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A comment to Matkowski's paper



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ABSTRACT

A lemma presented by Matkowski requires a proof.

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Let us recall Definition 6 from [2].

Definition 1. A metric space (X, d) is a bead space if the following is satisfied

$$\begin{aligned} &\text{for every } r > 0, \beta > 0 \text{ there exists a } \delta > 0 \text{ such that for each } x, y \in X \text{ with } d(x, y) \geq \beta \\ &\text{there exists a } z \in X \text{ such that } B(x, r + \delta) \cap B(y, r + \delta) \subset B(z, r - \delta). \end{aligned} \quad (1)$$

Clearly " \geq " in (1) can be replaced by " $>$ ".

It appears that a normed space is uniformly convex iff it is a bead space [3, Theorem 14].

A consequence of the following lemma was presented by Matkowski in [1] as Lemma 3 without a proof.

Lemma 2. Let X be a set in a linear space such that $(X + X)/2 \subset X$ and let $p : X - X \rightarrow \mathbb{R}$ be a mapping for which formula $d(x, y) = p(x - y)$, $x, y \in X$ defines a metric on X . Assume that the following condition holds

$$\begin{aligned} &\text{for every } r > 0, \beta > 0 \text{ there exists a } \delta > 0 \text{ such that for each } s, t, s - t \in X - X \\ &\text{the inequalities } p(s) < r + \delta, p(t) < r + \delta \text{ and } p(s - t) > \beta \text{ yield } p((s + t)/2) < r - \delta. \end{aligned} \quad (2)$$

Then (X, d) is a bead space.

Proof. Let us adopt $s = u - x$, $t = v - y$ for some $u, v, x, y \in X$. Then we obtain

$$(s + t)/2 = (u - x + v - y)/2 = (u + v)/2 - (x + y)/2 \in X - X,$$

as $(X + X)/2 \subset X$. Consequently, $p((s + t)/2)$ in (2) is well defined.

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Let $u, x, y \in X$ be such that $d(u, x) < r + \delta$, $d(u, y) < r + \delta$ and $d(x, y) > \beta$. Then for $s = u - x$, $t = u - y$ we have $p(s) < r + \delta$, $p(t) < r + \delta$ and for $s - t = y - x \in X - X$, $p(s - t) = p(u - x - (u - y)) = p(y - x) = p(x - y) > \beta$ holds. Now condition (2) yields

$$p(u - (x + y)/2) = p((u - x + u - y)/2) = p((s + t)/2) < r - \delta,$$

i.e. $u \in B(z, r - \delta)$ for $z = (x + y)/2 \in X$ and we obtain (1) as u is an arbitrary element of the respective balls. \square

Now the fixed point theorems for bead spaces are valid for (X, d) presented in Lemma 2. Therefore, Matkowski's Theorem 3 is a consequence of [2, Theorem 17] (see also [3, Theorem 4]).

References

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