Systems Theory

Laboratory 2: *Modelling and analysis of nonlinear systems.*

Purpose of the exercise:
Modelling and analysis of a complex system with nonlinearities, using MATLAB/Simulink environment.

1. Introduction

Wind turbine tower vibration may be analysed using a tower-nacelle beam model, in which all turbine components (a nacelle, blades, a hub, a shaft, a generator, and possibly a gearbox) are represented by beam tip mass. The main solutions utilised to reduce wind turbines towers vibration are: collective pitch control of the blades, generator torque control, and tuned vibration absorbers (TVAs) / tuned mass dampers (TMDs). TVAs are widely spread vibration reduction solutions for slender structures.

A standard (passive) TVA is being installed at/close to the top of the structure, and it consists of an additional moving mass, a spring and a viscous damper, which parameters are tuned to the selected (most often first) mode of the vibration. Passive TVAs work well at the load conditions characterised with a single frequency, but cannot adapt to a wide excitation spectrum. During the system/structure operation lifetime, its frequency spectra may vary, thus more advanced TVA solutions are required to enable TVA tuning to the vibration frequency. Among them, magnetorheological (MR) TVAs are placed, as using an MR damper instead of a viscous one guarantees a wide range of resistance force, millisecond response time, high operational robustness (including lower sensitivity to temperature change) and minor energy requirements (signal-level energy amount is needed for damper characteristics adjustment rather than direct force generation) as compared with active vibration reduction systems. Simulations and experiments show that implementation of an MR damper in the TVA system may lead to further vibration reduction in relation to passive TVA.

2. A regarded model

According to the design assumptions, the analysed model consists of a full circular cross-section rod aligned vertically, fixed to a ground, and a stiff body connected rigidly to the top of the rod, representing both nacelle and turbine assemblies.

A vibration reduction system that comprises a spring (of stiffness k_2) and an MR damper, built in parallel, with an additional stiff body of mass m_2, operating all together as a MR TVA system, is located at the top of the tower.

A horizontal excitation load may either be concentrated at the nacelle ($P(t)$, representing rotor interaction on the tower), or applied to the arbitrary tower section ($F(t)$), both enable to force tower bending modes of vibration. The diagram of the regarded system is presented in Fig. 1.

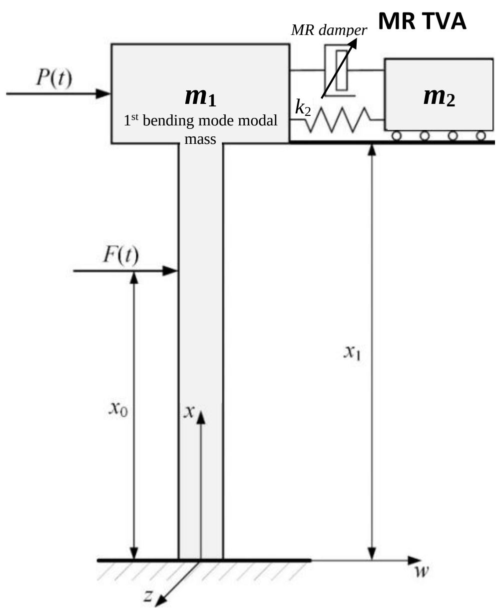

Fig. 1. Diagram of the regarded system

Assuming that the rod modelling a tower is a prismatic slender beam, and considering a fundamental bending mode of vibration only, the whole system may be regarded as a two mass-spring-damper system (1), see Fig. 2 (an excitation $F(t)$ and gravity forces are neglected):

$$\begin{cases} m_1\ddot{q}_1(t) = -k_1 q_1(t) - c_1\dot{q}_1(t) - k_2\big(q_1(t) - q_2(t)\big) - P_{MR}(t) + P(t) \\ m_2\ddot{q}_2(t) = k_2\big(q_1(t) - q_2(t)\big) + P_{MR}(t) \end{cases} \tag{1}$$

where $q_1(t)$ is a horizontal displacement of a tower tip associated with a tower-nacelle system 1st bending mode of vibration, $q_2(t)$ is an absorber absolute displacement, while $P_{MR}(t)$ is a force produced by the MR damper (Fig. 2). Designations m_1, c_1, k_1 state for modal mass, damping, and stiffness associated with the 1st bending mode. Assume m_1 and k_1 according to equations (3) from *Lab.1*, while $c_1 = c_1\big(q_1(t)\big)$ depending on the rod strain (thus on $q_1(t)$) value may be adopted to better reflect the real-world damping phenomenon. Assume: $c_1\big(q_1(t)\big) = 2m_1\omega_1\zeta_1\big(q_1(t)\big) \approx 8885 \cdot \zeta_1\big(q_1(t)\big)$ [Ns/m] with exemplary $\zeta_1\big(q_1(t)\big)$ relation as in Tab.1 (for *Lab. 1*, constant ζ_1=0.5% was assumed).

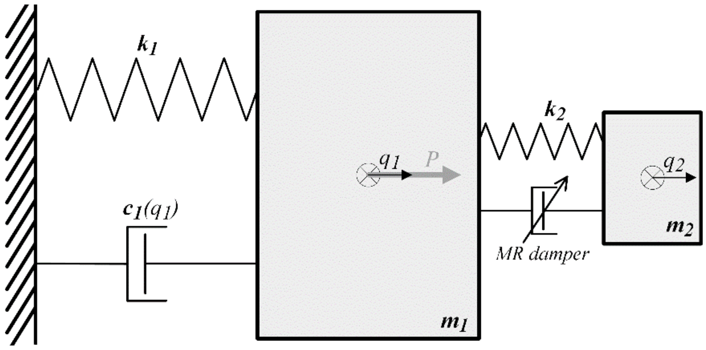

Fig. 2. Two mass-spring-damper system

Tab.1. $\boldsymbol{\zeta_1}(|\boldsymbol{q_1(t)}|)$ relationship

$\boldsymbol{\lvert q_1(t)\rvert}$ [mm]	$\boldsymbol{\zeta_1}$ [%]
0	0.02
1	0.10
4	0.20
16	0.60
32	1.40
64	4.50

The MR damper (Fig. 3) is an actuator in such a vibration reduction system. Its operating characteristics (Fig. 4) are strongly *nonlinear*, exhibiting:

- dependency of a maximum output force on control current i_{MR},
- presence of a hysteresis,
- presence of a yield point, corresponding to an MR fluid yield stress,
- force value constraints: minimum (due to, i.a., the residual magnetisation of a damper magnetic circuit, MR fluid viscosity), and maximum (due to, i.a., saturation magnetisation, a thermal limit of a damper electric coil).

Fig. 3. RD-1097-1 MR damper

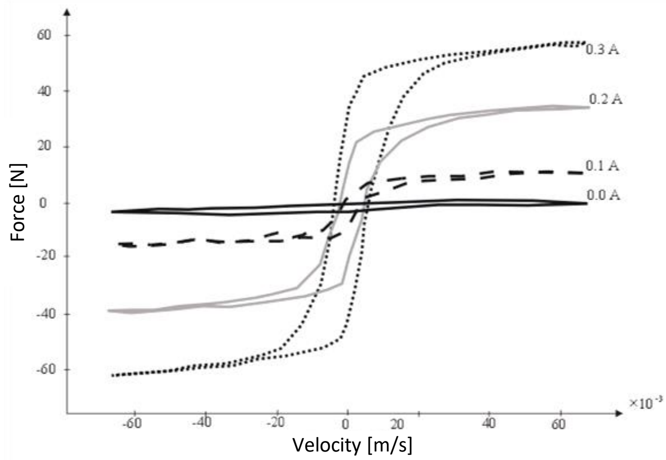

Fig. 4. RD-1097-1 force-velocity loops, sine exc.

To determine the MR damper resistance force, several phenomenological and non-phenomenological models are used, what is a subject of another studies. A reasonable balance between model accuracy and computational complexity is obtained with the use of hyperbolic tangent model in the form of (2):

$$P_{MR}(t) = P_c(i_{MR})\tanh\{v[(\dot{q}_1(t)-\dot{q}_2(t))+\mathrm{p}(q_1(t)-q_2(t))]\}+c_0(i_{MR})[(\dot{q}_1(t)-\dot{q}_2(t))+\mathrm{p}(q_1(t)-q_2(t))] \qquad (2)$$

where P_c and c_0 are coefficients that may be associated with friction force and viscous damping / stiffness (respectively) resistance force components, while v and p are scaling parameters. Values of parameters P_c and c_0 depend on control current; their simplified expressions may be given in affine forms: $P_c(i_{MR}) = C_1 i_{MR} + C_2$, $c_0(i_{MR}) = C_3 i_{MR} + C_4$.

To analyse and compare efficiency of a vibration reduction system, in particular (MR) TVA (for the assumed mass ratio m_2/m_1), a Dynamic Amplification Factor (*DAF*) is used, determined according to formula (3):

$$DAF = \frac{A(q_1)}{A(P)/k_1} \qquad (3)$$

where $A(\cdot)$ states for amplitude. In (3), amplitude of the displacement of a primary (protected) elastic system/structure of stiffness k_1 is scaled with its static displacement/deflection $A(P)/k_1$ under external force of amplitude $A(P)$.

3. Tasks

For the system parameters m_1, k_1, m_2, k_2 determined during *Lab. 1* along with $c_1(q_1(t))$ parameters given above (Tab.1), and MR damper RD-1097-1 model (2) parameters according to Tab.2:

Tab.2. RD-1097-1 model parameters

Parameter	Value
C_1	62 N/A
C_2	1.5 N
C_3	48 Ns/Am
C_4	14 Ns/m
v	130 s/m
p	1.0 1/s

1. build *Simulink* model representing system (1), including c_1 and MR damper nonlinearities (use *1-D Lookup Table* and *Trigonometric Function* blocks, respectively); assume $P(t)$ as a sine input of amplitude either $A(P)$=30.5 N, or $A(P)$=61 N, and angular frequency vector $\Omega=\omega_{1d}*$[0.50:0.05:1.50] rd/s according to relation (5b) from *Lab. 1*,

2. assume MR damper control current values vector: I_{MR}=[0.0, 0.1, 0.2, 0.5] A; for $A(P)$= 30.5 N and each of four control currents from the I_{MR} vector execute 21 simulations using each of the consecutive angular frequencies from the regarded Ω vector (see *Task 1*) and determine *DAF* (consider simulation time T_{sim} *long enough* to obtain steady state oscillations; T_{sim}>>10 s); repeat this task for $A(P)$=61 N,

3. print *DAF* frequency response characteristics (*DAF* [-] vs. angular frequency [rd/s] curve) for each of the $A(P)$ values and all the assumed control current values (I_{MR} elements) in one graph,

4. print *DAF* frequency responses for the system without the TVA using updated (with real-world damping c_1) *Simulink* model from *Lab. 1*. Use both $A(P)$= 30.5 N, and $A(P)$=61 N.

References:

[1] MATLAB/Simulink documentation: http://www.mathworks.com
[2] A. Erturk, D.J. Inman: *Piezoelectric Energy Harvesting*, Appendix C: *Modal Analysis of a Uniform Cantilever with a Tip Mass*, John Wiley & Sons, Ltd, 2011.
[3] P. Martynowicz *Vibration control of wind turbine tower-nacelle model with*

magnetorheological tuned vibration absorber, Journal of Vibration and Control 23(20), 2017.
[4] Lord Rheonetic, *MR Controllable Friction Damper RD-1097-01 Product Bulletin*, Lord Co. 2002.
[5] M. Maślanka, B. Sapiński and J. Snamina, *Experimental Study of Vibration Control of a Cable With an Attached MR Damper*, Journal of Theoretical and Applied Mechanics 45(4), 2007.