Laboratory 3

Operating characteristics of control systems
Basic structures of linear controllers

1. Purpose of the exercise:

- analysis of the control system operating characteristics
- learning the linear controllers structures

2. Analysis of the automatic system operating characteristics

Simulate the system presented in Fig. 1 with MATLAB/Simulink software:

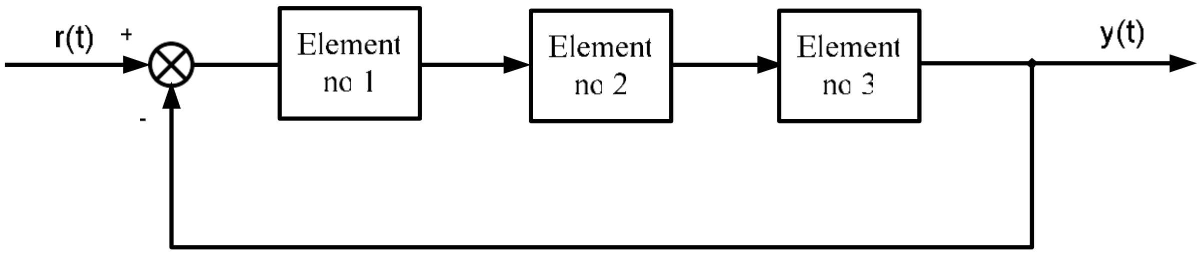


Fig. 1. Block diagram of the feedback system

While constructing the above-depicted system model consider:

- input signal: ***r*(t)** (step function)
- output signal: ***y*(t)**
- element no 1: **first order inertial element (K,T)**
- element no 2: **first order inertial element (K,T)**
- element no 3: **first order inertial element (K,T)**
- simulation time: 100 s

	Parameter	PC no. 1 / 6 / 11	PC no. 2 / 7 / 12	PC no. 3 / 8 / 13	PC no. 4 / 9 / 14	PC no.5 / 10 / 15
Element no 1	**K**	**1**	**1,5**	**2**	**2,5**	**3**
	T	**3**	**2,5**	**3**	**1,5**	**1**
Element no 2	**K**	**2,5**	**1**	**1,5**	**2**	**1**
	T	**1,5**	**3**	**2,5**	**3**	**3**
Element no 3	**K**	**3**	**2,5**	**1**	**1,5**	**2**
	T	**1**	**1,5**	**3**	**2,5**	**3**

On the basis of command input *r*(t) and output *y*(t) signals time runs, determine for the analyzed control system (see Appendix):

- steady-state error e_{SS}
- overshoot κ
- settling time t_S for 5% tolerance of steady-state output y_{SS}

3. Basic structures of the linear controllers

Block diagram of the **PID** controller structure is presented in Figure 2. Transfer functions of the PID controller family:

- Proportional controller (**P**): $G_R(s) = K_r$
- Integral controller (**I**): $G_R(s) = \frac{1}{T_i s}$
- Proportional-Integral controller (**PI**): $G_R(s) = K_r\left(1 + \frac{1}{T_i s}\right)$
- Proportional-Derivative controller (**PD**): $G_R(s) = K_r(1 + T_d s)$
- Proportional-Integral-Derivative controller (**PID**): $G_R(s) = K_r\left(1 + \frac{1}{T_i s} + T_d s\right)$

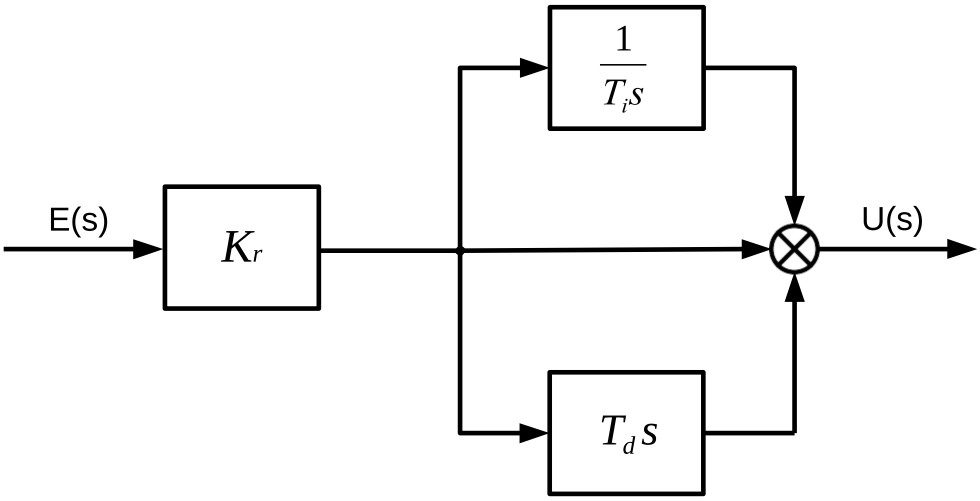


Fig. 2. General structure of the PID controller

Using MATLAB/Simulink, build the control system according to Fig 3:

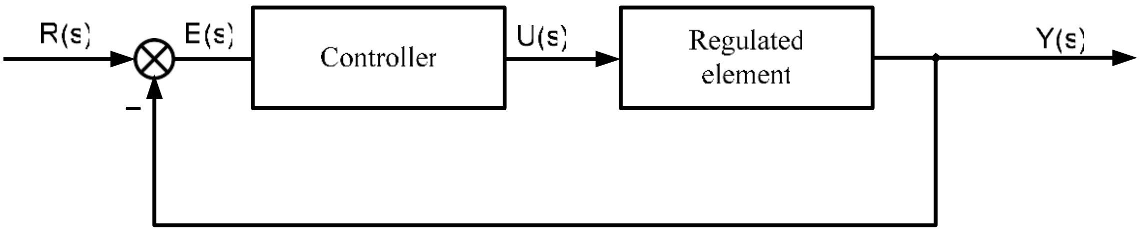


Fig. 3. Block diagram of the feedback control system

While constructing the above-depicted system model consider:

- input signal: ***r*(t)** (step function)
- output signal: ***y*(t)**
- regulated element:

		PC no. 1 / 6	PC no. 2 / 7 / 12	PC no. 3 / 8 / 13	PC no. 4 / 9 / 14	PC no.5 / 10 / 15
1st order inertial element	**K**	**1**	**1**	**1**	**1**	**1**
	T	**1**	**2**	**3**	**4**	**5**
2nd order oscillatory element	**K**	**1**	**2**	**3**	**4**	**5**
	T	**1**	**1**	**1**	**1**	**1**
	ξ	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**

Build the following feedback control systems:

1. control system with P controller,
2. control system with I controller,
3. control system with PI controller,
4. control system with PD controller,
5. control system with PID controller.

Find an influence of controllers parameters (K_r, T_i, T_d) on operating characteristics of the analyzed control system using following indexes: steady-state error $\boldsymbol{e_{SS}}$, overshoot $\boldsymbol{\kappa}$, settling time $\boldsymbol{t_S}$ (see Appendix).

4. Design of the control system

Build the control system for the elements given below and select a controller so that the settling time t_S is shortest (for steady-state error $e_{SS} = 0$):

a) $G_1(s) = \dfrac{1}{s^2+1}$, b) $G_2(s) = \dfrac{10}{s^2+s+1}$, c) $G_2(s) = \dfrac{0.228}{0.18s^2+1.18s+1}$

	PC no. 1 / 6 / 11	PC no. 2 / 7 / 12	PC no. 3 / 8 / 13	PC no. 4 / 9 / 14	PC no.5 / 10 / 15
Transfer function	**c**	**b**	**a**	**b**	**c**

Build the control system for the elements given below and select a controller so that the overshoot κ is smallest (for steady-state error $e_{SS} = 0$):

a) $G_1(s) = \dfrac{2.5}{2s^2+2s+1}$, b) $G_2(s) = \dfrac{1}{s^3+2s^2+2s+1}$

	PC no. 1 / 6 / 11	PC no. 2 / 7 / 12	PC no. 3 / 8 / 13	PC no. 4 / 9 / 14	PC no.5 / 10 / 15
Transfer function	**a**	**b**	**a**	**b**	**a**

References:

[1] G.F. Franklin, J.D. Powell, E. Emami-Naeini “Feedback control of dynamic systems”, Prentice Hall, New York, 2006.

[2] K. Ogata “Modern control engineering”, Prentice Hall, New York, 1997.

[3] R.H. Cannon “Dynamics of physical systems”, Mc-Graw Hill, 1967 (available in Polish as: R.H. Cannon “Dynamika układów fizycznych”, WNT, Warszawa, 1973).

[4] J. Kowal “Podstawy automatyki”, v.1 and 2, UWND, Kraków, 2006, 2007 (in Polish).

[5] W. Pełczewski “Teoria sterowania”, WNT, Warszawa, 1980 (in Polish).

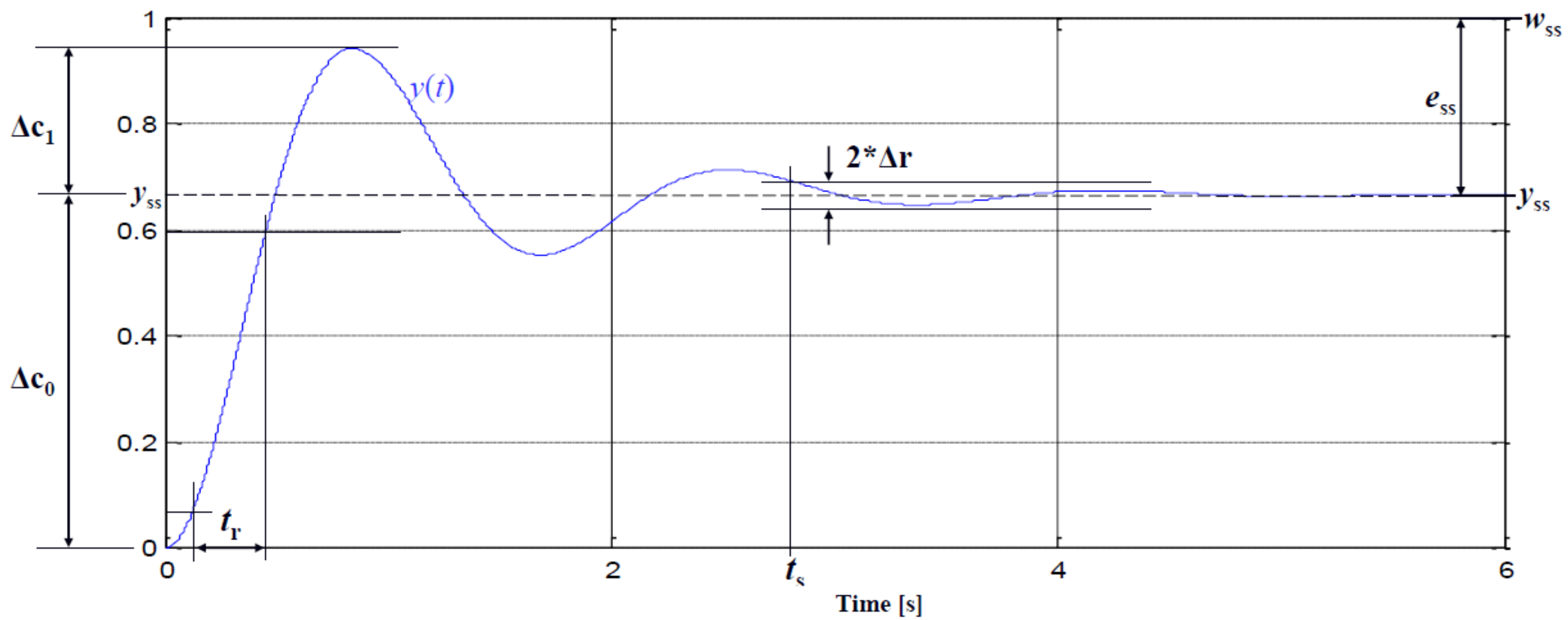
[6] Brzózka J., Ćwiczenia z Automatyki w MATLABIE i Simulinku, Wydawnictwo Mikon, Warszawa 1997 (in Polish).

[7] Zalewski A., Cegieła R., MATLAB: obliczenia numeryczne i ich zastosowania, Wydawnictwo Nakom, Poznań 1996 (in Polish).

[8] MATLAB/Simulink documentation: http://www.mathworks.com/help/

Appendix:

Setpoint value: $w(t) = 1(t)$



Steady-state error: $e_{ss} = w_{ss} - y_{ss}$ where: $y_{ss} = \lim_{t\to\infty} y(t)$ $w_{ss} = \lim_{t\to\infty} w(t)$

Rise time: t_r – time needed for response $y(t)$ change from 10% to 90% (or from 5% to 95%) of the steady-state value y_{ss}

Settling time: t_s – time needed for response $y(t)$ to reach y_{ss} neighbourhood of radius Δr
$\Delta r = 3 \div 5\% \, y_{ss}$

Overshoot: $\kappa = \Delta c_1 / \Delta c_0 \cdot 100\%$