## 2.25

$$\lim_{n\to\infty} \frac{(-1)^n}{2n-1} = \text{Tw. o trzech ciągach} = 0$$

$$-\frac{1}{2n-1} \le \frac{(-1)^n}{2n-1} \le \frac{1}{2n-1}$$

$$-\frac{1}{2n-1} \xrightarrow{n\to\infty} 0 \qquad \frac{1}{2n-1} \xrightarrow{n\to\infty} 0$$

## 2.31

$$\lim_{n\to\infty} \frac{(-0.8)^n}{2n-5} = \text{Tw o 3 ciągach} = 0$$

$$\frac{-1}{2n-5} < \frac{(-0.8)^n}{2n-5} < \frac{1}{2n-5}$$

$$\frac{-1}{2n-5} \xrightarrow{n\to\infty} 0 \qquad \frac{1}{2n-5} \xrightarrow{n\to\infty} 0$$

## 2.32

$$\lim_{n\to\infty} \frac{2-5n-10n^2}{3n+15} = \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}} \left( \frac{\frac{2}{n} - 5 - 10n}{3 + \frac{15}{n}} \right) =$$

$$\frac{2}{n} \to 0, \qquad \frac{15}{n} \to 0$$

$$= \lim_{n\to\infty} -10n = \underline{\underline{-\infty}}$$

## 2.34

$$\lim_{n\to\infty} \frac{\sqrt{1+2n^2} - \sqrt{1+4n^2}}{n} =$$

$$\left\{ \sqrt{1+2n^2} - \sqrt{1+4n^2} = \frac{1+2n^2-(1+4n^2)}{\sqrt{1+2n^2}+\sqrt{1+4n^2}} = \frac{-2n^2}{\sqrt{1+2n^2}+\sqrt{1+4n^2}} \right\}$$

$$= \lim_{n\to\infty} \frac{-2n^{\cancel{2}}}{\cancel{n}\left(\sqrt{1+2n^2}+\sqrt{1+4n^2}\right)} = \lim_{n\to\infty} \frac{-2\cancel{n}}{\cancel{n}\left(\sqrt{\frac{1}{n^2}+2}+\sqrt{\frac{1}{n^2}+4}\right)} =$$

wyłączamy spod $\sqrt{\cdot}$ $n^2$, na zewnątrz $n$

$$= \frac{-2}{\sqrt{2}+2} = \frac{-2(\sqrt{2}-2)}{2-4} = \sqrt{2}-2.$$

2.53

$$\lim_{n\to\infty}\left(\frac{3}{2}\right)^n \frac{2^{n+1}-1}{3^{n+1}-1} = \left\{ \begin{matrix}\text{wyłączamy}\\ 3^n \text{ i } 2^n \text{ przed}\\ \text{nawias}\end{matrix} \right\}$$

$$= \lim_{n\to\infty} \frac{\cancel{3^n}}{\cancel{2^n}} \frac{\cancel{2^n}(2-2^{-n})}{\cancel{3^n}(3-3^{-n})} =$$

$$= \lim_{n\to\infty} \frac{2-2^{-n}}{3-3^{-n}} = \frac{2}{3}$$

$$\lim_{n\to\infty} 2^{-n} = 0$$

$$\lim_{n\to\infty} 3^{-n} = 0$$

2.56

$$\lim_{n\to\infty} \sqrt[n]{10^{100}} - \sqrt[n]{10^{-100}} = \lim_{n\to\infty}\sqrt[n]{10^{100}} - \lim_{n\to\infty}\sqrt[n]{10^{-100}} =$$

$$= 1 - 1$$

$$\lim_{n\to\infty}\sqrt[n]{a} = 1 \quad , \quad a = \text{const.}$$

2.61, 2.62, 2.63 → nie obowiązuje

2.65

$$\lim_{n\to\infty}\left(1-\frac{1}{n^2}\right)^n =$$

$$k \cdot n^2 = n$$

$$\Downarrow$$

$$k = \frac{1}{n}$$

$$= \lim_{n\to\infty}\left[\left(1-\frac{1}{n^2}\right)^{n^2}\right]^{\frac{1}{n}} =$$

$$= \lim_{n\to\infty} e^{-\frac{1}{n}} = e^0 = 1$$

2.70

$$\lim_{n\to\infty} \left(\frac{n^2+2}{2n^2+1}\right)^{n^2} =$$

$$\frac{n^2+2}{2n^2+1} = \frac{1}{2} \cdot \frac{2n^2+1+3}{2n^2+1} = \frac{1}{2} \cdot \left(1 + \frac{3}{2n^2+1}\right)$$

$$= \lim_{n\to\infty} \left(\frac{1}{2}\right)^{n^2} \left(1 + \frac{3}{2n^2+1}\right)^{n^2} = \lim_{n\to\infty} \left(\frac{1}{2}\right)^{n^2} \left[\left(1 + \frac{3}{2n^2+1}\right)^{(2n^2+1)}\right]^{\frac{n^2}{2n^2+1}} =$$

$$\cancel{k n^2 = 2n^2+1} \qquad k \cdot (2n^2+1) = n^2$$

$$\cancel{k = \frac{2n^2+1}{n^2}} \qquad k = \frac{n^2}{2n^2+1}$$

$$= \left\{ \begin{array}{l} \lim_{n\to\infty} \left(1 + \frac{k}{u_n}\right)^{u_n} = e^k \\ k = \text{const.}, \quad \lim_{n\to\infty} u_n = +\infty \end{array} \right\}$$

$$= \lim_{n\to\infty} \underbrace{\left(\frac{1}{2}\right)^{n^2}}_{\to 0} \cdot \underbrace{\left(e^3\right)^{\frac{n^2}{2n^2+1}}}_{\xrightarrow{n\to+\infty} e^{\frac{3}{2}}} = 0.$$

2.71

$$\lim_{n\to\infty} \sqrt{n+\sqrt{n}} - \sqrt{n-\sqrt{n}} =$$

$$\sqrt{n+\sqrt{n}} - \sqrt{n-\sqrt{n}} = \frac{n+\sqrt{n} - n + \sqrt{n}}{\sqrt{n+\sqrt{n}} + \sqrt{n-\sqrt{n}}} =$$

$$= \frac{2\sqrt{n}}{\sqrt{n+\sqrt{n}} + \sqrt{n-\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{2\sqrt{n}}{\sqrt{n+\sqrt{n}} + \sqrt{n-\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\sqrt{n}}{\sqrt{n}\left(\sqrt{1+\underbrace{\tfrac{1}{\sqrt{n}}}_{\to 0}} + \sqrt{1-\underbrace{\tfrac{1}{\sqrt{n}}}_{\to 0}}\right)} = 1$$

2.74

$$\lim_{n\to\infty} \sqrt[n]{2n^3-3n^2+15} = \lim_{n\to\infty} \sqrt[n]{n^3}\cdot\sqrt[n]{2-\frac{3}{n}+\frac{15}{n^3}} =$$

$= 1$, gdyż

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

podobnie 2.75

2.76

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}} \cdot \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{1}{\sqrt{n^3}}}}} = 1$$

Zadania 2.77 – 2.79 nie obowiązują

1h

$$\lim_{x\to -5} \frac{2x^3+250}{x^2+4x-5} = 2 \lim_{x\to -5} \frac{x^3+125}{x^2+4x-5} =$$

$$= 2\cdot \lim_{x\to -5} \frac{(x+5)(x^2-5x+25)}{x^2+4x-5} =$$

$$\Delta = 16+20 = 36$$

$$x_1 = \frac{-4-6}{2} = -5$$

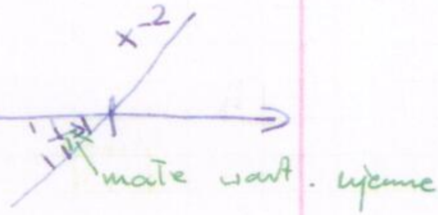
$$x_2 = 1$$

$$= 2\cdot \lim_{x\to -5} \frac{(x+5)(x^2-5x+25)}{(x+5)(x-1)} = 2\cdot\frac{75}{-6} = \frac{75}{-3} = -25$$

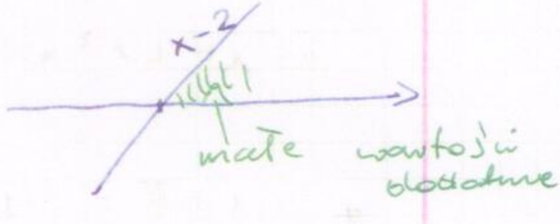
1i

$$\lim_{x\to 2} \frac{x^2-1}{x-2} = \left[\frac{3}{0}\right]$$

$$\lim_{x\to 2^-} \frac{x^2-1}{x-2} = \left[\frac{3}{0^-}\right] = -\infty$$




$$\lim_{x\to 2^+} \frac{x^2-1}{x-2} = \left[\frac{3}{0^+}\right] = +\infty$$




1o

$$\lim_{x\to -2} \frac{x+2}{x^5+32} = \lim_{x\to -2} \frac{x+2}{x^5+2^5} =$$

Wikipedia $a^5+b^5 = (a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$

$$= \lim_{x\to -2} \frac{x+2}{(x+2)(x^4-2x^3+4x^2-8x+16)} = \frac{1}{80}$$

## 2c

Analogicznie jak dla ciągów

$$\lim_{x \to +\infty} \frac{\sin x}{x} \underset{\text{fcjach}}{\overset{\text{Tw. o 3}}{=}} 0$$

$$\frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$$\downarrow \qquad \qquad \downarrow$$

$$0 \qquad \qquad 0$$

## 2d

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{x} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$$

$$\frac{\pi}{2} \Longleftrightarrow 90^\circ$$

miara łukowa
(w radianach)

$$\sin \frac{\pi}{2} = 1$$

## 2h

$$\lim_{x \to \pi} \frac{1 + \cos x}{\sin^2 x} =$$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x =$$
$$= (1 - \cos x)(1 + \cos x)$$

$$\cos \pi = -1$$

$$\sin \pi = 0 \qquad \sin^2 x = (\sin x)^2$$

$$= \lim_{x \to \pi} \frac{1 + \cos x}{(1 - \cos x)(1 + \cos x)} = \frac{1}{2}$$