Laboratory 6

Stability analysis of control systems

Purpose of the exercise:

Determination of the gain and phase margins on the basis of the frequency characteristics:
- amplitude and phase logarithmic characteristics (Bode plots),
- Nyquist plots.

MATLAB environment is used.

1. Determination of the gain and phase margins on the basis of frequency logarithmic characteristics (Bode plots)

Transfer functions of the open-loop systems:

$$(a)\ G_1(s) = \frac{K}{(s+\ 1)(s^2+s+1)} \qquad\qquad (b)\ G_2(s) = \frac{K}{2s^2+2s+1}$$

where: K= 1; 2; 3; 4; 5.

For each value of K:
1.1. Determine the step response in MATLAB. While inputting the transfer function into MATLAB, instruction ***conv*** may be used for denominator polynomial multiplication,
1.2. Determine the amplitude and phase frequency logarithmic characteristics using instruction ***margin*** without left hand side operands (observe $\Delta\boldsymbol{L}$ and $\Delta\boldsymbol{\varphi}$ margin values in graph header),
1.3. Determine the gain $\Delta\boldsymbol{K}$ and phase $\Delta\boldsymbol{\varphi}$ margin values using instruction ***margin*** with left hand side operands (type in Command Window: *help margin*),
1.4. Input the determined values of $\Delta\boldsymbol{K}$ and $\Delta\boldsymbol{\varphi}$ into the Table 1,
1.5. Determine $\Delta\boldsymbol{L}$ on the basis of the formula (1) (input the results into the Table 1).

$$\Delta L = 20\ log(\Delta K) \qquad (1)$$

2. Determination of the gain and phase margins on the basis of Nyquist characteristics

For the systems (a) and (b) introduced in the point 1, for each value of K:
2.1. Determine the Nyquist characteristics (instruction ***nyquist***) using angular frequency input vector with step 0.001 [rd/s] (type in Command Window: *help nyquist*),
2.2. Determine for the each case gain $\Delta\boldsymbol{K}$ and phase $\Delta\boldsymbol{\varphi}$ margins on the basis of drawn Nyquist characteristics; for the purpose of $\Delta\boldsymbol{\varphi}$ determination draw unit circle in the same coordinate plane,
2.3. Input the determined values into the Table 1,
2.4. Calculate $\Delta\boldsymbol{L}$ on the basis of the formula (1) (input the results into the Table 1).

Table 1.

		Bode plots			Nyquist plots		
G(s)	***K***	**ΔL [dB]**	**ΔK [-]**	**Δφ [º]**	**ΔL [dB]**	**ΔK [-]**	**Δφ [º]**
G_1(s)	**1**						
G_1(s)	**2**						
G_1(s)	**3**						
G_1(s)	**4**						
G_1(s)	**5**						
G_2(s)	**1**						
G_2(s)	**2**						
G_2(s)	**3**						
G_2(s)	**4**						
G_2(s)	**5**						

Remark

All step responses should be grouped in one coordinate plane for (a) case, and one (separate) coordinate plane for (b) case; the same applies for Bode plots, and for Nyquist plots.

3. Determine gain and phase margins for the following elements:
proportional, ideal / real integral and differential elements; 1^{st}, 2^{nd} and 3^{rd} order inertial element

References:

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[4] J. Kowal "Podstawy automatyki", v.1 and 2, UWND, Kraków, 2006, 2007 (in Polish).
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[6] Brzózka J., Ćwiczenia z Automatyki w MATLABIE i Simulinku, Wydawnictwo Mikon, Warszawa 1997 (in Polish).
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[8] MATLAB/Simulink documentation: http://www.mathworks.com/help/