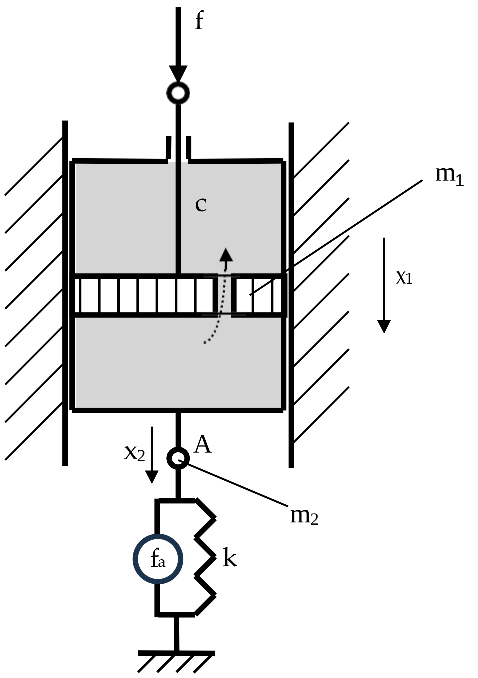
Modelling, Transfer Function, State Variables

Ex. 1.

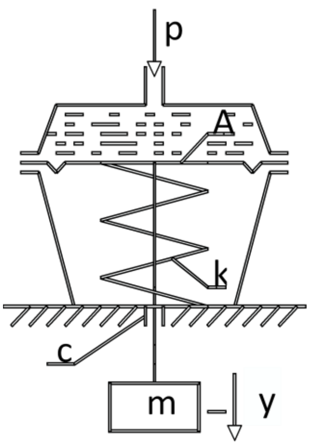
Formulate the equation of motion of the piston relative to the cylinder housing under the applied forces f and f_a, taking into account the effect of the inertia associated with the mass of the moving parts.



f, f_a – forces (inputs)
c – damping coefficient of the hydraulic damper
k – stiffness coefficient of the spring
m_1 – mass of the piston
m_2 – mass of the cylinder housing
(assumed to be concentrated in A)
x_1 – displacement of the piston (output 1)
x_2 – displacement of the cylinder housing (output 2)

Ex. 2

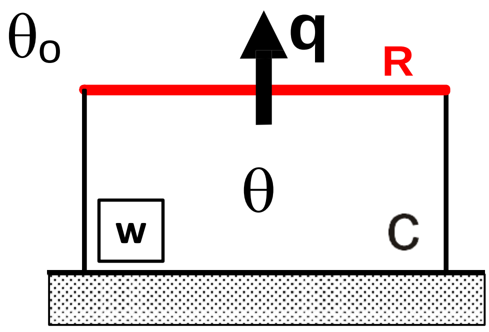
Derive the equation of motion for a pneumatic diaphragm actuator, considering the mass of the moving system and viscous friction. The input is the pressure p acting on the diaphragm, while the output is the displacement of the rod y.



A – diaphragm surface area
k – spring stiffness coefficient
c – viscous friction coefficient
m - mass of the moving system

Ex. 3.

Determine the model of the following system in the form of a transfer function as well as state-space equations. Assume a zero initial condition.



w – input (control) signal:
heater/air conditioner power

θ_o – input (disturbance) signal:
outside temperature

θ – output signal:
average indoor air temperature

R – thermal resistance of the ceiling/roof

C – thermal capacity of the building

Ex. 4.

Determine the model of a single loop of an RLC electrical circuit in the form of a transfer function and the state-space equation. Assume the external electromotive force e(t) as the input and the voltage drop across the capacitor u(t) as the output. Adopt zero initial conditions.

Ex. 5.

Determine the model (in the form of a transfer function and the state-space equation) of a mechanical oscillatory system with a single degree of freedom, consisting of a rigid body, a spring, and a viscous damper. Assume the external concentrated force f(t) as the input and the displacement of the body z(t) as the output. Adopt zero initial conditions.

Ex. 6.

Describe the system given by the transfer function:

$$G(s) = \frac{K}{s(Ts+1)}$$

using the state and output equations.