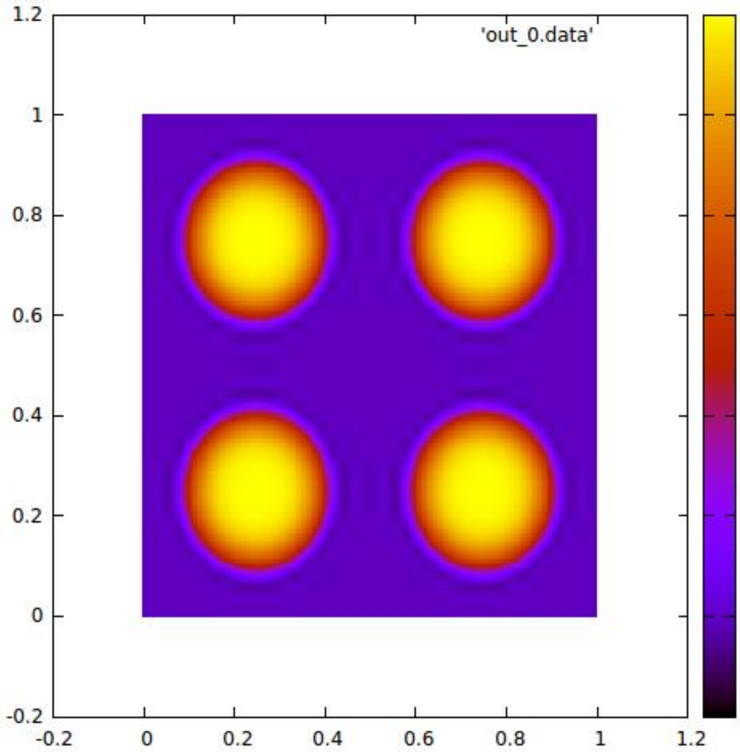
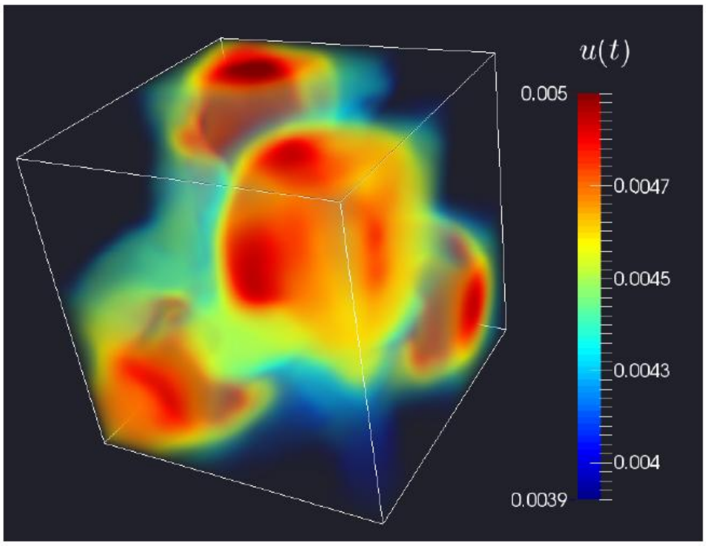
1. Please replace the initial state so we can four balls of heat at the beginning



Please run the simulation until the temperature stabilizes. What is the final uniform temperature?

Please generate GnuPlot or ParaView movies out of it.

2. Please add 4 central pumps (0.25,0.25,0.25), (0.25,0.75,0.75), (0.75,0.25,0.75), (0.75,0.75,0.25) in the non-linear flow in heterogeneous media problem. Please generate ParaView movie out of it.



3. In the simulation of propagation of elastic waves please add symmetric hit from the other side: let forcing represents two intermediate hits at points (1,1,1) and (0,0,1). Run the simulation and generate ParaView movie.

4. Please implement the wave equation

$\frac{\partial^2 u}{\partial t^2} - \Delta u = 0$ on $\Omega = [0,1]^2$

$u(0, x) = 0$

$\frac{\partial u}{\partial t}(0, x) = v_0(x)$= F(x)*(F(y)

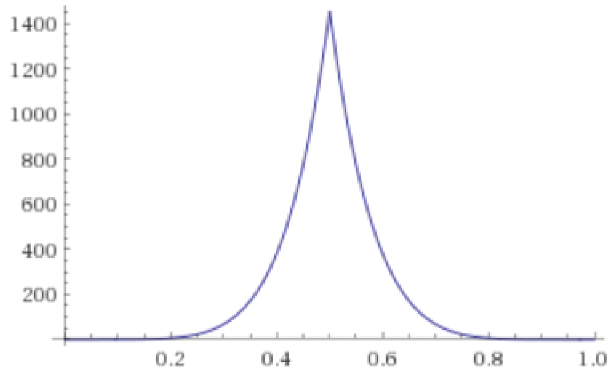
where F(x)= (6x)^6*(sgn(-x+1/2)+1)+ (6(x-1))^6*(sgn(x-1/2)+1)

$u(t, x) = 0$ on on $\partial\Omega$

Input interpretation:

plot	$(6\,x)^6\left(\operatorname{sgn}\left(-x+\frac{1}{2}\right)+1\right)+(6\,(x-1))^6\left(\operatorname{sgn}\left(x-\frac{1}{2}\right)+1\right)$

Plot:



The above equation is converted into a system of two equations

$$v = \frac{\partial u}{\partial t}$$

so we get

$$\frac{\partial v}{\partial t} - \Delta u = 0$$

$$\frac{\partial u}{\partial t} = v$$

We test with functions w and integrate by parts

$$\langle\frac{\partial v}{\partial t}, w\rangle + \langle\nabla u, \nabla w\rangle = 0$$

$$\langle\frac{\partial u}{\partial t}, w\rangle = \langle v, w\rangle$$

and emply Euler time integration scheme scheme

$$\langle v^{t+1}, w\rangle = \langle v^t, w\rangle - dt * \langle\nabla u^t, \nabla w\rangle$$

$$\langle u^{t+1}, w\rangle = \langle v^t + dt * v^t, w\rangle$$

$$u(0, x) = 0$$

$$v(0, x) = v_0(x\ )$$