Partial-Fraction Expansion (Method of Residues)

1) Distinct (simple) roots

$$F(s) = \frac{L(s)}{M(s)} = \frac{L(s)}{(s-p_1)(s-p_2)\ldots(s-p_n)} = \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2} + \cdots + \frac{A_m}{s-p_n}$$

$$A_1 = \left[(s-p_1)\cdot F(s)\right]\Big|_{s=p_1}$$

$$A_2 = \left[(s-p_2)\cdot F(s)\right]\Big|_{s=p_2}$$

$$\vdots$$

$$A_m = \left[(s-p_n)\cdot F(s)\right]\Big|_{s=p_n}$$

2) Repeated (multiple) roots

$$F(s) = \frac{L(s)}{M(s)} = \frac{L(s)}{(s-p_n)^m} = \frac{A_{11}}{(s-p_n)^m} + \frac{A_{12}}{(s-p_n)^{m-1}} + \frac{A_{13}}{(s-p_n)^{m-2}} + \cdots + \frac{A_{1m}}{s-p_n}$$

$$A_{11} = \left[(s-p_n)^m \cdot F(s)\right]\Big|_{s=p_n}$$

$$A_{12} = \frac{d}{ds}\left[(s-p_n)^m \cdot F(s)\right]\Big|_{s=p_n}$$

$$A_{13} = \frac{1}{2}\frac{d^2}{ds^2}\left[(s-p_n)^m \cdot F(s)\right]\Big|_{s=p_n}$$

$$\vdots$$

$$A_{1m} = \frac{1}{(m-1)!}\frac{d^{(m-1)}}{ds^{(m-1)}}\left[(s-p_n)^m \cdot F(s)\right]\Big|_{s=p_n}$$