$$\begin{cases} \hat{e}_r(t) = \hat{n}_x \cos\varphi(t) + \hat{n}_y \sin\varphi(t) \\ \hat{e}_\varphi(t) = -\hat{n}_x \sin\varphi(t) + \hat{n}_y \cos\varphi(t) \end{cases} \qquad \varphi = \varphi(t)$$

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$$\vec{v} = \frac{d\vec{r}}{dt} = \left\{ \vec{r} = r(t)\,\hat{e}_r(t) \right\} = \frac{dr}{dt}\cdot\hat{e}_r + r\cdot\underbrace{\frac{d\hat{e}_r}{dt}}_{=?}$$

$$\frac{d\hat{e}_r}{dt} = \hat{n}_x\cdot(-\sin\varphi)\cdot\frac{d\varphi}{dt} + \hat{n}_y\cos\varphi\cdot\frac{d\varphi}{dt} = \frac{d\varphi}{dt}\underbrace{\left(-n_x\sin\varphi + n_y\cos\varphi\right)}_{\hat{e}_\varphi}$$

$$\vec{v} = \underbrace{\frac{dr}{dt}}_{v_r}\hat{e}_r + \underbrace{r\cdot\frac{d\varphi}{dt}}_{v_\varphi}\hat{e}_\varphi$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2r}{dt^2}\hat{e}_r + \frac{dr}{dt}\cdot\overbrace{\frac{d\hat{e}_r}{dt}}^{\frac{d\varphi}{dt}\hat{e}_\varphi} + \frac{dr}{dt}\cdot\frac{d\varphi}{dt}\hat{e}_\varphi + r\frac{d^2\varphi}{dt^2}\hat{e}_\varphi + r\frac{d\varphi}{dt}\frac{d\hat{e}_\varphi}{dt}$$

$$\frac{d\hat{e}_\varphi}{dt} = -n_x\cos\varphi\cdot\frac{d\varphi}{dt} + \hat{n}_y(-\sin\varphi)\cdot\frac{d\varphi}{dt} = -\frac{d\varphi}{dt}\underbrace{\left(n_x\cos\varphi + n_y\sin\varphi\right)}_{e_r}$$

$$\vec{a} = \frac{d^2r}{dt^2}\hat{e}_r + \frac{dr}{dt}\frac{d\varphi}{dt}\hat{e}_\varphi + \frac{dr}{dt}\frac{d\varphi}{dt}\hat{e}_\varphi + r\frac{d^2\varphi}{dt^2}\hat{e}_\varphi + r\frac{d\varphi}{dt}\cdot\left(-\frac{d\varphi}{dt}\hat{e}_r\right)$$

$$= \hat{e}_r\underbrace{\left(\frac{d^2r}{dt^2} - r\left(\frac{d\varphi}{dt}\right)^2\right)}_{a_r} + \hat{e}_\varphi\underbrace{\left(r\frac{d^2\varphi}{dt^2} + 2\frac{dr}{dt}\frac{d\varphi}{dt}\right)}_{a_\varphi}$$