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FEM cd..

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- FEM theory
  - What is the base of FEM calculations
  - The procedure of FEM calculation

What is FEM?

- Finite Elements Method (FEM) is method of approximation (finding approximate solution) of the partial different equations
- Partial different equations describe a mathematical model of system or phenomena, which can be written as some function of the space.
- Partial differential equation is an equation which contains partial derivatives, such as the (wave) equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

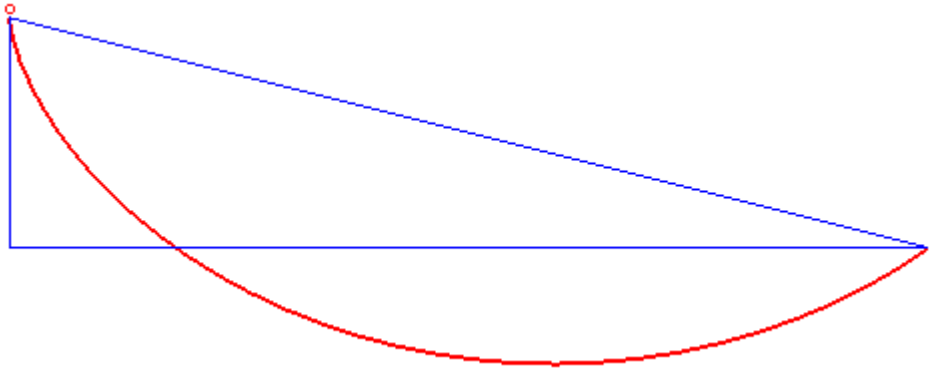
- Finding solution of such cases is a field of calculus of variations
- Typical (and historically first) example is finding a curve for material point to move under the gravity force, from one place to another, lower.
- This kind of curve is named brachistochrone

Calculus of variations

- The task is to find such a function for which the integral will have a minimum value
- In other words the minimum of the functionals, where functionals maps a functions domain to the number domain eg.:

$$I(f)=\int_a^b f(x)\,dx$$

Brachistochrone



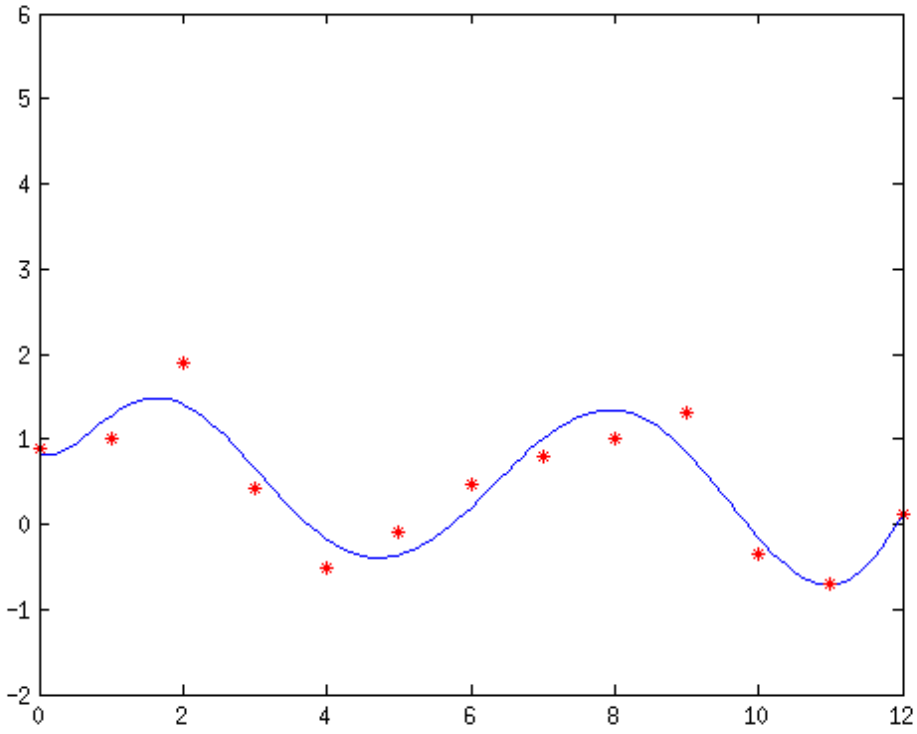
- Since, for the real objects finding a function that meets the stated assumption is generally not possible, modeled object is divided into small elements
- For each element we are looking for function close to the exact solution with sufficient accuracy. Shape of the functions for each element should be as simple as possible.

Example

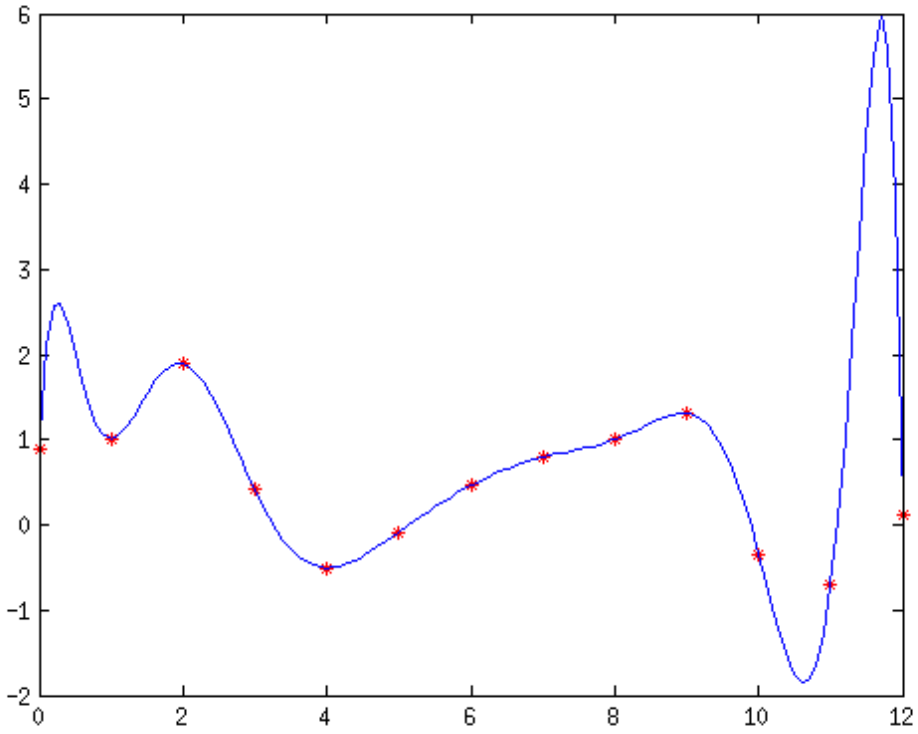
Approximation of measurement data using polynomials.

- We have the measured data of the relationship of two physical quantities, we are looking for the function describing this relationship.
- Using of polynomials of higher orders, we get better and better adjustment to the points, but between the points, we get fluctuations, having no justification in the nature of the studied phenomenon.

Approximation



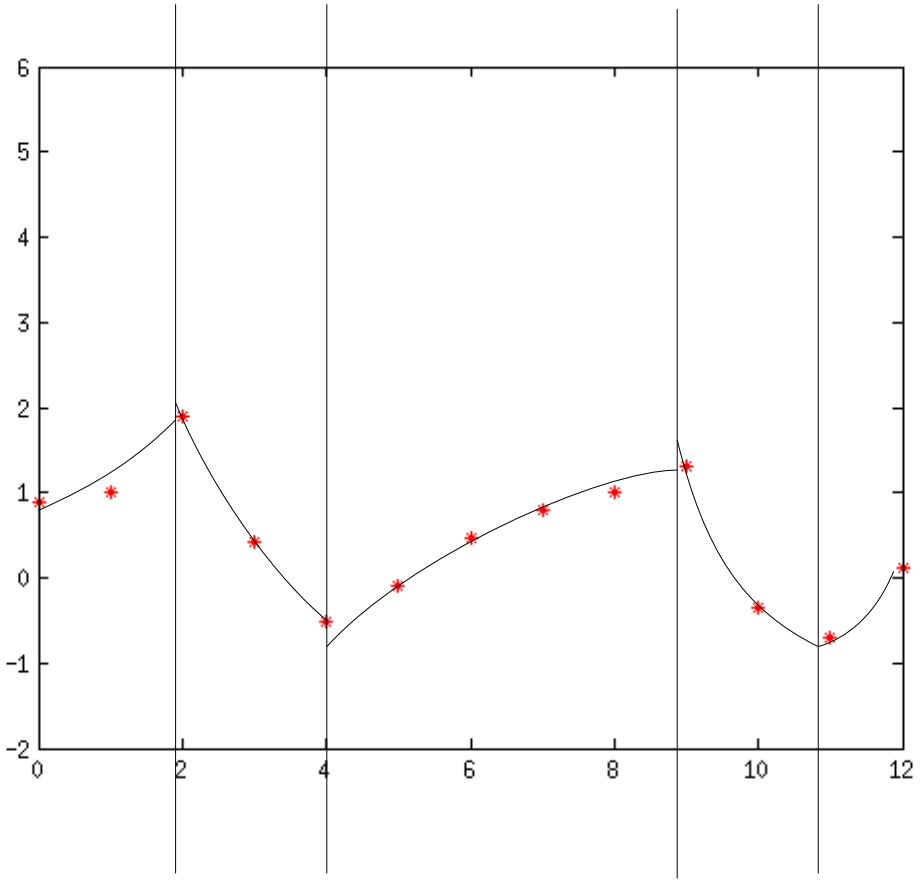
Higher polynomial approximation



Partition

- Better results we get by divide the data into smaller compartments and matching functions in these compartments.

Partition



- There is a problem of continuity when moving from compartment to compartment.
- It is therefore necessary to impose additional conditions such that the resultant approximation was continuous
- If we decide that (in every compartment) approximation functions are the same, we get a fixed number of parameters to find, in each compartment

The FEM problem formulation

- The integral equation related to the partial differential equation
- What kind of functions to use to construct the solution

All of this is usually fixed in the program that we use

Procedure ...

- Division of the area into sub-areas (elements). Elements share common nodes.
- In the elements, simple functions are defined called shape functions (these functions are clearly defining the distribution of the analyzed physical quantity inside the finite elements, dependent on the value of the physical quantities at the nodes)

Procedure ...

- The shape functions specified in the individual elements are used to construct functions defined in the whole area calculation (which is the sum of the parts). This process can be defined as the gluing of many functions on a small sub-areas as a single function in the entire area. The functions specified in the whole area are called basis functions

- Then it defines how the final solution is to be derived from base functions.
- The principle here is simple: approximate solution FEM is a linear combination of basis functions (e.g.: the sum of the coefficients different for each function). The set of all possible linear combinations of basis functions is the set of all possible approximate solutions of the problem.

- Coefficients of the linear combination for the approximated functions form a set of numbers.
- Knowing it, we are able to get approximate solution at any point in the area of calculation
- Often, by appropriate definitions of shape functions and base functions, a discrete set of numbers representing the FEM solution is a set of values in selected points (in nodes).

- The use of the previously mentioned components of the FEM problem as:
  - integral equation
  - definition of approximating functions

  Leads to the transformation of the integral equation into a linear system of equations.

- Solution to the problem using FEM boils down to solve the great system of linear equations.
- The number of unknowns is large, but quick ways to solutions are known.

  (and this is made by calculation program)

Our task in calculation program:

- Definition of the contours
- Definition of materials
- Definition of border conditions
- Solution
- Use and interpret of the solution

Additional information

Type of elements

- In the one-dimensional problems it is a line segment *(one-dimensional problems also exists !)*
- In the two-dimensional problems:
  - triangle with three nodes
  - rectangle
  - triangle with six nodes
  - quadrangle

Triangle with three nodes

- For the analysis of electromagnetic field triangular element with three nodes are very often used.
- It is good for representing areas of any shape, it is simple in constructions.

Other types of elements

- Axisymmetric elements are in the fact three-dimensional elements, but they are described by a one or two variables .
- These elements are used for cases of axial symmetry and cylindrical coordinates.

Degrees of freedom

- The element type is characterized (beside the shape) by the number of nodes and the number and type of variables in the nodes.

- Degrees of freedom (of the element) - variables in the nodes or parameters associated with the element.

Exercise:

- Try to model given object.
- Calculate change of the flux linked with the winding, respect to its place on the pole.
- Try to estimate how long it takes that the magnet drops down.
- Basing on this, try to calculate dfi/dt which is equal to the electromotive force.
  - (the winding consists of the 300 turns)

Some short FEM tutorials:

- http://www.idi.ntnu.no/~elster/tdt24/tdt24-f09/gagan-fem.pdf
- https://github.com/FluidityProject/training/raw/master/IntroToFEM/IntroToFEMJRP.pdf