Universal planar graphs

Florian Lehner

Graz University of Technology

Let $\mathcal{G}$ be a class of graphs. Call a graph $G \in \mathcal{G}$ subgraph-universal, minor-universal, or topological-minor-universal if it contains every graph in $\mathcal{G}$ as a subgraph, minor, or topological minor, respectively.

Answering a question posed by Ulam, Pach proved in 1981 that there is no subgraph-universal planar graph. In contrast to this result, Diestel and Kühn showed that a minor-universal planar graph exists and asked whether a universal planar graph exists for the topological minor relation.

We answer their question in two different ways: we show that the class of planar graphs does not contain a topological-minor-universal graph, but the class locally finite planar graphs does.