsilon_i}} \left(1+\varepsilon_i\right)^{m_5 t} e^{m_7 \varepsilon_i} \xi_i^{m_8 t} t^{m_9} \qquad (12)$$

where A, m_1 - m_9 are empirical coefficients.

The coefficients in equation (12) were determined using the inverse approach with the least squares method. The objective function was formulated as the root-mean-square difference between experimental and predicted loads. The following values of coefficients were obtained:

MgCa0.8: A=447.4; m_1=0.0007542; m_2=0.4485; m_3=0.2867; m_4=-0.0001899; m_5=-0.009392; m_6=2; m_7=0.8318; m_8=-0.0004359; m_9=0.007962.

ZEK100: A=656.5; m_1=0.001210; m_2=0.4445; m_3=0.05207; m_4=-0.0006153; m_5=-0.009350; m_6=2; m_7=0.5107; m_8=0.0002455; m_9=0.01805.

A relative error in the objective function was 0.055 (MgCa0.8) and 0.052 (ZEK100).

Ductility Model. The key parameter, which presents fracture is called ductility function. This parameter is defined by the following formula:

$$\psi = \frac{\varepsilon_i}{\varepsilon_p\left(k,t,\xi_i\right)} < 1 \qquad (13)$$

where k is the triaxility factor, $k = \sigma_0 / \sigma_s$.

Critical deformation function $\varepsilon_p\left(k,t,\xi_i\right)$ is obtained on the basis of experimental results for the upsetting and the tension tests. In the Drawing2d FEM code [10] equation (13) is implemented as an integral:

$$\psi = \int_0^{\tau} \frac{\xi_i}{\varepsilon_p\left(k,t,\xi_i\right)} d\tau \approx \sum_{m=1}^{m=m_\tau} \frac{\xi_i^{(m)}}{\varepsilon_p\left(k,t,\xi_i\right)} \Delta\tau^{(m)} \qquad (14)$$

where τ is the time of deformation, $\Delta\tau^{(m)}$ the time increment, $\xi_i^{(m)}$ is the values of the strain rate in the current time and m is a index number of time step during numerical integration along the flow line.

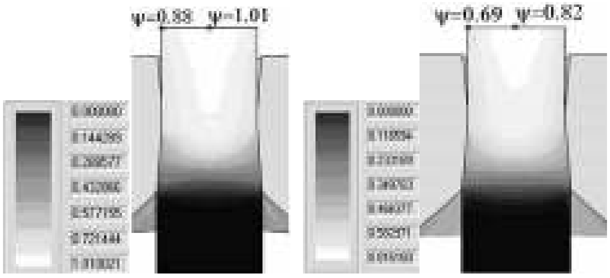
The numerical integration of Eq. (14) along the flow lines is carried out according to Eqs. (4)-(7). The following function of critical deformation is proposed:

$$\varepsilon_p = d_1 \exp\left(-d_2 k\right) \exp\left(d_3 t\right) \xi_i^{d_4} \qquad (15)$$

The parameters d_1-d_4 in Eq. (15) were obtained using ductility test for different values of k,t,ξ_i, which are described in previous section of this paper. Interpretation of results of tensile and upsetting tests was done using the inverse algorithm. The FEM models of tests were created for determining conditions of ductility (k,t,ξ_i). Change of values k,t,ξ_i during deformation was calculated for that part of the test, where initiation of fracture occurred. Ductility function for each test was calculated on the basis of Eqs. (14) and (15). The difference between experimental and calculated value of ductility function at the moment of the fracture is used as the objective function. The minimum of the objective function is reached by a variation of the coefficients. The following values of coefficients were obtained: MgCa0.8: d_1 = 0.01531; d_2 = 0.1288; d_3 = 0.01576; d_4= -0.2354. ZEK100: d_1= 0.05503; d_2= 0.1388; d_3= 0.01036; d_4= -0.1216. A relative error in the objective function was 0.04 (MgCa0.8) and 0.025 (ZEK100).

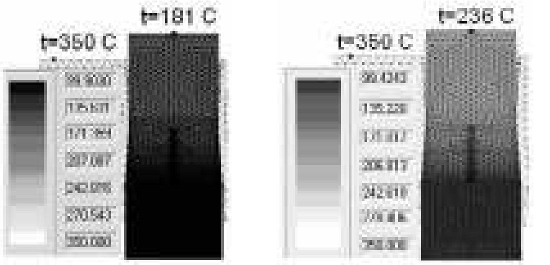
Results of FEM Modelling of Wire Drawing

After the implementation of the models (12) - (15) into the code of Drawing2d the simulation of the wire drawing was performed. The initial wire diameter was 0.5 mm. The following parameters of the process were changed: velocity 0.02 - 0.1 m /s, final diameter of wire 0.38 - 0.46 mm (elongation 1.73 - 1.18 respectively). The value of ductility function is accepted as the objective function. Examples of the obtained non-optimal (a) and optimal (b) solutions are shown in **Figures 3-6**. Since the alloy MgCa0.8 is characterized by smaller ductility at a low temperature, the lower velocity of wire drawing is proposed for it, Figures 3-4. The large plasticity of alloy ZEK100 makes it possible to draw it with the higher velocity (0.05 m /s) and with the larger elongation per pass (1.235) (see Figures 5 and 6).



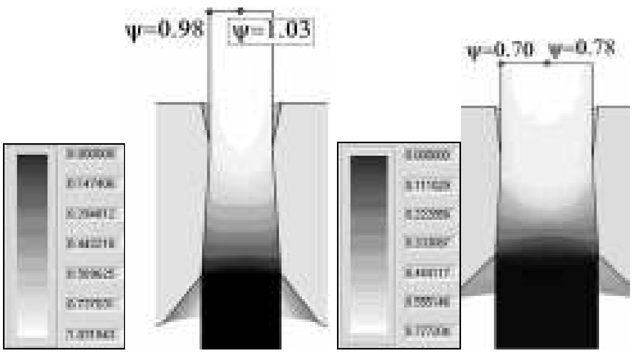
(a) (b)

Figure 3. Distribution of ductility function during drawing of MgCa0.8 alloy for deformation from ø0.5 mm to ø0.46 mm and drawing velocity a) 0.05 m /s and b) 0.02 m /s.



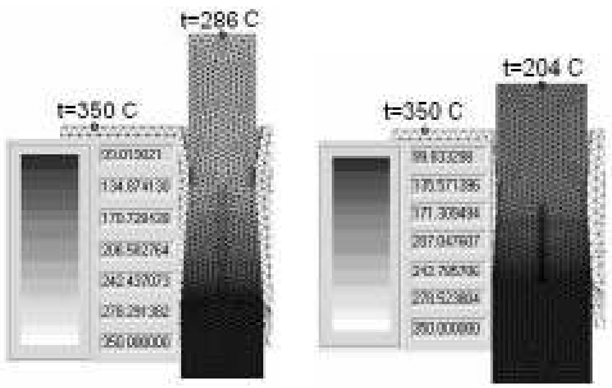
(a) (b)

Figure 4. Distribution of temperature during drawing of MgCa0.8 alloy for deformation from ø0.5 mm to ø0.46 mm and drawing velocity a) 0.05 m /s and b) 0.02 m /s.



(a) (b)

Figure 5. Distribution of ductility function during drawing of ZEK100 alloy for drawing velocity 0.05 m /s and deformation: a) from ø0.5 mm to ø0.4 mm; b) from ø0.5 mm to ø0.45 mm.



(a) (b)

Figure 6. Distribution of temperature during drawing of ZEK100 alloy for drawing velocity 0.05 m /s and deformation: a) from ø0.5 mm to ø0.4 mm; b) from ø0.5 mm to ø0.45 mm.

Modelling of multipass drawing technology assumes that the value of the ductility function should not exceed 0.9 in each pass. In the case of the optimal variant MgCa0.8 alloy drawing is composed of 23 passes with a coefficient of elongation equal to 1.15. Proposed drawing schedule for MgCa0.8 alloy is: 0.5→ 0.464→ 0.431→ 0.4→0.371 → 0.344→ 0.32→ 0.298→ 0.277 → 0.258→ 0.24→ 0.223→ 0.208→ 0.194→ 0.181→ 0.169→ 0.158→ 0.148→ 0.138→ 0.129→ 0.121→ 0.113→ 0.16→ 0.1, with drawing velocity 0.02 m /s. Proposed drawing schedule for ZEK100 (15 passes) is: 0.5→ 0.449→ 0.403→ 0.362→ 0.326→ 0.292→ 0.263→ 0.236 → 0.212→ 0.190→ 0.171→ 0.154→ 0.138→ 0.124→ 0.111→ 0.100 with drawing velocity 0.05 m/s. The angle of die was 4º and friction coefficient 0.03. The experimental verification of drawing schedule is the follow-up objective of the authors, which they will carry out together with the German partner of the project within the framework of program DFG-SFB.

Conclusions

1. The mathematical models of yield stress and ductility function for MgCa0.8 and ZEK100 alloys were proposed. For identification of empirical parameters of material models the tensile and upsetting tests were performed on testing machine Zwick Z250.
2. The results of experiment showed that the MgCa0.8 alloy is more plastic than the ZEK100 at a temperature above 250°C. However, at a low temperature alloy ZEK100 is more plastic. The essential influence of the deformation rate on the plasticity also is observed.
3. The FEM model of wire drawing processes in heated die was developed. The simulations of drawing processes were helpful for determination of technological parameters of drawing.

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