Systems Theory

Laboratory 1: *Modelling and analysis of linear systems.*

Purpose of the exercise:
Modelling and analysis of complex linear system using MATLAB/Simulink environment.

1. Introduction

Wind turbines are crucial renewable energy solutions nowadays. The wind load (and also sea waves load for offshore structures) that is varying in time as well as rotation of turbine elements, are major contributors to the structural vibration of tower and blades. This vibration is generally lightly damped. Damping ratio for the first two tower bending modes is usually less than or equal to 0.5%, excluding aerodynamic damping.

Wind turbine tower vibration may be analysed using a tower-nacelle beam model, in which all turbine components (a nacelle, blades, a hub, a shaft, a generator, and possibly a gearbox) are represented by beam tip mass.

The main solutions utilised to reduce wind turbines towers vibration are: collective pitch control of the blades, generator torque control, and tuned vibration absorbers (TVAs) / tuned mass dampers (TMDs). TVAs are widely spread structural vibration reduction solutions for slender structures.

In a standard approach, a TVA is being installed at/close to the top of the structure, and it consists of an additional moving mass, a spring and viscous damper, which parameters are tuned to the selected (most often first) mode of structure vibration.

2. A regarded model

According to the design assumptions, the analysed model consists of a full circular cross-section rod of diameter d and length L aligned vertically, fixed to the ground (representing a tower), and a stiff body of mass M connected rigidly to the top of the rod (representing both nacelle and turbine assemblies). A vibration reduction system that comprises a spring and a damper (built in parallel) with an additional stiff body, operating all together as a TVA system, is located at the top of the tower. A diagram of the regarded system is presented in Fig. 1.

A horizontal disturbance load may either be concentrated at the nacelle ($P(t)$, representing rotor or rotating machinery unbalance interaction), or applied to the arbitrary tower section ($F(t)$, representing wind / blade pass or sea interaction), both enable to force tower bending modes of vibration. The TVA direction of operation is the same as a direction of the applied excitation (assuming small bending angles).

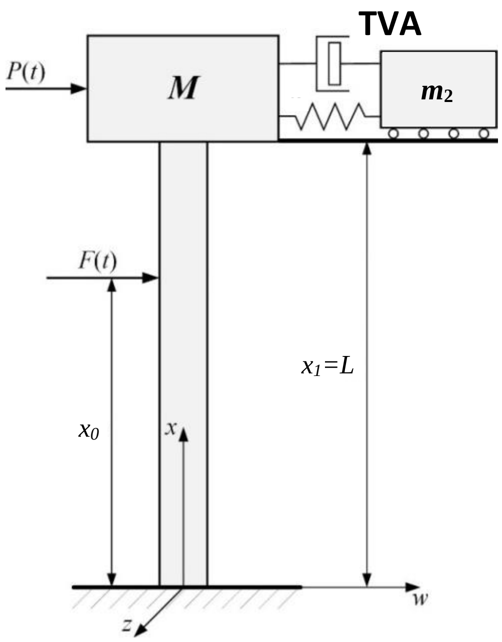

Fig. 1. Diagram of the regarded system

The dependences describing the wind, sea, etc., excitations, as well as tower dynamics, geometry, internal damping and stiffness are usually nonlinear. However, when displacements are small, often linearized model is used. Assuming constant mass and cross-section distribution along a beam that is regarded to be slender, as well as small bending deflections and angles, the tower-nacelle subsystem is modelled as a prismatic *Euler-Bernoulli* cantilever beam, which does not include sections shear deformations (sections assumed as planes perpendicular to the beam neutral axis), *fixed* at the bottom and *free* at the top, with a tip mass *M* (gravity components *neglected* for simplicity):

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + EJ\alpha_i \frac{\partial^5 w(x,t)}{\partial x^4 \partial t} + EJ \frac{\partial^4 w(x,t)}{\partial x^4} = F(t)\delta(x - x_0) + P(t)\delta(x - x_1) \tag{1}$$

where:

ρ is a beam density,

$\alpha_i = 2\zeta_i/\omega_i$ – retardation time of viscoelastic material (*Voigt-Kelvin* beam material model assumed) for the *i*-th bending mode of natural frequency ω_i; ζ_i – damping ratio,

E – modulus of elasticity of beam material,

A – cross-section area,

$J = \pi d^4/64$ – area moment of inertia,

δ – Dirac's delta.

Considering a fundamental bending mode of vibration only, and *neglecting* the excitation $F(t)$, the whole system may be regarded as a two mass-spring-damper system (2) (see Fig. 2):

$$\begin{cases} m_1\ddot{q}_1(t) = -k_1 q_1(t) - c_1\dot{q}_1(t) - k_2\left(q_1(t) - q_2(t)\right) - c_2\left(\dot{q}_1(t) - \dot{q}_2(t)\right) + P(t) & \text{(2a)} \\ m_2\ddot{q}_2(t) = k_2\left(q_1(t) - q_2(t)\right) + c_2\left(\dot{q}_1(t) - \dot{q}_2(t)\right) & \text{(2b)} \end{cases}$$

where $q_1(t)=w_1(L,t)$ (1) is a displacement of tower tip associated with tower-nacelle system 1st bending mode of vibration as $w(x,t)=q(t)X(x)$ (Fourier's separation of variables), thus: $w_1(L,t)=q_1(t)X_1(L)$, where $X_1(x)$ is the 1st bending mode shape normalised such as $X_1(L)=1$. In (2ab) and Fig. 2, designations m_1, c_1, k_1 represent modal mass, damping, and stiffness associated with the 1st bending mode, according to dependencies (3):

$$m_1 = \int_0^L \rho A X_1^2(x)dx + M$$

$$c_1 = \int_0^L \alpha_1 EJ\left(X_1''(x)\right)^2 dx \tag{3}$$

$$k_1 = \int_0^L EJ\left(X_1''(x)\right)^2 dx$$

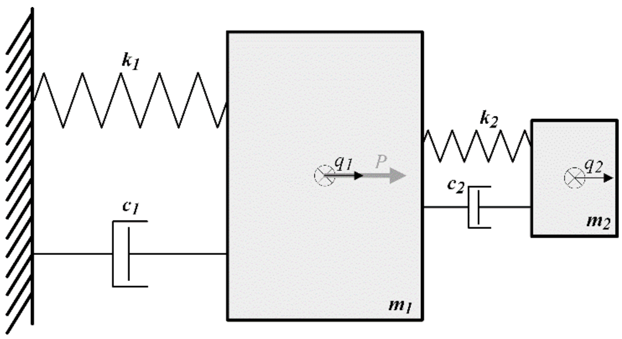

Fig. 2. Two mass-spring-damper system

The 1st mode shape $X_1(x)$ is given by (4):

$$X_1(x) = A_1\sin\beta_1 x + B_1\cos\beta_1 x + C_1\sinh\beta_1 x + D_1\cosh\beta_1 x \tag{4}$$

with: β_1=0.5474, A_1=2.7292, B_1=–2.2174, C_1=–2.7292, D_1=2.2174 being a solution of a beam equation (1) for the assumed mass-geometry parameters and boundary conditions. Value of β for the particular bending mode (1st in this case, i.e. β_1) yields natural angular frequency ω_1:

$$\omega_1 = \beta_1^2 \sqrt{\frac{EJ}{\rho A}} \tag{5a}$$

whereas damped natural angular frequency ω_{1d} is:

$$\omega_{1d} = \omega_1 \sqrt{1-\varsigma_1^2} \tag{5b}$$

In (2ab) and Fig. 2, designations: m_2, c_2, k_2 are mass, damping, and stiffness coefficients of the TVA, given after *Den Hartog*:

$$k_2 = m_2 \omega_2^2 \tag{6}$$

$$c_2 = 2\zeta_2 m_2 \omega_2 \tag{7}$$

where:

$$\omega_2 = \frac{1}{1+\mu}\omega_1$$

$$\zeta_2 = \sqrt{\frac{3\mu}{8(1+\mu)^3}}$$

$\mu = \frac{m_2}{m_1}$ is a mass ratio.

3. Tasks

For the assumed system parameters: ρ=4430 kg/m^3, M=162.64 kg, E=110 GPa, d=70.5 mm, L=1.507 m, ζ_1=0.5 %:

1. calculate m_1, c_1, k_1 according to (3), using e.g. *integral* evaluation function along with properly built MATLAB functions representing $X_1(x)$ (4) and $X_1''(x)$; otherwise, *int* or *vpaintegral* integration of symbolic expressions $X_1(x)$ and $X_1''(x)$ may be used,
2. tune the TVA according to (5a)(6)(7) assuming μ=6.37 % mass ratio,
3. build *Simulink* model representing the system (2ab), using *Integrator* blocks; assume $P(t)$ as a sine input of amplitude 61 N and angular frequency vector $\Omega=\omega_{1d}*[0.50:0.05:1.50]$ rd/s (5b) (execute *21* simulations in total, use each of the consecutive angular frequencies from the regarded vector Ω); for each of the angular frequencies determine amplitude of $q_1(t)$ (consider simulation time T_{sim} *long enough* to obtain steady state oscillations; T_{sim}>>10 s),
4. plot $q_1(t)$ amplitude frequency response characteristic (amplitude of $q_1(t)$ [mm] vs. angular frequency [rd/s] graph),
5. repeat steps 3-4 for the system (2a) without the TVA.
6. *SUPPLEMENTARY*: plot $q_1(t)$ amplitude frequency response characteristic using system (2ab), Fig. 2, and *cosine* representation of all $q_1(t)$, $q_2(t)$, and $P(t)$ (assume different amplitudes and phase shifts). Build a system of 4 equations with 4 unknown amplitudes and solve it with e.g. MATLAB *inv* command (as it was presented at the lecture).

References:

[1] MATLAB/Simulink documentation: http://www.mathworks.com
[2] Erturk A, Inman DJ: *Piezoelectric Energy Harvesting*, Appendix C: *Modal Analysis of a Uniform Cantilever with a Tip Mass*, John Wiley & Sons, Ltd, 2011.
[3] Den Hartog JP *Mechanical Vibrations*, Mineola: Dover Publications, 1985.

[4] Martynowicz P, *Vibration control of wind turbine tower-nacelle model with magnetorheological tuned vibration absorber*, Journal of Vibration and Control 23(20), 2017.