NOTE ON STRONG EDGE-COLOURING

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A proper edge-colouring of a graph $G = (V, E)$ is an assignment of colours to the edges of the graph such that two adjacent edges do not use the same colour. A *strong edge-colouring* of a graph G is a proper edge-colouring of G, such that for every path $uvxy$ of length 3, we have $c(uv) \neq c(xy)$. We denote by $\chi'_s(G)$ the *strong chromatic index* of G which is the smallest integer k such that G can be strong edge-coloured with k colours.

The notion of strong edge-colouring was introduced by Fouquet and Jolivet in 1983 [2, 3].

Let Δ denote the maximum degree of a graph. In 1985, during a seminar in Prague, Erdős and Nešetřil gave a construction of graphs having strong chromatic index equal to $\frac{5}{4}\Delta^2$ when Δ is even and $\frac{1}{4}(5\Delta^2 - 2\Delta + 1)$ when Δ is odd. They conjectured that the strong chromatic index is bounded by these values and it was verified for $\Delta \leqslant 3$ [1, 4].

In this talk, we will present results on strong edge-colouring in terms of planarity, maximum average degree, maximum degree, complexity...We will give the sketch of some results using discharging method.

Literatura

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