WZORY NA POCHODNE

$(c)' = 0$

$(x^n)' = nx^{n-1}$

$(x)' = 1$

$\left(\frac{a}{x}\right)' = -\frac{a}{x^2}$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$(a^x)' = a^x \ln a$

$(e^x)' = e^x$

$(\log_a x)' = \frac{1}{x \ln a}$

$(\ln x)' = \frac{1}{x}$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(ctg\, x)' = \frac{-1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$

$(arctg\, x)' = \frac{1}{x^2+1}$

$(arcctg\, x)' = \frac{-1}{x^2+1}$

WZORY NA CAŁKI

$\int dx = x + C$

$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \qquad n \neq 1$

$\int x dx = \frac{1}{2}x^2 + C$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$

$\int e^x dx = e^x + C$

$\int \sin x\, dx = -\cos x + C$

$\int \cos x\, dx = \sin x + C$

$\int tg\, x\, dx = -\ln|\cos x| + C$

$\int ctg\, x\, dx = -\ln|\sin x| + C$

$\int \frac{dx}{cos^2 x} = tg\ x + C$

$\left[\int \frac{dx}{sin^2 x} = -ctg\, x + C\right]$

$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} arctg \frac{x}{a} + C$

$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = arcsin \frac{x}{a} + C$

$\int \frac{dx}{\sqrt{x^2+q}} = \ln\left|x + \sqrt{x^2+q}\right| + C$

ASYMPTOTY:

PIONOWA:

$\lim_{x\to a^\pm} f(x) = \pm\infty$

UKOŚNA:

$\lim_{x\to\pm\infty} \frac{f(x)}{x} = a$

$\lim_{x\to\pm\infty} (f(x) - ax) = b$

CIĄGI:

ARYTMETYCZNY - SUMA:

$S_n = \frac{a_1 + a_n}{2} \cdot n$

GEOMETRYCZNY - SUMA:

$S_n = a_1 \frac{1-q^n}{1-q}$

GRANICE:

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$

$\lim_{x\to 0} \frac{\arcsin x}{x} = 1$

$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x} = 1$

$\lim_{x\to\infty} \left(1 + \frac{z}{x}\right)^x = e^z$!

$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$

$\lim_{x\to 0} \frac{\log_a(1+x)}{x} = \log_a e$

TRYGONMETRIA:

$\sin 2\alpha = 2 \cdot \sin\alpha \cdot \cos\alpha$

$\sin\alpha + \sin\beta = 2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2 \cdot \sin\frac{\alpha-\beta}{2} \cdot \cos\frac{\alpha+\beta}{2}$

$\cos\alpha + \cos\beta = 2 \cdot \cos\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}$

$\sin\alpha \cdot \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\sin\alpha \cdot \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha \cdot \cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$A\sin x + B\cos x = \sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\sin x + \frac{B}{\sqrt{A^2+B^2}}\cos x\right)$

SZEREG TAYLORA W PUNKCIE a

$$f(x) = f(a) + \frac{1}{1!} \cdot f'(a)(x-a) + \frac{1}{2!} \cdot f''(a)(x-a)^2 + \cdots + \frac{1}{n!} \cdot f^{(n)}(a)(x-a)^n + \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

CAŁKOWANIE PRZEZ CZĘŚCI

$\int f(x) \cdot g'(x)\, dx = f(x)\, g(x) - \int f'(x) \cdot g(x)\, dx$

METODA WSPÓŁCZYNNKÓW NIEOZNACZONYCH

$\int \frac{W_n(x)}{\sqrt{ax^2+bx+c}} dx = P_{n-1}(x)\sqrt{ax^2+bx+c} + \lambda \int \frac{dx}{\sqrt{ax^2+bx+c}}$

REGUŁA DE L'HOSPITALA

$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

SYMBOLE NIEOZNACZONE

$\frac{0}{0}, \frac{\pm\infty}{\pm\infty}, \infty - \infty, 0 \cdot \infty, 0^0, 1^\infty, \infty^0$

WZORY SKRÓCONEGO MNOŻENIA

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

WŁASNOŚCI POCHODNEJ

$[f(x) + g(x)]' = f'(x) + g'(x)$

$[f(x) - g(x)]' = f'(x) - g'(x)$

$[a \cdot f(x)]' = a \cdot f'(x)$

$[f(x) \cdot g(x)]' = f'(x)g(x) + f(x)g'(x)$

$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

DEFINICJA POCHODNEJ

$f'(x_0) = \lim_{\Delta x\to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$

FUNKCJA KWADRATOWA - POSTAĆ KANONICZNA:

$ax^2 + bx + c = a\left(\left(x + \frac{b}{2a}\right)^2 - \left[\frac{\Delta}{4a}\right]\right)$, $x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a}$