

# Laboratory 4

## Designing automatic systems with MATLAB / Simulink

### 1. Purpose of the exercise:

- learning methods of creating linear automatic systems models & transforming model forms
- creating block diagrams of automatic systems
- drawing time responses and frequency responses

### 2. Theoretical introduction

#### 2.1. Automatic systems models

MATLAB often uses two types of linear dynamic models:

- state-space and output equation

To achieve a complete definition of the model, you should define matrices A, B, C and D. For instance:

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = [1 \quad 2] \quad D = [0]$$

Using `ss(A, B, C, D)` we get model presentation in *Command Window*.

- matrix transfer function (for SIMO systems only – Single-Input Multi-Output).

To define a transfer function you should input two vectors, containing coefficients of numerator and denominator. Coefficients are put in descending order of operator *s* powers. For instance: vectors NUM=[1 2], DEN=[1 3 2] give the transfer function shown below:

$$G(s) = \frac{s + 2}{s^2 + 3s + 2}$$

Using `printsys(NUM,DEN)` command, the transfer function is printed on the screen.

#### 2.2. Transforming model forms

- `ss2tf` and `tf2ss` functions

Syntax :

$$\begin{aligned} [\text{NUM}, \text{DEN}] &= \text{ss2tf}(\text{A}, \text{B}, \text{C}, \text{D}, \text{Ui}) && \text{– state-space to transfer function conversion} \\ [\text{A}, \text{B}, \text{C}, \text{D}] &= \text{tf2ss}(\text{NUM}, \text{DEN}) && \text{– transfer function to state-space conversion} \end{aligned}$$

A – state (system) matrix, B – input (control) matrix, C – output matrix, D – feedthrough matrix, Ui – input number (for multi-input systems only).

For more information use `help ss2tf` or `help tf2ss` in MATLAB.

### 2.3. Block diagrams modeling

Several functions that allow to receive resultant models for systems with feedback, serial and parallel connections, are shown below:

- ***feedback*** - feedback connection of the two models  
[NUM, DEN]=*feedback*(NUM1, DEN1, NUM2, DEN2, SIGN)
- ***series*** - series connection of the two models  
[NUM, DEN]=*series*(NUM1, DEN1, NUM2, DEN2)
- ***paralell*** - parallel connection of the two models  
[NUM, DEN]=*paralell*(NUM1, DEN1, NUM2, DEN2)

If SIGN = 1 then positive feedback is used. If SIGN = -1 or SIGN is omitted, negative feedback is used. For more information use HELP in MATLAB.

### 2.4. Time response determination

Unit impulse response of a linear system is calculated by ***impulse***. The impulse response is a response to a Dirac input for continuous-time systems. Syntax:

- *impulse*(A,B,C,D,Ui)
- *impulse*(NUM, DEN)
- [Y,X,T]=*impulse*(A,B,C,D,Ui)
- [Y,X,T]=*impulse*(NUM, DEN)

**step** calculates the unit step response of a linear system. Syntax:

- *step*(A,B,C,D,Ui)
- *step*(NUM, DEN)
- [Y,X,T]= *step*(A,B,C,D,Ui)
- [Y,X,T]= *step*(NUM, DEN)

### 2.5 Frequency response determination

***nyquist*** calculates the Nyquist frequency response of the model. Syntax:

- *nyquist*(A,B,C,D,Ui)
- *nyquist*(NUM, DEN)
- [re,im,w]=*nyquist*(NUM, DEN)

***bode*** computes the magnitude and phase of the frequency response. Syntax:

- *bode*(A,B,C,D,Ui)
- *bode*(NUM, DEN)
- [amplitude,phase,w]=*bode*(NUM, DEN)

### 3. Procedure of the laboratory

#### 3.1. For matrices given below, convert state-space model to a transfer function

$$\text{a) } \mathbf{A} = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathbf{C} = [0 \ 0] \quad \mathbf{D} = [0]$$

$$\text{b) } \mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{C} = [1 \ 1 \ 0] \quad \mathbf{D} = [0]$$

#### 3.2. For transfer functions given below, convert models to state-space

$$\text{a) } G(s) = \frac{4s}{2s+1}, \quad \text{b) } G(s) = \frac{2}{s^2+4s+6}, \quad \text{c) } G(s) = \frac{1}{5s}, \quad \text{d) } G(s) = 3$$

#### 3.3. Determine time and frequency responses for the following automatics elements:

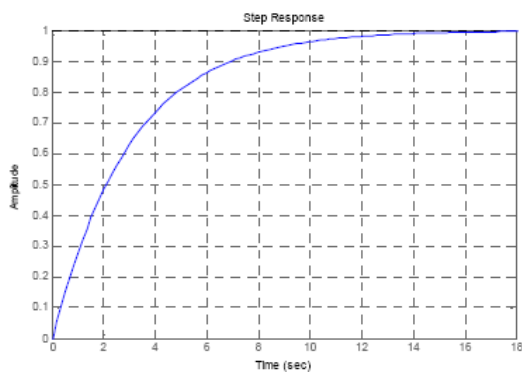
- a) proportional element:  $K=2$ ;
- b) ideal integrator / integrating element:  $K=3$ ;
- c) ideal differentiator / differentiating element:  $K=5$ ;
- d) real differentiator with 1<sup>st</sup> order inertia:  $K=0.1$ ,  $T=8$ ;
- e) 1<sup>st</sup> order inertial element:  $K=3$ ,  $T=1$ ;
- f) 2<sup>nd</sup> order inertial element:  $K=2$ ,  $T_1=2$ ,  $T_2=4$ ;
- g) 2<sup>nd</sup> order oscillatory element:  $K=1$ ,  $\omega=1$ ,  $\zeta=0.2$ ;
- h) 2<sup>nd</sup> order oscillatory element:  $K=1$ ,  $\omega=2$ ,  $\zeta=0.2$ ;

#### Example 1

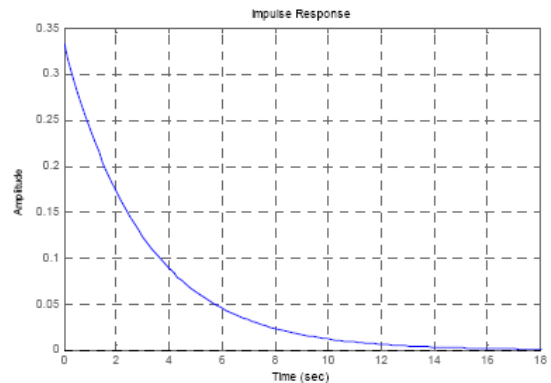
Determine time and frequency responses for 1st order inertial element:

$$G(s) = \frac{K}{Ts+1} \quad K=1, T=3$$

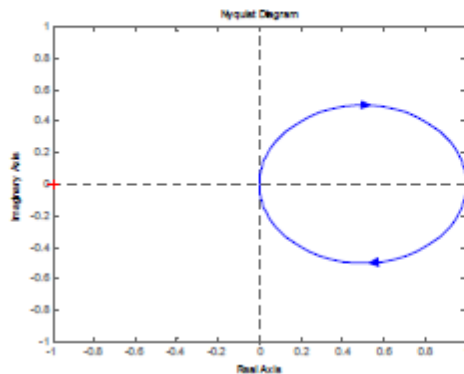
```
>> l=[1];
>> m=[3, 1];
>> step(l,m)
>> grid
```



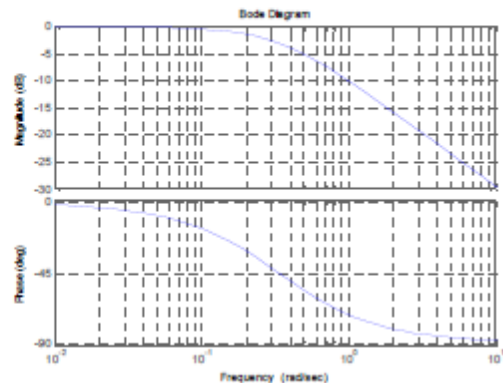
```
>> impulse(l,m)
>> grid
```



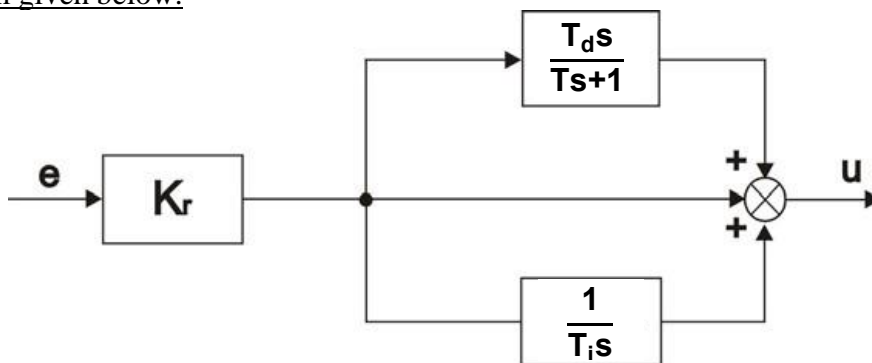
>> nyquist(l,m)



>> bode(l,m)  
>> grid

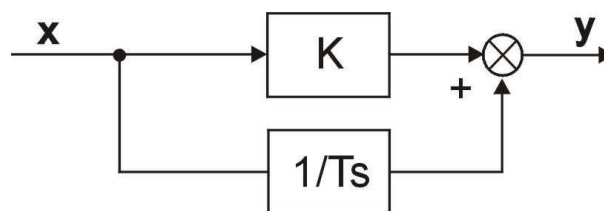


3.4. Assuming:  $K_r=1.5$  ;  $T_d=3$  ;  $T_i=2$  ;  $T=1$ , write the m-file that plots step and impulse responses, Nyquist frequency response, and Bode magnitude and phase frequency responses for a system given below.



### Example 2

Assuming:  $K=2$  ;  $T=4$ , write the m-file that plots Nyquist frequency response, and Bode magnitude and phase frequency responses for a system given below.



*%Data*

*k=1.5;T=3;*

*%Numerator and denominator of: K+1/Ts*

*[NUM, DEN]=parallel([k],[1],[1],[T 0]); w=0:0.01:200;*  
*[mod,phase,w]=bode(NUM, DEN);*

*% magnitude and phase of the frequency response*

*nyquist(NUM,DEN,w);axis([-1 5 -5 2]);grid;pause*

*% logarithmic magnitude characteristics*

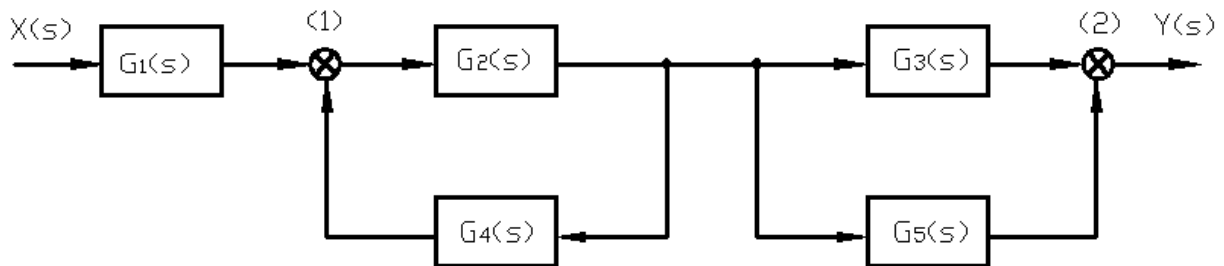
*semilogx(w,20\*log10(mod)); grid;ylabel('Lm [dB]');pause*

*% logarithmic phase characteristics*

*semilogx(w,phase); grid;ylabel('phase [degrees]');pause;*

3.5. For the system below:

- write the MATLAB m-file to calculate a resultant transfer function; present the results with *printsys*,
- draw the step response of the system using *step* MATLAB command,
- build a Simulink model of the system with *Step* input block, and compare its output with the result obtained with *step* MATLAB command,
- calculate the resultant transfer function analytically and compare it with the transfer function (a) calculated in MATLAB (enclose the solution).



PC no.	1 / 8 / 15	2 / 9	3 / 10	4 / 11	5 / 12	6 / 13	7 / 14
$G_1(s)$	$\frac{2}{s+1}$	$\frac{4}{2s+1}$	$\frac{1}{4s+1}$	$\frac{1}{3s+2}$	10	$\frac{4}{2s+1}$	$\frac{2}{s+1}$
$G_2(s)$	$\frac{4}{2s+1}$	10	$\frac{2}{s+1}$	$\frac{1}{4s+1}$	$\frac{1}{3s+2}$	$\frac{1}{4s+1}$	10
$G_3(s)$	10	$\frac{1}{4s+1}$	$\frac{1}{3s+2}$	$\frac{4}{2s+1}$	$\frac{2}{s+1}$	10	$\frac{4}{2s+1}$
$G_4(s)$	$\frac{1}{3s}$	$-\frac{2}{s}$	$\frac{4}{2s+1}$	-10	$\frac{-1}{4s+1}$	$\frac{2}{s}$	$\frac{1}{3s}$
$G_5(s)$	$\frac{1}{4s+1}$	$\frac{1}{3s+2}$	10	$\frac{2}{s+1}$	$\frac{4}{2s+1}$	$\frac{1}{3s+2}$	$\frac{1}{4s+1}$
(1)							
(2)							

3.6. Observe the influence of  $\xi$ ,  $k$ ,  $\omega_o$  on the logarithmic plots of 2nd order oscillatory element:

$$G(s) = \frac{k\omega_o^2}{s^2 + 2\xi\omega_o s + \omega_o^2}$$

Write the m-file that allows to draw logarithmic plots on the assumption that:

- $\xi = \text{var}$ ,  $k, \omega_o = \text{const}$
- $k = \text{var}$ ,  $\xi, \omega_o = \text{const}$
- $\omega_o = \text{var}$ ,  $k, \xi = \text{const}$

Use instructions like *input*, *pause*, *hold on*, ...

Task assignment:

PC no.		Observe the influence of	
7 / 14	4 / 11	$\xi$	k
2 / 9	5 / 12	k	$\omega_o$
3 / 10	6 / 13	$\xi$	$\omega_o$
1 / 8 / 15		$\xi$	k

Matching parameters:

$\xi_1 = 0.2$	$k_1 = 1.5$	$\omega_1 = 1.0$
$\xi_2 = 0.5$	$k_2 = 4.5$	$\omega_2 = 2.5$
$\xi_3 = 0.8$	$k_3 = 7.5$	$\omega_3 = 4.5$
$\xi_4 = 0.3$	$k_4 = 2.0$	$\omega_4 = 2.5$
$\xi_5 = 0.6$	$k_5 = 5.5$	$\omega_5 = 5.0$
$\xi_6 = 0.9$	$k_6 = 8.0$	$\omega_6 = 7.5$

Determine time and frequency responses and formulate comments about parameter influence.

## References:

- [1] G.F. Franklin, J.D. Powell, E. Emami-Naeini "Feedback control of dynamic systems", Prentice Hall, New York, 2006.
- [2] K. Ogata "Modern control engineering", Prentice Hall, New York, 1997.
- [3] R.H. Cannon "Dynamics of physical systems", Mc-Graw Hill, 1967 (available in Polish as: R.H. Cannon "Dynamika układów fizycznych", WNT, Warszawa, 1973).
- [4] J. Kowal "Podstawy automatyki", v.1 and 2, UWND, Kraków, 2006, 2007 (in Polish).
- [5] W. Pełczewski "Teoria sterowania", WNT, Warszawa, 1980 (in Polish).
- [6] Brzózka J., Ćwiczenia z Automatyki w MATLABIE i Simulinku, Wydawnictwo Mikon, Warszawa 1997 (in Polish).
- [7] Zalewski A., Cegięła R., MATLAB: obliczenia numeryczne i ich zastosowania, Wydawnictwo Nakom, Poznań 1996 (in Polish).
- [8] MATLAB/Simulink documentation: <http://www.mathworks.com/help/>