

Informatyka

E-podręczniki

Moduły

Informacja

Strona zapisana (wersja 6).

Non-stationary problems **Brak plików do pobrania.**

Note that the Euler methods used come down to making the L2 isogeometric sequence of the projection, similar to the calculation of the bitmap projection. This time, however, our "bitmap" is the state of the system in the previous moment, plus changes caused by the "physics" of the modeled phenomenon during the time step, plus changes caused by the force acting on our system during the time step.

The method of treating the non-stationary equation as sequences of isogeometric projections has been described in the paper [1]

In order to develop a solver using the Euler method in the isogeometric finite element method, we need to transform a strong formulation into a weak formulation.

So we multiply our equation by the test functions

$$(u_{t+1}, w) = (u_t + dt * \mathcal{L}(u_t) + dt * f_t, w)$$

We will use a linear combination of the B-spline function to approximate the state of our system at a given moment in time. For this purpose, we select the basis of two-dimensional B-spline functions, specifying the node vectors in the direction of the axis of the coordinate system. To establish attention, we can choose a two-dimensional basis of the second order B-spline function

$$\{B_{i,2}^x(x)B_{j,2}^y(y)\}_{i=1,\dots,N_x;j=1,\dots,N_y}$$

They will be used to approximate the simulated scalar field to the current time instant

$$u(x, y; t + 1) \approx \sum_{i=1, \dots, N_x; j=1, \dots, N_y} u_{i,j}^{t+1} B_i^x(x) B_j^y(y)$$

Similarly, for testing, we will use the B-spline base functions:

$$\{B_k^x(x) B_l^y(y)\}_{k=1, \dots, N_x; l=1, \dots, N_y}$$

Our equation now looks identical to the isogeometric L2 projection problem of two dimensions

$$\sum_{i=1, \dots, N_x; j=1, \dots, N_y} u_{i,j}^{t+1} (B_i^x(x) B_j^y(y), B_k^x(x) B_l^y(y)) = RHS \quad \forall k = 1, \dots, N_x; l = 1, \dots, N_y$$

Our right side, however, is not a bitmap, but a projection of the sum of three elements:

1. The state of our system at the previous moment in time $(u_t, w) = \sum_{i=1, \dots, N_x; j=1, \dots, N_y} u_{i,j}^t (B_i^x(x) B_j^y(y), B_k^x(x) B_l^y(y))$ (also multiplied by the test function and cut off by area). Of course, we also use a linear combination of B-spline base functions to represent the state of our system in the previous time step

$$u(x, y; t) = u_t = \sum_{i=1, \dots, N_x; j=1, \dots, N_y} u_{i,j}^t B_i^x(x) B_j^y(y)$$

2. Changes induced by the "physics" of the simulated phenomenon during the time step. These changes are calculated by applying the differential operator representing the modeled phenomenon to the state of the system at the previous moment. The state of the system is, of course, represented by a linear combination of the B-spline function. So the differential operator is applied to the B-spline function

$$(dt * \mathcal{L}(u_t), w) = \sum_{i=1, \dots, N_x; j=1, \dots, N_y} dt * u_{i,j}^t (\mathcal{L}(B_i^x(x) B_j^y(y)), B_k^x(x) B_l^y(y)).$$

For example, for the heat transport problem we have $\sum_{i=1, \dots, N_x; j=1, \dots, N_y} dt * u_{i,j}^t \left(\frac{\partial^2 (B_i^x(x) B_j^y(y))}{\partial x^2} + \frac{\partial^2 (B_i^x(x) B_j^y(y))}{\partial y^2}, B_k^x(x) B_l^y(y) \right)$. Dzięki sformułowaniu słabemu możemy odcałkować przez części

$$\sum_{i=1, \dots, N_x; j=1, \dots, N_y} dt * u_{i,j}^t \left(\left(\frac{\partial (B_i^x(x) B_j^y(y))}{\partial x} B_j^y(y), \frac{\partial (B_k^x(x) B_l^y(y))}{\partial x} B_l^y(y) \right) + \left(B_i^x(x) \frac{\partial (B_i^x(x) B_j^y(y))}{\partial y}, B_k^x(x) \frac{\partial (B_k^x(x) B_l^y(y))}{\partial y} \right) \right)$$

which due to the structure of the tensor product of the B-spline function, and due to the fact that the derivative in the y direction of the B-spline function of the variable x gives 0, and the derivative in the x direction of the B-spline function of the variable y gives, so we get

$$\sum_{i=1, \dots, N_x; j=1, \dots, N_y} dt * u_{i,j}^t \left(\left(\frac{\partial B_i^x(x)}{\partial x} B_j^y(y), \frac{\partial B_k^x(x)}{\partial x} B_l^y(y) \right) + \left(B_i^x(x) \frac{\partial B_j^y(y)}{\partial y}, B_k^x(x) \frac{\partial B_l^y(y)}{\partial y} \right) \right).$$

3. Changes caused by a force acting on the system during the time step $(f_t, w) = (f_t(x, y), B_k^x(x) B_l^y(y))$

In the explicit method, the system matrix is a mass matrix

$$\{M_x \otimes M_y\}_{i,j,k,l} = \int B_i^x(x)B_k^x(x)B_j^y(y)B_l^y(y) = \int B_i^x(x)B_j^y(y)B_k^x(x)B_l^y(y) = \mathbf{M}_{i,j,k,l}$$

which has a Kronecker product structure and can be factored in linear time by a variable-direction solver.

The disadvantage of this method is that a single time step cannot be too long, otherwise the method will be unstable. It will manifest itself in an explosion of numerical simulation. This is due to the so-called Courant-Friedrichs-Lewy condition (CFL condition). In general, it says numerical simulation in which we model a scalar field $u(x, t)$ is stable if $C = \frac{u*dt}{h} < C_{max}$ where $u = \max u(x, t)$ is the maximum value of the simulated field, dt is the length of the time step h is the size of the mesh element (which in the case of isogeometric analysis results from the value of the point in the node vector), and C_{max} is a certain constant resulting from the problem being solved. The practical significance of this condition is as follows. If we increase the number of mesh elements (increase the number of points in the node vector) then h it will decrease and so that our simulation does not "explode" we have to decrease the time step. This is a big limitation because it means that increasing the accuracy in space forces the size of the time steps to decrease, which means a higher simulation cost. One solution to this problem is the use of implicit simulations with diagrams that enable the use of variable direction solvers.

Bibliografia

1. Marcin M.Łoś, Maciej Woźniak, Maciej Paszyński, Andrew Lenharth, Muhammm Amber Hassaan, Keshav Pingali: IGA-ADS: Isogeometric analysis FEM using ADS solver, Computer & Physics Communications, Elsevier 2017, dostęp:18.10.2019

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