SEMINARIUM
MATEMATYKA DYSKRETNA

wtorek, 29 października 2024 r., godz. 12:30, s. 612 C7

On the distinguishing chromatic number in claw-free graphs

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Let G be a graph and $c : V(G) \to C$ be a proper vertex colouring. An automorphism with respect to G and c is a bijective mapping $\varphi : V(G) \to V(G)$ such that $c(v) = c(\varphi(v))$ for each $v \in V(G)$ and $vw \in E(G)$ if and only if $\varphi(v)\varphi(w) \in E(G)$ for each $v, w \in V(G)$. The set of automorphisms with respect to G and c is denoted by $\mathrm{Aut}(G, c)$. A vertex of G is fixed if it is a fixed point of every automorphism of $\mathrm{Aut}(G, c)$. Furthermore, c is distinguishing if it fixes every vertex of G.

It is known since 2006 (Collins, Trenk), that the general upper bound of a minimal number of colours in the distinguishing colouring, called the chromatic distingushing number of a graph and denoted by $\chi_D(G)$ is $2\Delta(G)$. And it is achieved by the cycle C_6 and by bipartite balanced complete graphs $K_{p,p}$.

For several classes of graphs it was shown that this upper bound could be reduced to $\Delta(G) + \mathrm{const}$. Last result due to Cranston, that it is enough $\Delta(G) + 1$ colours for any graph with girth at least 5 different from C_6.

We consider graphs without induced $K_{1,3}$, called claw-free graphs. And, we prove, that if G is a connected claw-free graph of order n, then $\chi_D(G) \leqslant \Delta(G) + 2$ with equality if and only if $G = C_6$ or $G = K_{n/2}[2K1]$.

It is joint work with Christoph Brause, Rafal Kalinowski and Ingo Schiermeyer.