Systems Theory

Laboratory 3: *Selected continuous-time control algorithms.*

Purpose of the exercise:
Design, analysis, and verification of various continuous control solutions for a complex system using MATLAB/Simulink environment.

1. Introduction

Tuned vibration absorbers (TVAs) are widely spread vibration reduction solutions for slender structures. A standard (passive) TVA is being installed at/close to the top of the structure. It consists of an additional moving mass, a spring and a viscous damper, which parameters are tuned to the selected (most often first) mode of the vibration. Passive TVAs work well at the load conditions characterised with a single frequency but cannot adapt to a broad excitation spectrum. During the system/structure operation lifetime, its frequency spectra may vary; thus, more advanced TVA solutions are required to enable TVA tuning to the vibration frequency. Among them, semiactive, magnetorheological (MR), and active TVAs are placed. Using an MR damper instead of a viscous one guarantees a wide range of resistance force, millisecond response time, and high operational robustness (including lower sensitivity to temperature change); furthermore, MR damper has minor energy requirements (signal-level energy amount is needed for damper characteristics adjustment rather than direct force generation) as compared with active solutions.

Most MR damper real-time control solutions are based on bang-bang or fuzzy logic approach, or two-stage concepts with the calculation of an MR damper required force (1st stage) and precise force tracking algorithms (2nd stage). The latter concepts suffer from the inability to produce the required (by the 1st stage algorithm) MR damper force pattern due to e.g. the impossibility to generate active forces, and force value limitations: lower constraint imposed by the residual force at zero current, and upper constraint imposed by the piston velocity and maximum current that can be fed through an MR damper coil along with magnetic circuit saturation. Some adaptive 1st stage algorithms need real-time oscillation frequency determination.

2. A regarded system

A vibration reduction system that comprises a spring (of stiffness k_2) and an MR damper, built in parallel, with an additional stiff body of mass m_2, operating all together as an MR TVA system, is regarded. Alternatively, an active element is considered in place of an MR damper.

The analysed system may be regarded a two mass-spring-damper system (1) (Fig. 1) with an external excitation $P(t)$:

$$\begin{cases} m_1\ddot{q}_1(t) = -k_1 q_1(t) - c_1(q_1)\dot{q}_1(t) - k_2\big(q_1(t) - q_2(t)\big) - P_{MR}(t) + P(t) \\ m_2\ddot{q}_2(t) = k_2\big(q_1(t) - q_2(t)\big) + P_{MR}(t) \end{cases} \tag{1}$$

where $q_1(t)$ is a horizontal displacement of a protected system/structure (e.g. corresponding to a tower-nacelle system 1st bending mode of vibration), $q_2(t)$ is an absorber absolute displacement, while $P_{MR}(t)$ is a force produced by the MR damper (alternatively by an active cylinder). Designations m_1, c_1, k_1 state for (modal) mass, damping, and stiffness of the protected system/structure.

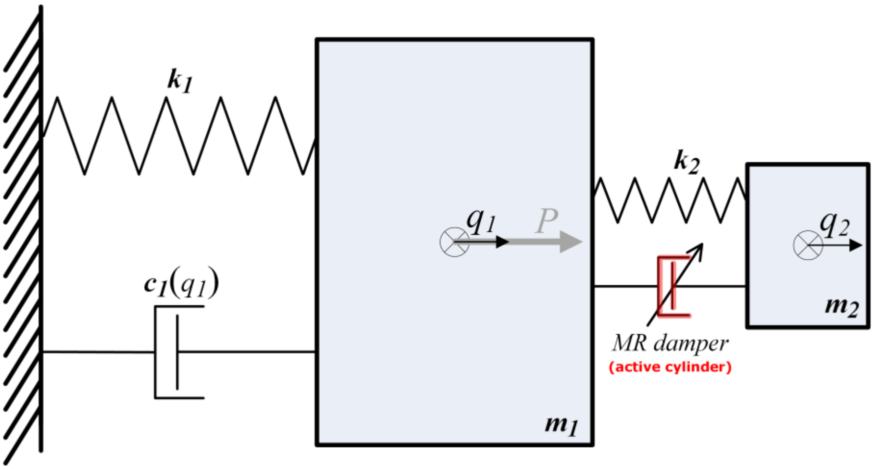

Fig. 1. A regarded system diagram

To determine the MR damper resistance force P_{MR} (t) hyperbolic tangent model with its parameters given during *Lab. 2* will be used.

3. Reproducing the required force by an MR damper

Two approaches of reproducing an MR damper force may be used. For both of them, *saturation* of output control current with $0 \div i_{MR}^{\max}$ limits is necessary.

a) MR damper inverse model

Based on the required $P_{MR}{}^*$ force value, MR damper control signal i_{MR} may be calculated according to hyperbolic tangent inverse model (see equation (2), *Laboratory 2*):

$$i_{MR}\left(P_{MR}{}^*, \mathbf{q}(t), t\right) = \frac{P_{MR}{}^* - C_2 \tanh\left[\mu\left(\dot{q}_{12}(t) + pq_{12}(t)\right)\right] - C_4\left(\dot{q}_{12}(t) + pq_{12}(t)\right)}{C_1 \tanh\left[\mu\left(\dot{q}_{12}(t) + pq_{12}(t)\right)\right] + C_3\left(\dot{q}_{12}(t) + pq_{12}(t)\right)} \tag{2}$$

where:

$$\mathbf{q}(t) = \left[q_1(t) \quad \dot{q}_1(t) \quad q_2(t) \quad \dot{q}_2(t)\right]^T$$

is a system state vector, while:

$$q_{12}(t) = q_1(t) - q_2(t).$$

<u>Remark</u>: p=0 is a case with neglected hysteresis (may be assumed for i_{MR} unambiguity).

b) MR damper force tracking algorithm

The MR damper force tracking algorithm is presented in Fig. 2. *Current Driver* block is responsible for forcing the demanded MR damper current value through the damper coil (may be omitted during these exercises along with coil dynamics).

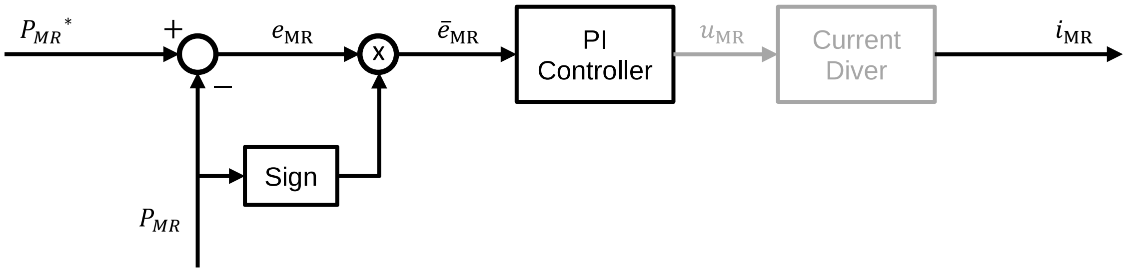

Fig. 2. MR damper force tracking algorithm diagram

4. Control solutions

(a) Ground-hook control law

According to standard, two-level displacement ground-hook law, control signal i_{MR} is defined by a formula:

$$i_{MR}^{*} = \begin{cases} i_{MR}^{\max}, & q_1\dot{q}_{12} \geq 0 \\ 0, & q_1\dot{q}_{12} < 0 \end{cases} \tag{3}$$

where $i_{MR}^{\max}$ is a maximum MR damper control signal (electric current) value. The control law switches the current between 0 and $i_{MR}^{\max}$ values with regard to q_1 and $\dot{q}_{12}$ signs.

(b) Modified ground-hook control law

A modified, two-level displacement ground-hook law is a simple implementation of the optimal control for the case when primary system/structure displacement/deflection amplitude minimisation is the sole objective. This control law switches the current between 0 and $i_{MR}^{\max}$ with regard to q_1 and P_{MR} signs:

$$i_{MR}^{*} = \begin{cases} i_{MR}^{\max}, & q_1 P_{MR} \geq 0 \\ 0, & q_1 P_{MR} < 0 \end{cases} \tag{4}$$

(c) Adaptive stiffness

An undamped vibration absorber that tracks an excitation frequency is emulated using an MR damper. The MR damper should generate positive or negative stiffness force in such a way that MR TVA stiffness k_2^{*} is tuned to the actual operational (excitation) frequency ω_{exc} rather than to the system/structure damped natural frequency. Based on this assumption, (real-time) determination of ω_{exc} is followed by the calculation of the TVA desired force P_{MR}^{*} (stiffness component), while the damping component is assumed to be zero (for the most *efficient* vibration mitigation at the frequency of tuning), leading to the MR damper required force formula:

$$P_{MR}{}^{*} = \gamma\left(k_2^{*} - k_2\right)q_{12} \tag{5}$$

where: $k_2^{*} = m_2\omega_{exc}^2$, while $\gamma\in[1, 2]$ is a correction factor that is present as an MR damper cannot deliver energy to the system, thus the force P_{MR}^{*} pattern cannot be mapped during two quarters of each sine oscillation period. As this solution yields P^*_{MR} rather than i^*_{MR}, for *MR-damper-based* configuration, inverse model (Fig. 3) or force tracking algorithm (Fig. 4) is necessary.

(d) Adaptive stiffness and damping

A damped vibration absorber that tracks an excitation frequency is implemented using an MR damper. The MR damper is used to emulate controllable (positive and negative) stiffness and viscous damping in such a way, that TVA stiffness, and also TVA damping (for *robust* vibration mitigation at the frequency of tuning) is tuned to the actual operational (excitation) frequency ω_{exc} rather than to the system/structure damped natural frequency. Based on this assumption, (real-time) ω_{exc} determination is followed by calculation of TVA required stiffness and damping force components according to [2]:

$$P^*_{MR} = \left(k^*_2 - k_2\right) q_{12} + c^*_2 \dot{q}_{12} \tag{6}$$

where:

$$k^*_2 = \frac{m_2 \omega^2_{exc}}{(1+\mu)^2}$$

$$c^*_2 = \frac{2\zeta_2 m_2 \omega_{exc}}{1+\mu}$$

$$\zeta_2 = \sqrt{\frac{3\mu}{8(1+\mu)^3}}$$

As this solution yields P^*_{MR} rather than i^*_{MR}, for *MR-damper-based* configuration, inverse model (Fig. 3) or force tracking algorithm (Fig. 4) is necessary.

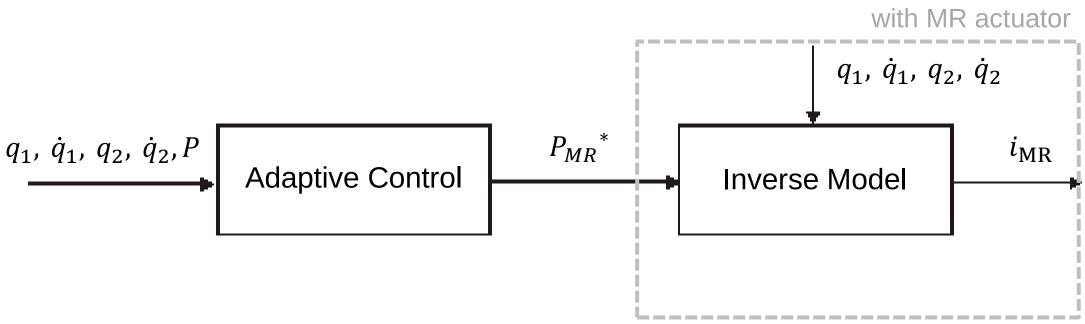

Fig. 3. Adaptive control with MR damper inverse model

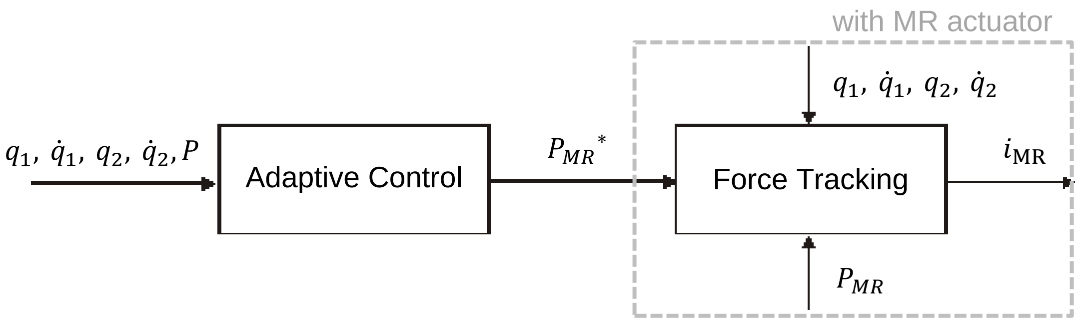

Fig. 4. Adaptive control with MR damper force tracking algorithm

(e) Fuzzy logic controller

Fuzzy logic may be used to design *fuzzy* algorithm (e1) from scratch (as in, e.g. [7,8]), or (e2) to smoothen the transition region of the control law that is confirmed to be favourable concerning the protected system/structure displacement/deflection minimisation – the optimal-based *modified ground-hook control law (b).*

Concerning the approach **(e1)**, below are possible input membership functions:

- q_1 is negative (N) / positive (P),
- $\dot{q}_1$ is negative (N) / positive (P),
- P_{MR} is negative large-in-value (NL) / negative small-in-value (NS) / positive small-in-value (PS) / positive large-in-value (PL).

Possible output membership functions:

- i_{MR} zero (Z) / small (S) / large (L).

A framework of a rule-base table for computing the output current i_{MR} may be (you may consult papers [7,8]):

q_1 $\dot{q}_1$ \ P_{MR}	NL	NS	PS	PL
N N	L			Z
N P				
P N				
P P	Z			L

Concerning the approach **(e2)**, below are input membership functions:

- q_1 is negative (N) / positive (P),
- P_{MR} is negative (N) / positive (P),

output membership functions:

- i_{MR} zero (Z) / maximum (M),

and a rule-base table for computing the output current i_{MR}:

q_1 \ P_{MR}	N	P
N	M	Z
P	Z	M

The membership functions may be assumed of e.g. trapezoid or triangular shape with 50% overlap.

5. Tasks

For system (1) with parameters m_1, k_1 m_2, k_2 determined on *Lab. 1*, along with $c_1(q_1)$ and MR damper model given on *Lab. 2* (or using *Simulink* models build during *Lab. 2*), with γ=1.5, and $i_{MR}^{\max}$=0.5 [A]:

1. build all the controllers according to *section 4*, points *(a)* to *(d)* (including MR damper inverse model and force tracking algorithm, *section 3*) using *MATLAB/Simulink* environment; assume excitation $P(t)$ as a sine input of amplitude 61 N and angular frequency vector $\Omega=\omega_{1d}*[0.50:0.05:1.50]$ rd/s with ω_{1d} according to relation (5b) from *Lab. 1*,

2. for each of the control solutions from *section 4*, i.e. points *(a)(b)*, as well as *(c)(d)* combined with MR damper inverse model and force tracking algorithm (*section 3*), execute 21 simulations using each of the consecutive excitation angular frequencies from the regarded vector Ω (*Task 1*), and determine *DAF* (see *Lab. 2*) for steady-state oscillations; consider the simulation time long enough to obtain steady-state oscillations (check this by observing time patterns for each excitation angular frequency from vector Ω);

   Remark: for each of the adaptive solutions *(c)(d)* use *both MR-damper-based* configurations (according to Fig. 3 and Fig. 4), as well as *one additional* configuration with an ideal *active cylinder/actuator* of [–100, 100] N output force range and γ=1.0, instead of the MR damper (Fig. 1),

3. print *DAF* frequency response characteristics (*DAF* [-] vs. angular frequency [rd/s] curve) for all of the regarded control solutions in one graph,

4. compare *DAF* frequency responses of *Task 3* with those obtained for constant MR damper current values (from the vector I_{MR}=[0.0, 0.1, 0.2, 0.5] A), using graph/data (or *Simulink* model) from *Lab. 2*,

5. SUPPLEMENTARY: execute *tasks 1–4* using controller *(e1)* (1 pt.) or *(e2)* (2 pt.) (maximum of 2 additional points).

References:

[1] MATLAB/Simulink documentation: http://www.mathworks.com

[2] J.P. Den Hartog, *Mechanical Vibrations*. Mineola: Dover Publications, 1985.

[3] P. Martynowicz *Vibration control of wind turbine tower-nacelle model with magnetorheological tuned vibration absorber*, Journal of Vibration and Control 23(20), 2017.

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[6] M. Maślanka, B. Sapiński and J. Snamina, *Experimental Study of Vibration Control of a Cable With an Attached MR Damper*, Journal of Theoretical and Applied Mechanics 45(4), 2007.

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