Higher-order Ck basis functions in 1D

Brak plików do pobrania.

Wyprowadzenie 1: Higher order basic B-spline functions

Knots vector $[0 \quad 0 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 5 \quad 5]$ in which the first and last knot are repeated 3 = 2 + 1 times, it defines us the second order basis functions.

In order to illustrate the basis functions resulting from the various knot vectors, we recommend the attached MATLAB code.

So we have now

$\xi_1 = \xi_2 = \xi_3 = 0, \xi_4 = 1, \xi_5 = 2, \xi_6 = 3, \xi_7 = 4, \xi_8 = \xi_9 = \xi_{10} = 5$

Since the number of knots the vector has changed, we need to redefine the new zero and first order basis functions (because there will be more of them and they will have different numbering).

Zero-degree basis functions:

$B_{1,0} = 1 \text{ dla } x \in [\xi_1, \xi_2 = [0, 0] = \{0\}$, 0 in other points,

$B_{2,0}=1$ dla $x\in[\xi_2,\xi_3]=[0,0]=\{0\}$, 0 in other points,
$B_{3,0}=1$ dla $x\in[\xi_3,\xi_4]=[0,1]$, 0 in other points,
$B_{4,0}=1$ dla $x\in[\xi_4,\xi_5]=[1,2]$, 0 in other points,
$B_{5,0}=1$ dla $x\in[\xi_5,\xi_6]=[2,3]$, 0 in other points,
$B_{6,0}=1$ dla $x\in[\xi_6,\xi_7]=[3,4]$, 0 in other points,
$B_{7,0}=1$ dla $x\in[\xi_7,\xi_8]=[4,5]$, 0 in other points,
$B_{8,0}=1$ dla $x\in[\xi_8,\xi_9]=[5,5]=\{5\}$, 0 in other points,
$B_{9,0}=1$ dla $x\in[\xi_9,\xi_{10}]=[5,5]=\{5\}$, 0 in other points

Similarly, we need to redefine the first order basis functions for the new knot vector. We recall the formula for $p=1$

$$B_{i,1}(\xi)=\frac{\xi-\xi_i}{\xi_{i+1}-\xi_i}B_{i,0}(\xi)+\frac{\xi_{i+2}-\xi}{\xi_{i+2}-\xi_{i+1}}B_{i+1,0}(\xi)$$

where we insert the next knots:

$$B_{1,1}(\xi)=\frac{\xi-\xi_1}{\xi_2-\xi_1}B_{1,0}(\xi)+\frac{\xi_3-\xi}{\xi_3-\xi_2}B_{2,0}(\xi)=\frac{\xi-0}{0-0}B_{1,0}(\xi)+\frac{0-\xi}{0-0}B_{2,0}(\xi)=0$$
$$B_{2,1}(\xi)=\frac{\xi-\xi_2}{\xi_3-\xi_2}B_{2,0}(\xi)+\frac{\xi_4-\xi}{\xi_4-\xi_3}B_{3,0}(\xi)=\frac{\xi-0}{0-0}B_{2,0}(\xi)+\frac{1-\xi}{1-0}B_{3,0}(\xi)=1-\xi \text{ dla } \xi\in[0,1]$$
$$B_{3,1}(\xi)=\frac{\xi-\xi_3}{\xi_4-\xi_3}B_{3,0}(\xi)+\frac{\xi_5-\xi}{\xi_5-\xi_4}B_{4,0}(\xi)=\frac{\xi-0}{1-0}B_{3,0}(\xi)+\frac{2-\xi}{2-1}B_{4,0}(\xi)=\xi \text{ dla } \xi\in[0,1],2-\xi \text{ dla } \xi\in[1,2]$$
$$B_{4,1}(\xi)=\frac{\xi-\xi_4}{\xi_5-\xi_4}B_{4,0}(\xi)+\frac{\xi_6-\xi}{\xi_6-\xi_5}B_{5,0}(\xi)=\frac{\xi-1}{2-1}B_{4,0}(\xi)+\frac{3-\xi}{3-2}B_{5,0}(\xi)=\xi-1 \text{ dla } \xi\in[1,2],3-\xi \text{ dla } \xi\in[2,3]$$
$$B_{5,1}(\xi)=\frac{\xi-\xi_5}{\xi_6-\xi_5}B_{5,0}(\xi)+\frac{\xi_7-\xi}{\xi_7-\xi_6}B_{6,0}(\xi)=\frac{\xi-2}{3-2}B_{5,0}(\xi)+\frac{4-\xi}{4-3}B_{6,0}(\xi)=\xi-2 \text{ dla } \xi\in[2,3],4-\xi \text{ dla } \xi\in[3,4]$$
$$B_{6,1}(\xi)=\frac{\xi-\xi_6}{\xi_7-\xi_6}B_{6,0}(\xi)+\frac{\xi_8-\xi}{\xi_8-\xi_7}B_{7,0}(\xi)=\frac{\xi-3}{4-3}B_{6,0}(\xi)+\frac{5-\xi}{5-4}B_{7,0}(\xi)=\xi-3 \text{ dla } \xi\in[3,4],5-\xi \text{ dla } \xi\in[4,5]$$
$$B_{7,1}(\xi)=\frac{\xi-\xi_7}{\xi_8-\xi_7}B_{7,0}(\xi)+\frac{\xi_9-\xi}{\xi_9-\xi_8}B_{8,0}(\xi)=\frac{\xi-4}{5-4}B_{7,0}(\xi)+\frac{5-\xi}{5-5}B_{8,0}(\xi)=\xi-4 \text{ dla } \xi\in[4,5]$$
$$B_{8,1}(\xi)=\frac{\xi-\xi_8}{\xi_9-\xi_8}B_{8,0}(\xi)+\frac{\xi_{10}-\xi}{\xi_{10}-\xi_9}B_{9,0}(\xi)=\frac{\xi-5}{5-5}B_{8,0}(\xi)+\frac{5-\xi}{5-5}B_{9,0}(\xi)=0$$

Second degree basis functions are obtained again using the formula for $p=2$, assuming that the subsequent knots inserted into the denominator must be different, and if they are not different, then the given term is changed to zero. The elements that disappear are marked in red. In the final stage of the derivation, we insert the formulas calculated before $B_{1,1}(\xi)=0$, $B_{2,1}=1-\xi$ dla $\xi\in[0,1]$, $B_{3,1}=\xi$ dla $\xi\in[0,1], 2-\xi$ dla $\xi\in[1,2]$, $B_{4,1}=\xi-1$ dla $\xi\in[1,2], 3-\xi$ dla $\xi\in[2,3]$,

$B_{5,1}=\xi-2 \text{ dla } \xi\in[2,3], 4-\xi \text{ dla } \xi\in[3,4]$, $B_{7,1}=\xi-4 \text{ dla } \xi\in[4,5]$, i $B_{8,1}=0$. At this point, we write down the formulas for up to three elements on which the second-order B-spline function is determined.

$$B_{1,2}(\xi)=\frac{\xi-\xi_1}{\xi_3-\xi_1}B_{1,1}(\xi)+\frac{\xi_4-\xi}{\xi_4-\xi_2}B_{2,1}(\xi)=\frac{\xi-0}{0-0}B_{1,1}(\xi)+\frac{1-\xi}{1-0}B_{2,1}(\xi)=$$
$$=(1-\xi)^2 \text{ dla } \xi\in[0,1]$$
$$B_{2,2}(\xi)=\frac{\xi-\xi_2}{\xi_4-\xi_2}B_{2,1}(\xi)+\frac{\xi_5-\xi}{\xi_5-\xi_3}B_{3,1}(\xi)=\frac{\xi-0}{1-0}B_{2,1}(\xi)+\frac{2-\xi}{2-0}B_{3,1}(\xi)=$$
$$=\frac{\xi-0}{1-0}[1-\xi \text{ dla } \xi\in[0,1]]+\frac{2-\xi}{2-0}[\xi \text{ dla } \xi\in[0,1], 2-\xi \text{ dla } \xi\in[1,2]]$$
$$=\xi(1-\xi)+\frac{1}{2}(2-\xi)\xi \text{ dla } \xi\in[0,1], \frac{1}{2}(2-\xi)^2 \text{ dla } \xi\in[1,2]]$$
$$B_{3,2}(\xi)=\frac{\xi-\xi_3}{\xi_5-\xi_3}B_{3,1}(\xi)+\frac{\xi_6-\xi}{\xi_6-\xi_4}B_{4,1}(\xi)=\frac{\xi-0}{2-0}B_{3,1}(\xi)+\frac{3-\xi}{3-1}B_{4,1}(\xi)=$$
$$=\frac{1}{2}\xi\,[\xi \text{ dla } \xi\in[0,1], 2-\xi \text{ dla } \xi\in[1,2]]+\frac{3-\xi}{2}[\xi-1 \text{ dla } \xi\in[1,2], 3-\xi \text{ dla } \xi\in[2,3]]=$$
$$=\frac{1}{2}\xi^2 \text{ dla } \xi\in[0,1], \frac{1}{2}\xi(2-\xi)+\frac{1}{2}(3-\xi)(\xi-1) \text{ dla } \xi\in[1,2], \frac{1}{2}(3-\xi)^2 \text{ dla } \xi\in[2,3]$$
$$B_{4,2}(\xi)=\frac{\xi-\xi_4}{\xi_6-\xi_4}B_{4,1}(\xi)+\frac{\xi_7-\xi}{\xi_7-\xi_5}B_{5,1}(\xi)=\frac{\xi-1}{3-1}B_{4,1}(\xi)+\frac{4-\xi}{4-2}B_{5,1}(\xi)=$$
$$=\frac{\xi-1}{3-1}[\xi-1 \text{ dla } \xi\in[1,2], 3-\xi \text{ dla } \xi\in[2,3]]+\frac{4-\xi}{4-2}[\xi-2 \text{ dla } \xi\in[2,3], 4-\xi \text{ dla } \xi\in[3,4]]=$$
$$=\frac{1}{2}(\xi-1)^2 \text{ dla } \xi\in[1,2], \frac{1}{2}(\xi-1)(3-\xi)+\frac{1}{2}(4-\xi)(\xi-2) \text{ dla } \xi\in[2,3], \frac{1}{2}(4-\xi)^2 \text{ dla } \xi\in[3,4]$$
$$B_{5,2}(\xi)=\frac{\xi-\xi_5}{\xi_7-\xi_5}B_{5,1}(\xi)+\frac{\xi_8-\xi}{\xi_8-\xi_6}B_{6,1}(\xi)=\frac{\xi-2}{4-2}B_{5,1}(\xi)+\frac{5-\xi}{5-3}B_{6,1}(\xi)=$$
$$=\frac{1}{2}(\xi-2)\,[\xi-2 \text{ dla } \xi\in[2,3], 4-\xi \text{ dla } \xi\in[3,4]]+\frac{1}{2}(5-\xi)\,[\xi-3 \text{ dla } \xi\in[3,4], 5-\xi \text{ dla } \xi\in[4,5]]=$$
$$=\frac{1}{2}(\xi-2)^2 \text{ dla } \xi\in[2,3], \frac{1}{2}(\xi-2)(4-\xi)+\frac{1}{2}(5-\xi)(\xi-3) \text{ dla } \xi\in[3,4], (5-\xi) \text{ dla } \xi\in[4,5]$$
$$B_{6,2}(\xi)=\frac{\xi-\xi_6}{\xi_8-\xi_6}B_{6,1}(\xi)+\frac{\xi_9-\xi}{\xi_9-\xi_7}B_{7,1}(\xi)=\frac{\xi-3}{5-3}B_{6,1}(\xi)+\frac{5-\xi}{5-4}B_{7,1}(\xi)=$$
$$=\frac{\xi-3}{5-3}[\xi-3 \text{ dla } \xi\in[3,4], 5-\xi \text{ dla } \xi\in[4,5]]+\frac{5-\xi}{5-4}[0]=$$
$$=\frac{1}{2}(\xi-3)^2 \text{ dla } \xi\in[3,4], \frac{1}{2}(\xi-3)(5-\xi) \text{ dla } \xi\in[4,5]$$

$$B_{7,2}(\xi)=\frac{\xi-\xi_7}{\xi_9-\xi_7}B_{7,1}(\xi)+\frac{\xi_{10}-\xi}{\xi_{10}-\xi_8}B_{8,1}(\xi)=\frac{\xi-4}{5-4}B_{7,1}(\xi)+\frac{5-\xi}{5-5}B_{8,1}(\xi)=$$
$$=\frac{\xi-4}{5-4}[0]+0=0$$

We got six second order basis functions (leaving the last seventh equal to zero), $B_{1,2},\ldots,B_{6,2}$.

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