Laboratory 2

State space modeling of the linear systems

<u>Prepare state-space models (i.e.find matrices A, B, C, D) using symbolic expressions before the classes!</u>

1. Purpose of the exercise:

Modeling of the dynamic systems using state space approach. MATLAB/Simulink environment is used.

2. Mechanical system modeling

Simulate the system presented in Fig. 1 with MATLAB/Simulink software:

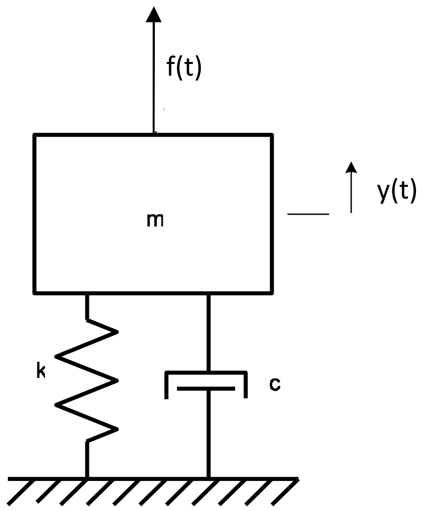

Fig. 1. Mechanical system

While constructing this system model consider:

- *input signal* – force **f(t)** (step and sine functions)
- *output signal* – displacement **y(t)**
- parameters values:

Parameter	Unit	PC no. 1 / 6 / 11	PC no. 2 / 7 / 12	PC no. 3 / 8 / 13	PC no. 4 / 9 / 14	PC no. 5 / 10 / 15
c	**Ns/m**	**1.5**	**1**	**0.5**	**2**	**2.5**
k	**N/m**	**10**	**5**	**2.5**	**15**	**20**
m	**kg**	**1**	**1.5**	**2**	**2.5**	**3**

<u>State space modeling of the mechanical system</u>

General form of a state space model is as follows: (2)

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where:
x (n x 1) – vector representing the state,
u (m x 1) – vector representing the input,
y (r x 1) – vector representing the output.

The matrices **A** (n x n), **B** (n x m), **C** (r x n) and **D** (r x m) determine the relationships between state variables and inputs or outputs. Note that the state equation corresponds to n first-order differential equations. While modeling the above system assume the state vector:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ where: } x_1 = y, \; x_2 = dy/dt$$

Tasks:
(a) simulate the system described by the state space model with MATLAB/Simulink software using *Step* input block (*Sources* library), *State-Space* block (*Continuous* library), and *Scope* (*Sinks* library); plot the response in a graphical window with *plot* command,
(b) generate *bode* characteristics and determine system resonance angular frequency ω_r,
(c) simulate the system using *Sine* input block at the angular frequency ω_r with input amplitude of 1 [N]; use also *State-Space*, and *Scope* blocks – assume simulation time long enough to obtain (more or less) constant amplitude of sine response (at least 10-15 periods); plot the response in a graphical window,
(d) assume damping coefficient value of **0.5c** and repeat steps (a)÷(c).

3. Electrical system modeling

Simulate the system presented in Fig. 2 with MATLAB/Simulink software:

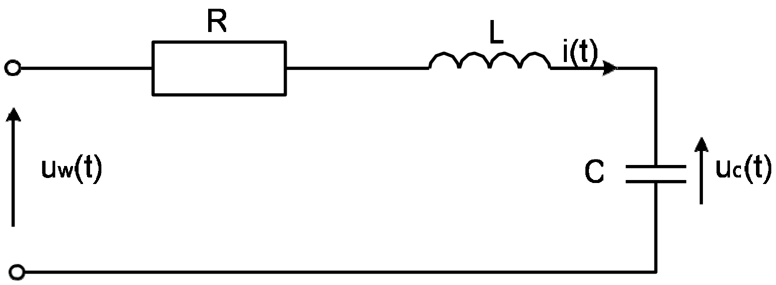

Fig. 2. Electrical system

While constructing this system model consider:
- *input signal* – electromotive force $\mathbf{u_w(t)}$ (step and sine functions),
- *output signal 1* – voltage drop across the induction coil $\mathbf{u_L(t)}$
- *output signal 2* – voltage drop across the capacitor $\mathbf{u_C(t)}$
- parameters values (note the units!):

Parameter	Unit	PC no. 1 / 6 / 11	PC no. 2 / 7 / 12	PC no. 3 / 8 / 13	PC no. 4 / 9 / 14	PC no. 5 / 10 / 15
R	**Ω**	**5**	**7.5**	**10**	**12.5**	**15**
L	**mH**	**1**	**1.5**	**2**	**2.5**	**3**
C	**μF**	**3**	**2.5**	**2**	**1.5**	**1**

State space modeling of the electrical system

While modeling the electrical system given above, assume the state vector:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ where: } x_1 = u_c,\ \ x_2 = i$$

Note that two versions of output equation (i.e. two versions of matrices C, D) are needed (using either *output signal 1*, or *output signal 2*).

Tasks – **only** *output signal 2* (**u_C(t)**) should be used below:

a) simulate the system described by the state space model with MATLAB/Simulink software using *Step* input block, *State-Space*, and *Scope* blocks (plot the response in a graphical window),

b) generate *bode* characteristics and determine system resonance angular frequency ω_r,

c) simulate the system using *Sine* input block at the angular frequency ω_r, $0.5\omega_r$ and $1.5\omega_r$ with input amplitude of 1 [V]; use also *State-Space*, and *Scope* blocks – assume simulation time long enough to obtain (more or less) constant amplitude of sine response (at least 10-15 periods); plot the responses in three separate graphical windows.

References:

[1] G.F. Franklin, J.D. Powell, E. Emami-Naeini "Feedback control of dynamic systems", Prentice Hall, New York, 2006.

[2] K. Ogata "Modern control engineering", Prentice Hall, New York, 1997.

[3] R.H. Cannon "Dynamics of physical systems", Mc-Graw Hill, 1967 (available in Polish as: R.H. Cannon "Dynamika układów fizycznych", WNT, Warszawa, 1973).

[4] J. Kowal "Podstawy automatyki", v.1 and 2, UWND, Kraków, 2006, 2007 (in Polish).

[5] W. Pełczewski "Teoria sterowania", WNT, Warszawa, 1980 (in Polish).

[6] Brzózka J., Ćwiczenia z Automatyki w MATLABIE i Simulinku, Wydawnictwo Mikon, Warszawa 1997 (in Polish).

[7] Zalewski A., Cegieła R., MATLAB: obliczenia numeryczne i ich zastosowania, Wydawnictwo Nakom, Poznań 1996 (in Polish).

[8] MATLAB/Simulink documentation: http://www.mathworks.com/help/