$$\begin{cases} 4x_1 + 3x_2 + 2x_3 - 2x_4 = -2 \leftarrow W_2 \\ x_1 + 2x_2 + 3x_3 + 3x_4 = 0 \leftarrow W_1 - 4W_2 \\ x_1 - 2x_2 + 2x_3 - x_4 = -5 \leftarrow W_3 - W_2 \\ 2x_1 + 2x_2 + x_3 - 2x_4 = -1 \leftarrow W_4 - 2W_2 \end{cases}$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 3x_4 = 0 \\ -5x_2 - 10x_3 - 14x_4 = -2 \leftarrow W_4 \\ -4x_2 - x_3 - 4x_4 = -5 \leftarrow W_3 - 2W_4 \\ -2x_2 - 5x_3 - 8x_4 = -1 \leftarrow W_2 - \frac{5}{2}W_4 \end{cases}$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 3x_4 = 0 \\ -2x_2 - 5x_3 - 8x_4 = -1 \\ 9x_3 + 12x_4 = -3 \leftarrow \frac{1}{3}W_3 \\ \frac{5}{2}x_3 + 6x_4 = \frac{1}{2} \leftarrow 2W_4 - \frac{5}{9}W_3 \end{cases}$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 3x_4 = 0 \\ -2x_2 - 5x_3 - 8x_4 = -1 \\ \cancel{9x_3 + 12x_4 = -3} \\ 3x_3 + 4x_4 = -1 \\ \frac{16}{3}x_4 = \frac{8}{3} \end{cases}$$

Układ rów. z macierzą trójkątną górną

$$\begin{cases} \underline{x_4 = \frac{1}{2}} \\ 3x_3 = -1 - 4\cdot\frac{1}{2} = -3 \Rightarrow \underline{x_3 = -1} \\ -2x_2 = -1 + 8\cdot\frac{1}{2} + 5\cdot(-1) = -2 \Rightarrow \underline{x_2 = 1} \\ \underline{x_1} = 0 - 3\cdot\frac{1}{2} - 3(-1) - 2\cdot 1 = \underline{-\frac{1}{2}} \end{cases}$$

$$\begin{bmatrix} -\frac{1}{11} & \frac{20}{11} & \frac{25}{11} & \frac{7}{11} \\ \frac{3}{11} & -\frac{16}{11} & -\frac{20}{11} & \frac{1}{11} \\ \frac{2}{11} & -\frac{7}{11} & -\frac{6}{11} & -\frac{3}{11} \\ 0 & -1 & -1 & 0 \end{bmatrix}$$

Znaleźć macierz odwrotną

Metoda operacji prostych (Gaussa - Jordana)

$$\left[\begin{array}{cccc|cccc} \boxed{-\frac{1}{11}} & \frac{20}{11} & \frac{25}{11} & \frac{7}{11} & 1 & 0 & 0 & 0 \\ \frac{3}{11} & -\frac{16}{11} & -\frac{20}{11} & \frac{1}{11} & 0 & 1 & 0 & 0 \\ \frac{2}{11} & -\frac{7}{11} & -\frac{6}{11} & -\frac{3}{11} & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow (-11)\cdot W_1 \\ \leftarrow W_2 + 3W_1 \\ \leftarrow W_3 + 2W_1 \\ \leftarrow W_4 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & -20 & -25 & -7 & -11 & 0 & 0 & 0 \\ 0 & \boxed{4} & 5 & 2 & 3 & 1 & 0 & 0 \\ 0 & 3 & 4 & 1 & 2 & 0 & 1 & 0 \\ 0 & \boxed{-1} & -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow W_1 - 20W_4 \\ \leftarrow W_4\cdot(-1) \\ \leftarrow W_3 + 3W_4 \\ \leftarrow W_2 + 4W_4 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -5 & -7 & -11 & 0 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \boxed{1} & 1 & 2 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 & 3 & 1 & 0 & 4 \end{array}\right] \begin{array}{l} \leftarrow W_1 + 5W_3 \\ \leftarrow W_2 - W_3 \\ \leftarrow W_3 \\ \leftarrow W_4 - W_3 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2 & -1 & 0 & 5 & -5 \\ 0 & 1 & 0 & -1 & -2 & 0 & -1 & -4 \\ 0 & 0 & 1 & 1 & 2 & 0 & 1 & 3 \\ 0 & 0 & 0 & \boxed{1} & 1 & 1 & -1 & 1 \end{array}\right] \begin{array}{l} \leftarrow W_1 + 2W_4 \\ \leftarrow W_2 + W_4 \\ \leftarrow W_3 - W_4 \\ \leftarrow W_4 \end{array}$$

Po tych przekształceniach otrzymamy już macierz odwrotną

$$\left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & 2 & 3 & -3 \\ 0 & 1 & 0 & 0 & -1 & 1 & -2 & -3 \\ 0 & 0 & 1 & 0 & 1 & -1 & 2 & 2 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 & 1 \end{array}\right]$$

Macierz odwrotna do danej:

$$\begin{bmatrix} 1 & 2 & 3 & -3 \\ -1 & 1 & -2 & -3 \\ 1 & -1 & 2 & 2 \\ 1 & 1 & -1 & 1 \end{bmatrix}$$

Znajdź macierz odwrotną

$$A = \begin{bmatrix} 1 & -\frac{7}{18} & -\frac{4}{3} & \frac{13}{18} \\ 1 & -\frac{2}{9} & -\frac{4}{3} & \frac{5}{9} \\ 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \\ -2 & \frac{7}{6} & 3 & -\frac{7}{6} \end{bmatrix}$$

W tym wypadku zastosujemy metodę dopełnień algebraicznych

det A =

$$= (-1)^{1+1} \cdot 1 \cdot \det \begin{bmatrix} -\frac{2}{9} & -\frac{4}{3} & \frac{5}{9} \\ -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \\ \frac{7}{6} & 3 & -\frac{7}{6} \end{bmatrix} M1$$

$$+ (-1)^{1+2} \cdot 1 \cdot \det \begin{bmatrix} -\frac{7}{18} & -\frac{4}{3} & \frac{13}{18} \\ -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \\ \frac{7}{6} & 3 & -\frac{7}{6} \end{bmatrix} M2 + \left\{ \begin{array}{l} \text{rozwinięcie względem} \\ \text{elementu } a_{31} \\ \text{daje zero} \end{array} \right\}$$

$$+ (-1)^{1+4} \cdot (-2) \cdot \det \begin{bmatrix} -\frac{7}{18} & -\frac{4}{3} & \frac{13}{18} \\ -\frac{2}{9} & -\frac{4}{3} & \frac{5}{9} \\ -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \end{bmatrix} M3 =$$

Wyznaczamy poszczególne wyznaczniki macierzy oznaczonych M1, M2, M3

$$\det(M1) = \left(-\frac{2}{9}\right)\left(\frac{1}{3}\right)\left(-\frac{7}{6}\right) + \left(-\frac{1}{9}\right)\cdot 3\cdot\left(\frac{5}{9}\right) + \left(\frac{7}{6}\right)\left(-\frac{4}{3}\right)\left(-\frac{2}{9}\right)$$

$$-\left[\left(\frac{7}{6}\right)\left(\frac{1}{3}\right)\left(\frac{5}{9}\right) + 3\left(-\frac{2}{9}\right)\left(-\frac{2}{9}\right) + \left(-\frac{7}{6}\right)\left(-\frac{1}{9}\right)\left(-\frac{4}{3}\right)\right] =$$

Wymnażamy i sprowadzamy do wspólnego mianownika:

$$= \frac{14}{3^4\cdot 2} + \frac{-30}{3^4\cdot 2} + \frac{56}{3^4\cdot 2} - \frac{35}{3^4\cdot 2} - \frac{24}{3^4\cdot 2} + \frac{28}{3^4\cdot 2} = \frac{9}{3^4\cdot 2} = \frac{1}{3^2\cdot 2} = \frac{1}{18}$$

$$\det(M2) = \left(-\frac{7}{18}\right)\left(\frac{1}{3}\right)\left(-\frac{7}{6}\right) + \left(-\frac{1}{9}\right)\cdot 3\cdot\left(\frac{13}{18}\right) + \left(\frac{7}{6}\right)\left(-\frac{4}{3}\right)\left(-\frac{2}{9}\right)$$

$$-\left[\left(\frac{7}{6}\right)\left(\frac{1}{3}\right)\left(\frac{13}{18}\right) + 3\cdot\left(-\frac{2}{9}\right)\left(-\frac{7}{18}\right) + \left(-\frac{7}{6}\right)\left(-\frac{1}{9}\right)\left(-\frac{4}{3}\right)\right] =$$

$$= \frac{49}{324} + \frac{-39\cdot 2}{162\cdot 2} + \frac{56\cdot 2}{162\cdot 2} - \frac{91}{324} - \frac{42\cdot 2}{162\cdot 2} + \frac{28\cdot 2}{162\cdot 2} = -\frac{36}{324} = -\frac{1}{9}$$

$$\det(M3) = \left(-\frac{7}{18}\right)\left(-\frac{4}{3}\right)\left(-\frac{2}{9}\right) + \left(-\frac{2}{9}\right)\left(\frac{1}{3}\right)\left(\frac{13}{18}\right) + \left(-\frac{1}{9}\right)\left(-\frac{4}{3}\right)\left(\frac{5}{9}\right)$$

$$-\left[\left(-\frac{1}{9}\right)\left(-\frac{4}{3}\right)\left(\frac{13}{18}\right) + \left(\frac{1}{3}\right)\left(\frac{5}{6}\right)\left(-\frac{7}{18}\right) + \left(-\frac{2}{9}\right)\left(-\frac{2}{9}\right)\left(-\frac{4}{3}\right)\right] =$$

$$= -\frac{56}{486} + \frac{-26}{486} + \frac{20\cdot 2}{243\cdot 2} - \frac{52}{486} + \frac{35}{486} + \frac{16\cdot 2}{243\cdot 2} = -\frac{27}{486} = -\frac{1}{18}$$

Wstawiamy wyniki do wzoru ze str. 4:

$$\det A = \frac{1}{18} - \left(-\frac{1}{9}\right) - (-2)\cdot\left(-\frac{1}{18}\right) = \frac{1}{18} + \frac{1}{9} - \frac{1}{9} = \frac{1}{18}$$

$$\det A = \frac{1}{18}$$

Aby móc skorzystać ze wzoru:

$$A^{-1} = \frac{1}{\det A} \cdot \left(A^D\right)^T$$

wyznaczamy A^D:

$\cancel{A_{11} =}$

$$A_{11} = \det \underbrace{\begin{bmatrix} -\frac{2}{9} & -\frac{4}{3} & \frac{5}{9} \\ -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \\ \frac{7}{6} & 3 & -\frac{7}{6} \end{bmatrix}}_{\text{to macierz } M1} = \frac{1}{18}$$

$$A_{21} = \det \underbrace{\begin{bmatrix} -\frac{7}{18} & -\frac{4}{3} & \frac{13}{18} \\ -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \\ \frac{7}{6} & 3 & -\frac{7}{6} \end{bmatrix}}_{\text{macierz } M2} = -\frac{1}{9}$$

$$A_{41} = \det \underbrace{\begin{bmatrix} -\frac{7}{18} & -\frac{4}{3} & \frac{13}{18} \\ -\frac{2}{9} & -\frac{4}{3} & \frac{5}{9} \\ -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} \end{bmatrix}}_{\text{macierz } M3} = -\frac{1}{18}$$

Tu było łatwiej, teraz trzeba liczyć kolejne minory:

$$A_{31} = \det \begin{bmatrix} -\frac{7}{18} & -\frac{4}{3} & \frac{13}{18} \\ -\frac{2}{9} & -\frac{4}{3} & \frac{5}{9} \\ \frac{7}{6} & 3 & -\frac{7}{6} \end{bmatrix}$$

Można liczyć jak poprzednio lub próbować zastosować wzór:

dla macierzy A wymiaru $n \times n$ zachodzi:

$$\det(k \cdot A) = k^n \cdot \det A, \quad k \in \mathbb{R}$$

$$A_{31} = \left(\frac{1}{18}\right)^3 \cdot \det\begin{bmatrix}-7 & -24 & 13\\ -4 & -24 & 10\\ 21 & 54 & -21\end{bmatrix} = \left| \text{Dla używających kalkulatora} \right| (*)$$

$$= (-7)(-24)(-21) + (-4)(54)(13) + (21)(-24)\cdot(10) - \left[(21)(-24)(13) + (54)\cdot(10)(-7) + (-21)(-4)(-24)\right] = 972$$

$$(*) = \frac{1}{18^3} \cdot 972 = \frac{972}{5832} = \frac{1}{6}$$

$$A_{\cancel{21}\,12} = \det\begin{bmatrix}\cancel{1 \quad -\frac{4}{3} \quad \frac{13}{18}} & & \\ 1 & -\frac{4}{3} & \frac{5}{9}\\ 0 & \frac{1}{3} & -\frac{2}{9}\\ -2 & 3 & -\frac{7}{6}\end{bmatrix} = \left(\frac{1}{18}\right)^3 \cdot \det\begin{bmatrix}18 & -24 & 10\\ 0 & 6 & -4\\ -36 & 54 & -21\end{bmatrix} =$$

$$= \left[18 \cdot 6 \cdot (-21) + (-36)(-24)(-4) - \left[(-36)(6)(10) + (54)(-4)(18)\right]\right] \cdot \left(\frac{1}{18}\right)^3 =$$

$$= \frac{324}{5832} = \frac{1}{18}$$

$$A_{22} = \det\begin{bmatrix}1 & -\frac{4}{3} & \frac{13}{18}\\ 0 & \frac{1}{3} & -\frac{2}{9}\\ -2 & 3 & -\frac{7}{6}\end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix}18 & -24 & 13\\ 0 & 6 & -4\\ -36 & 54 & -21\end{bmatrix} =$$

$$= \left(\frac{1}{18}\right)^3 \left\{(18)(6)(-21) + (-36)(-24)(-4) - \left[(-36)\cdot 6 \cdot (13) + (54)(-4)(18)\right]\right\} =$$

$$= \frac{972}{5832} = \frac{1}{6}$$

$$A_{32} = \det\begin{bmatrix} 1 & \cancel{-\frac{7}{18}} & -\frac{4}{3} & \frac{13}{18} \\ 1 & \cancel{-\frac{7}{18}} & -\frac{4}{3} & \frac{5}{9} \\ -2 & \cancel{\frac{7}{6}} & 3 & -\frac{7}{6} \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -24 & 13 \\ 18 & -24 & 10 \\ -36 & 54 & -21 \end{bmatrix} =$$

$$= \Big\{ (18)\cdot(-24)(-21) + 18\cdot 54\cdot 13 + (-36)(-24)\cdot 10 -$$

$$- \big[(-36)(-24)\cdot 13 + (54)\cdot 10\cdot 18 + (-21)(18)(-24)\big]\Big\} \cdot \left(\frac{1}{18}\right)^3$$

$$= \frac{324}{5832} = \frac{1}{18}$$

$$A_{42} = \det\begin{bmatrix} 1 & -\frac{4}{3} & \frac{13}{18} \\ 1 & -\frac{4}{3} & \frac{5}{9} \\ 0 & \frac{1}{3} & -\frac{2}{9} \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -24 & 13 \\ 18 & -24 & 10 \\ 0 & 6 & -4 \end{bmatrix} =$$

$$= \left(\frac{1}{18}\right)^3 \cdot \Big\{ (-4)(-24)18 + 6\cdot 18\cdot 13 - \big[6\cdot 10\cdot 18 + (18)(-4)(-24)\big]\Big\} =$$

$$= \frac{324}{5832} = \frac{1}{18}$$

$$A_{13} = \det\begin{bmatrix} 1 & -\frac{2}{9} & \frac{5}{9} \\ 0 & -\frac{1}{9} & -\frac{2}{9} \\ -2 & \frac{7}{6} & -\frac{7}{6} \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -4 & 10 \\ 0 & -2 & -4 \\ -36 & 21 & -21 \end{bmatrix} = (**)$$

$$\Big\{ 18(-2)(-21) + (-36)(-4)(-4) - \big[(-36)(-2)(10) + 21(-4)18\big]\Big\} = 972$$

$$(**) = \frac{972}{5832} = \frac{1}{6}$$

$$A_{23} = \det\begin{bmatrix} 1 & -\frac{7}{18} & \frac{13}{18} \\ 0 & -\frac{1}{9} & -\frac{2}{9} \\ -2 & \frac{7}{6} & -\frac{7}{6} \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -7 & 13 \\ 0 & -2 & -4 \\ -36 & 21 & -21 \end{bmatrix} =$$

$$18(-2)(-21) + (-36)(28) - \big[(-36)(-2)13 + 21(-4)18\big] = 324$$

$$A_{23} = \frac{324}{5832} = \frac{1}{18}$$

$$A_{33} = \det\begin{bmatrix} 1 & -\frac{7}{18} & \frac{13}{18} \\ 1 & -\frac{2}{9} & \frac{5}{9} \\ -2 & \frac{7}{6} & -\frac{7}{6} \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -7 & 13 \\ 18 & -4 & 10 \\ -36 & 21 & -21 \end{bmatrix} =$$

$$18(-4)(-21) + 18\cdot 21\cdot 13 + (-36)\cdot(-7)(10) - \left[(-36)(-4)13 + 210\cdot 18 + (-21)18(-7)\right]$$

$$= 648$$

$$A_{33} = \frac{648}{5832} = \frac{1}{9}$$

$$A_{43} = \det\begin{bmatrix} 1 & -\frac{7}{18} & \frac{13}{18} \\ 1 & -\frac{2}{9} & \frac{5}{9} \\ 0 & -\frac{1}{9} & -\frac{2}{9} \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -7 & 13 \\ 18 & -4 & 10 \\ 0 & -2 & -4 \end{bmatrix}$$

$$18\cdot 16 + (-2)(18)(13) - \left[(-20)\cdot 18 + 18\cdot 28\right] = -324$$

$$A_{43} = -\frac{324}{5832} = -\frac{1}{18}$$

$$A_{14} = \det\begin{bmatrix} 1 & -\frac{2}{9} & -\frac{4}{3} \\ 0 & -\frac{1}{9} & \frac{1}{3} \\ -2 & \frac{7}{6} & 3 \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -4 & -24 \\ 0 & -2 & 6 \\ -36 & 21 & 54 \end{bmatrix}$$

$$18(-2)54 + (-36)(-4)6 - \left[(-36)(-2)(-24) + (21)6\cdot 18\right] = -1620$$

$$A_{14} = -\frac{1620}{5832} = -\frac{5}{18}$$

$$A_{24} = \det\begin{bmatrix} 1 & -\frac{7}{18} & -\frac{4}{3} \\ 0 & -\frac{1}{9} & \frac{1}{3} \\ -2 & \frac{7}{6} & 3 \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & 7 & 24 \\ 0 & -2 & 6 \\ -36 & 21 & 54 \end{bmatrix}$$

$$18(-2)54 + (-36)(-7)\cdot 6 - \left[(-36)(-2)\cdot(-24) + 21\cdot 6\cdot 18\right] = -972$$

$$A_{24} = -\frac{972}{5832} = -\frac{1}{6}$$

$$A_{34} = \det\begin{bmatrix} 1 & -\frac{7}{18} & -\frac{4}{3} \\ 1 & -\frac{2}{9} & -\frac{4}{3} \\ -2 & \frac{7}{6} & 3 \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -7 & -24 \\ 18 & -4 & -24 \\ -36 & 21 & 54 \end{bmatrix}$$

$$18\cdot(-4)54 + 18\cdot 21(-24) + (-36)(-7)(-24) - \left[(-36)(-4)(-24) + 21\cdot(-24)\cdot 18 + 54\cdot 18\cdot(-\right.$$

$$= 324$$

$$A_{34} = \frac{324}{5832} = \frac{1}{18}$$

$$A_{44} = \det\begin{bmatrix} 1 & -\frac{7}{18} & -\frac{4}{3} \\ 1 & -\frac{2}{9} & -\frac{4}{3} \\ 0 & -\frac{1}{9} & \frac{1}{3} \end{bmatrix} = \left(\frac{1}{18}\right)^3 \det\begin{bmatrix} 18 & -7 & -24 \\ 18 & -4 & -24 \\ 0 & -2 & 6 \end{bmatrix}$$

$$18\cdot(-24) + (-2)\cdot 18\cdot(-24) - \left[(-2)\cdot(-24)\cdot 18 + 6\cdot 18(-7)\right] = 324$$

$$A_{44} = \frac{324}{5832} = \frac{1}{18}$$

w macierzy A^D uwzględniamy zmianę znaków

$$\left[A^D\right]^T = \begin{bmatrix} \frac{1}{18} & -\frac{1}{18} & \frac{1}{6} & +\frac{5}{18} \\ +\frac{1}{9} & \frac{1}{6} & -\frac{1}{18} & -\frac{1}{6} \\ \frac{1}{6} & -\frac{1}{18} & \frac{1}{9} & -\frac{1}{18} \\ -\frac{1}{18} & \frac{1}{18} & +\frac{1}{18} & \frac{1}{18} \end{bmatrix}^T = \begin{bmatrix} \frac{1}{18} & +\frac{1}{9} & \frac{1}{6} & +\frac{1}{18} \\ -\frac{1}{18} & \frac{1}{6} & -\frac{1}{18} & \frac{1}{18} \\ \frac{1}{6} & -\frac{1}{18} & \frac{1}{9} & +\frac{1}{18} \\ +\frac{5}{18} & -\frac{1}{6} & -\frac{1}{18} & \frac{1}{18} \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\left[A^D\right]^T = \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 3 & -1 & 1 \\ 3 & -1 & 2 & 1 \\ 5 & -3 & -1 & 1 \end{bmatrix}$$

Jak widać pomimo uwzględnienia własności operacji brania wyznacznika macierzy jest to metoda bardzo czasochłonna.

Dla kontrastu zastosujemy metodę eliminacji Gaussa-Jordana

$$\left[\begin{array}{cccc|cccc} \boxed{1} & -\frac{7}{18} & -\frac{4}{3} & \frac{13}{18} & 1 & 0 & 0 & 0 \\ 1 & -\frac{2}{9} & -\frac{4}{3} & \frac{5}{9} & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} & 0 & 0 & 1 & 0 \\ -2 & \frac{7}{6} & 3 & -\frac{7}{6} & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow W_1 \\ \leftarrow W_2 - W_1 \\ \leftarrow W_3 \\ \leftarrow W_4 + 2W_1 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & -\frac{7}{18} & -\frac{4}{3} & \frac{13}{18} & 1 & 0 & 0 & 0 \\ 0 & \boxed{\frac{1}{6}} & 0 & -\frac{1}{6} & -1 & 1 & 0 & 0 \\ 0 & -\frac{1}{9} & \frac{1}{3} & -\frac{2}{9} & 0 & 0 & 1 & 0 \\ 0 & \frac{7}{18} & \frac{1}{3} & \frac{5}{18} & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow (18W_1 + 42W_2)\cdot\frac{1}{18} \\ \leftarrow 6 \cdot W_2 \\ \leftarrow 9W_3 + 6W_2 \\ \leftarrow 18W_4 - 42W_2 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -\frac{4}{3} & \frac{1}{3} & -\frac{4}{3} & \frac{7}{3} & 0 & 0 \\ 0 & 1 & 0 & -1 & -6 & 6 & 0 & 0 \\ 0 & 0 & \boxed{3} & -3 & -6 & 6 & 9 & 0 \\ 0 & 0 & 6 & 12 & 78 & -42 & 0 & 18 \end{array}\right] \begin{array}{l} \leftarrow W_1 + \frac{4}{9}W_3 \\ \leftarrow W_2 \\ \leftarrow \frac{1}{3}W_3 \\ \leftarrow W_4 - 2W_3 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & -4 & 5 & 4 & 0 \\ 0 & 1 & 0 & -1 & -6 & 6 & 0 & 0 \\ 0 & 0 & 1 & -1 & -2 & 2 & 3 & 0 \\ 0 & 0 & 0 & \boxed{18} & 90 & -54 & -18 & 18 \end{array}\right] \begin{array}{l} \leftarrow W_1 + \frac{1}{18}W_4 \\ \leftarrow W_2 + \frac{1}{18}W_4 \\ \leftarrow W_3 + \frac{1}{18}W_4 \\ \leftarrow \frac{1}{18}W_4 \end{array}$$

$$A^{-1} = \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & 3 & -1 & 1 \\ 3 & -1 & 2 & 1 \\ 5 & -3 & -1 & 1 \end{bmatrix}$$

Powyższe przykłady dowodzą, że metoda oparta na konstrukcji macierzy dopełnień algebraicznych jest znacznie bardziej czasochłonna.

W przypadku obliczeń praktycznych (ręcznych oraz komputerowych) stosujemy zwykle metody eliminacji.

Metody bazujące na ~~na~~ wartościach wyznaczników stosuje się do problemów wymiaru $n \leq 3$.

Opracował

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